Dynamics in n-dimensional Gauss-Bonnet gravity

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論点

＊4dim, 5dim, 6dim, … ダイナミクスはどう変化するか

＊Gauss-Bonnet項は，ダイナミクスにどう影響するか

Part I　Field Equations (dual-null formulation)

Part II　Colliding Scalar Waves

Part III　Wormhole dynamics

Dynamics in Gauss-Bonnet gravity?

- Action

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\} + \mathcal{L}_{\rm matter}\Big]$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu}$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- **has GR correction terms from String Theory**
- **has two solution branches (GR/non-GR).**
- **is expected to have singularity avoidance feature.**
  (but has never been demonstrated.)

- **new topic in numerical relativity.**
  S Golod & T Piran, PRD 85 (2012) 104015
  N Deppe+, PRD 86 (2012) 104011
  F Izaurieta & E Rodriguez, 1207.1496

- **much attentions in WH community**
  H Maeda & M Nozawa, PRD 78 (2008) 024005
  P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101
  P Kanti, B Kleihaus & J Kunz, PRD 85 (2012) 044007

Formulation for evolution [N+1]

PHYSICAL REVIEW D **78,** 084037 (2008)

$N+1$ formalism in Einstein-Gauss-Bonnet gravity

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Towards the investigation of the full dynamics in a higher-dimensional and/or a stringy gravitational model, we present the basic equations of the Einstein-Gauss-Bonnet gravity theory. We show the $(N+1)$-dimensional version of the Arnowitt-Deser-Misner decomposition including Gauss-Bonnet terms, which shall be the standard approach to treat the space-time as a Cauchy problem. Because of the quasilinear property of the Gauss-Bonnet gravity, we find that the evolution equations can be in a treatable form in numerics. We also show the conformally transformed constraint equations for constructing the initial data. We discuss how the constraints can be simplified by tuning the powers of conformal factors. Our equations can be used both for timelike and spacelike foliations.

- **Initial Value Construction via Conformal approach**

  Black hole initial data: H Yoshino , PRD 83 (2011) 104010

- **Set of Equations**

  ready, but complicated

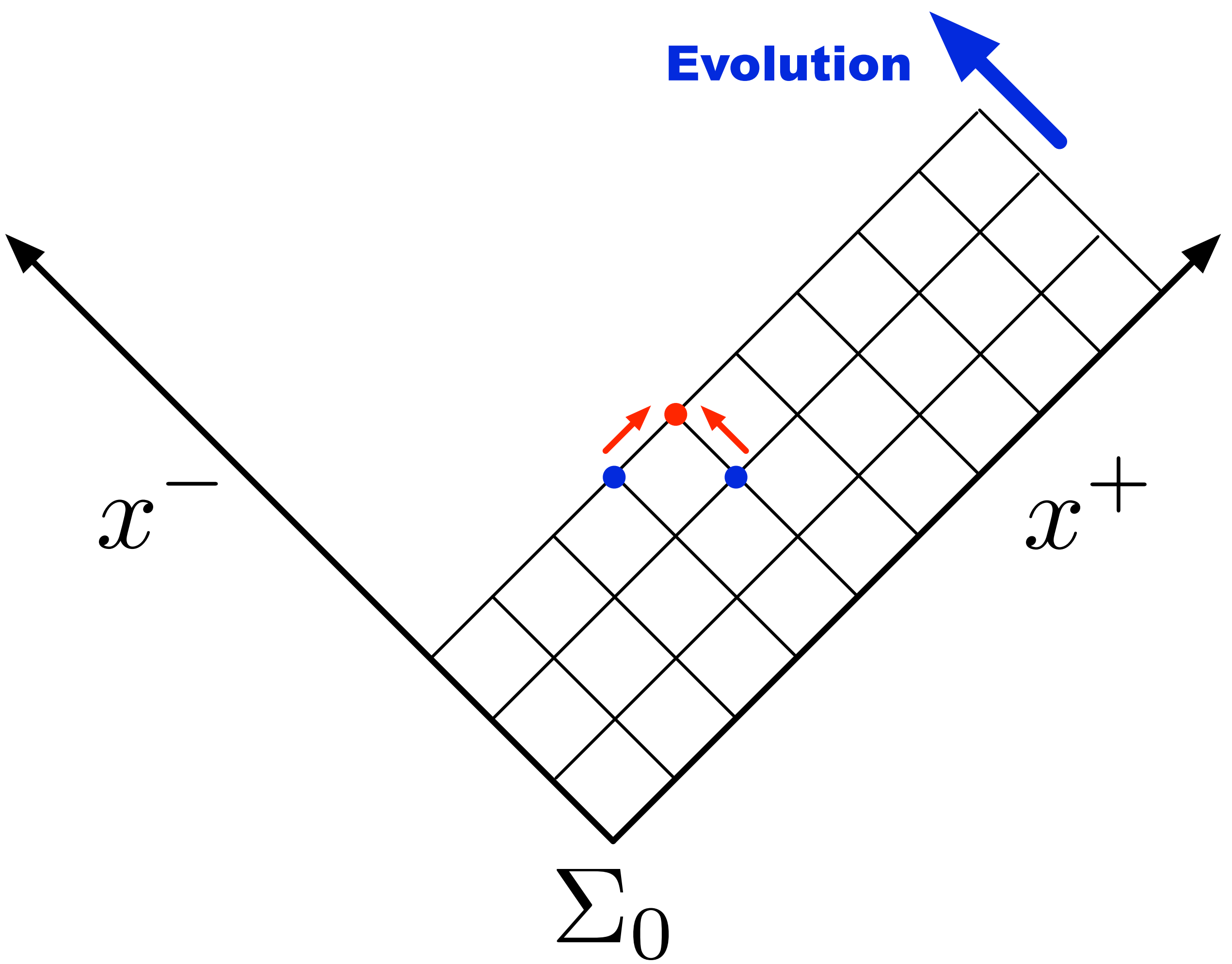
Formulation for evolution [dual null]

Metric n-dimensional, dual-null coordinate, $2+(n-2)$ decomposition

$$ds^2 = -2e^{-f(x^+,x^-)}\,dx^+\,dx^- + r^2(x^+,x^-)\gamma_{ij}dx^i dx^j \tag{1}$$

Variables

$\Omega = \frac{1}{r}$	Conformal factor
$\vartheta_\pm = (n-2)\partial_\pm r$	expansion
f	lapse function
$\nu_\pm = \partial_\pm f$	inaffinity (shift)



ψ	scalar field (normal)
$\pi_\pm = r\partial_\pm\psi$	scalar momentum
ϕ	scalar field (ghost)
$p_\pm = r\partial_\pm\phi$	scalar momentum

Formulation for evolution [dual null]

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Parameters

n	dimension
k	curvature
Λ	cosmological constant

For simplicity, we define

$$\tilde{\alpha} = (n-3)(n-4)\alpha_2, \tag{2}$$

$$A = \alpha_1 + 2\tilde{\alpha}\Omega^2 Z, \tag{3}$$

$$W = \frac{2e^f}{(n-2)^2}\vartheta_+\vartheta_-, \tag{4}$$

$$Z = k + W, \tag{5}$$

$$\eta = \Omega^2\frac{(n-2)(n-3)}{2}e^{-f}Z, \tag{6}$$

matter variables

normal field $\psi(u,v)$ and/or ghost field $\phi(u,v)$

$$\begin{aligned} T_{\mu\nu} &= T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi) \\ &= \left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] + \left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right] \end{aligned}$$

this derives Klein-Gordon equations

$$\Box\psi = \frac{dV_1}{d\psi}, \qquad \Box\phi = \frac{dV_2}{d\phi}.$$

Scalar field variables

$$\begin{aligned} \pi_\pm &\equiv r\partial_\pm\psi = \frac{1}{\Omega}\partial_\pm\psi \\ p_\pm &\equiv r\partial_\pm\phi = \frac{1}{\Omega}\partial_\pm\phi \end{aligned}$$

Klein-Gordon eqs.

$$\begin{aligned} \Box\phi &= -\frac{e^f}{r}\left(2r\phi_{uv} + (n-2)r_u\phi_v + (n-2)r_v\phi_u\right) \\ &= -2e^f\phi_{uv} - e^f\Omega^2\left(\vartheta_- p_+ + \vartheta_+ p_-\right) \end{aligned}$$

Energy-momentum tensor

$$\begin{aligned} T_{++} &= \Omega^2(\pi_+^2 - p_+^2) \\ T_{--} &= \Omega^2(\pi_-^2 - p_-^2) \\ T_{+-} &= -e^{-f}\left(V_1(\psi) + V_2(\phi)\right) \\ T_{zz} &= e^f(\pi_+\pi_- - p_+p_-) - \frac{1}{\Omega^2}\left(V_1(\psi) - V_2(\phi)\right) \end{aligned}$$

evolution equations (1)

Equations for x^+ direction

$$
\begin{aligned}
\partial_+\Omega &= -\frac{1}{n-2}\vartheta_+\Omega^2 && (7)\\
\partial_+\vartheta_+ &= -\vartheta_+\nu_+ - \frac{1}{\Omega A}\kappa^2 T_{++} = -\vartheta_+\nu_+ - \frac{1}{A}\kappa^2\Omega(\pi_+^2 - p_+^2) && (8)\\
\partial_+\vartheta_- &= \frac{1}{A}\frac{e^{-f}}{\Omega}\left[-\alpha_1\Omega^2\frac{(n-2)(n-3)}{2}Z + \Lambda + \kappa^2(V_1+V_2)\right] - \frac{\tilde{\alpha}}{A}\Omega^3 e^{-f}\frac{(n-2)(n-5)}{2}\left[Z^2+W\right] && (9)\\
\partial_+ f &= \nu_+ && (10)\\
\partial_+\nu_+ &= \text{no evolution eq. exists}\\
\partial_+\nu_- &= \frac{\alpha_1}{A}Ze^{-f}\Omega^2\frac{(n-3)}{2}\left\{-\frac{\alpha_1}{A}2(n-3)+n-4\right\}\\
&\quad +\frac{1}{A}\Omega^2 e^{-f}\kappa^2(\pi_+\pi_- - p_+p_-) + \frac{1}{A}e^{-f}\left\{\frac{\alpha_1}{A}\frac{2(n-3)}{(n-2)} - 1\right\}\{\Lambda + \kappa^2(V_1+V_2)\}\\
&\quad -\frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\left[\frac{\alpha_1}{A}\Omega^2(n-3)\left\{k^2+2WZ+2Z^2\right\} + \frac{\tilde{\alpha}}{A}\Omega^4 2(n-5)\left\{k^2+2WZ\right\}Z\right]\\
&\quad +\frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\left[\frac{1}{2}\Omega^2\left\{(n-2)k^2+2WZ-4Z^2\right\} + \frac{1}{A}\frac{4}{n-2}Z\left\{\Lambda+\kappa^2(V_1+V_2)\right\}\right]\\
&\quad -\frac{\tilde{\alpha}}{A}e^{f}\Omega^2\frac{4}{(n-2)^2}\left\{\nu_+\vartheta_+(\partial_-\vartheta_-) + \nu_-\vartheta_-(\partial_+\vartheta_+) + (\partial_+\vartheta_+)(\partial_-\vartheta_-) + \nu_+\nu_-\vartheta_+\vartheta_- - (\partial_-\vartheta_+)^2\right\} && (11)\\
\partial_+\psi &= \Omega\pi_+ && (12)\\
\partial_+\phi &= \Omega p_+ && (13)\\
\partial_+\pi_+ &= \text{no evolution eq. exists}\\
\partial_+\pi_- &= \left(\frac{1}{n-2}-\frac{1}{2}\right)\Omega\vartheta_+\pi_- - \frac{1}{2}\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} && (14)\\
\partial_+ p_+ &= \text{no evolution eq. exists}\\
\partial_+ p_- &= \left(\frac{1}{n-2}-\frac{1}{2}\right)\Omega\vartheta_+ p_- - \frac{1}{2}\Omega\vartheta_- p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} && (15)
\end{aligned}
$$

evolution equations (2)

Equations for x^- direction

$$
\begin{aligned}
\partial_-\Omega &= -\frac{1}{n-2}\vartheta_-\Omega^2 && (16)\\
\partial_-\vartheta_+ &= (9) && (17)\\
\partial_-\vartheta_- &= -\vartheta_-\nu_- - \frac{1}{\Omega A}\kappa^2 T_{--} = -\vartheta_-\nu_- - \frac{1}{A}\Omega\kappa^2(\pi_-^2 - p_-^2) && (18)\\
\partial_- f &= \nu_- && (19)\\
\partial_-\nu_+ &= (11) && (20)\\
\partial_-\nu_- &= \text{no evolution eq. exists} &&\\
\partial_-\psi &= \Omega\pi_- && (21)\\
\partial_-\phi &= \Omega p_- && (22)\\
\partial_-\pi_+ &= -\frac{1}{2}\Omega\vartheta_+\pi_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} && (23)\\
\partial_-\pi_- &= \text{no evolution eq. exists} &&\\
\partial_- p_+ &= -\frac{1}{2}\Omega\vartheta_+ p_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega\vartheta_- p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} && (24)\\
\partial_- p_- &= \text{no evolution eq. exists} &&
\end{aligned}
$$

This constitutes the first-order dual-null form, suitable for numerical coding.

GR 5d: small amplitude waves

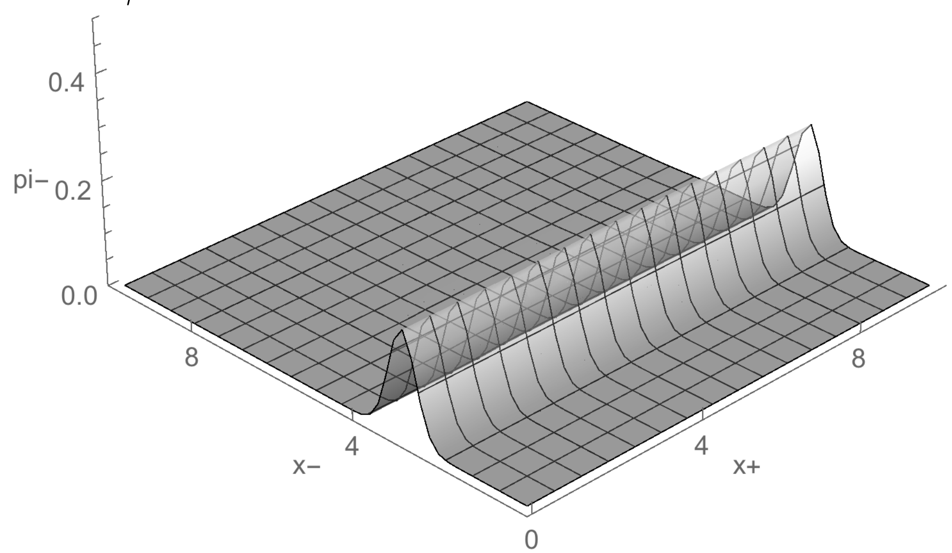
flat background, normal scalar field

Initial data:

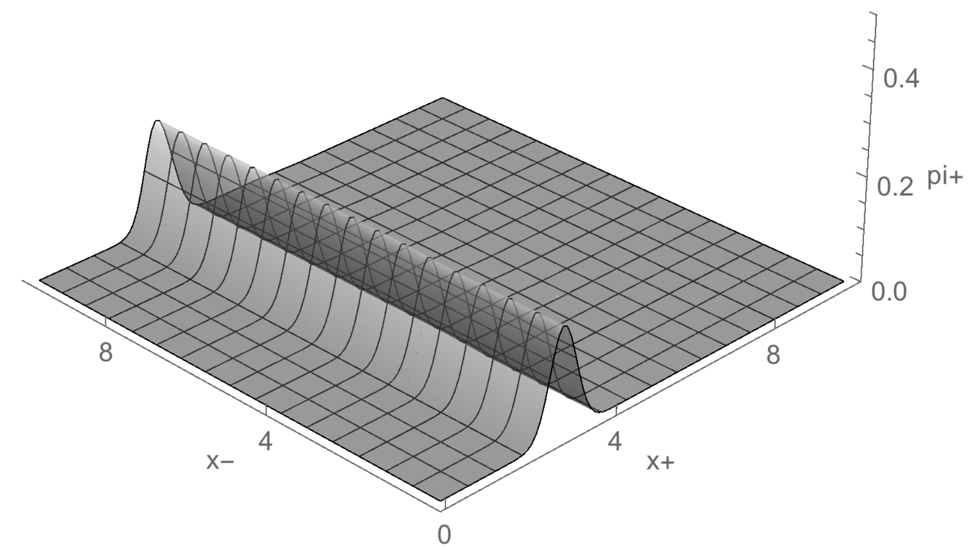
$\psi = 0,\ \pi_+ = a\exp(-b(z-c)^2)$ on $x_- = 0$ surface, where $z = x^+/\sqrt{2}$

$\psi = 0,\ \pi_- = a\exp(-b(z-c)^2)$ on $x_+ = 0$ surface, where $z = x^-/\sqrt{2}$

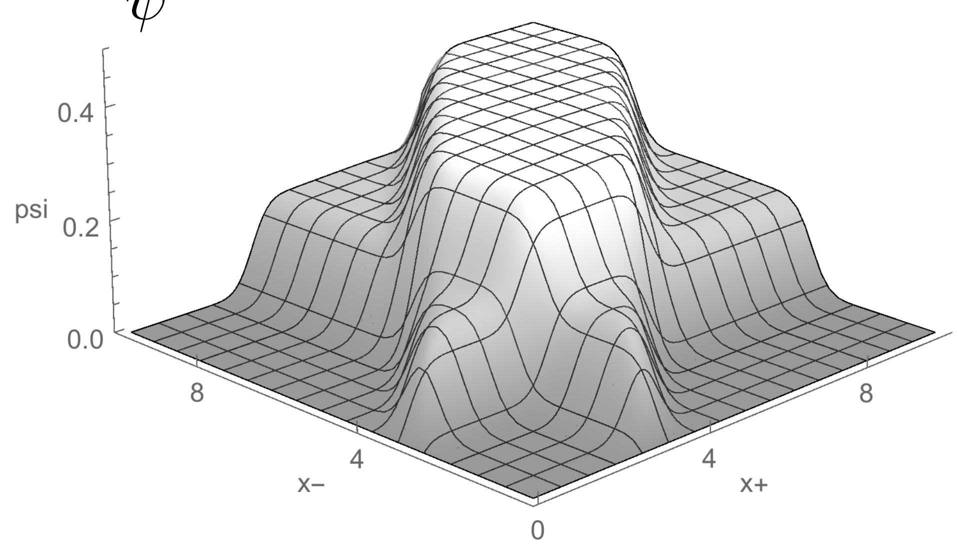
$\partial_-\psi$



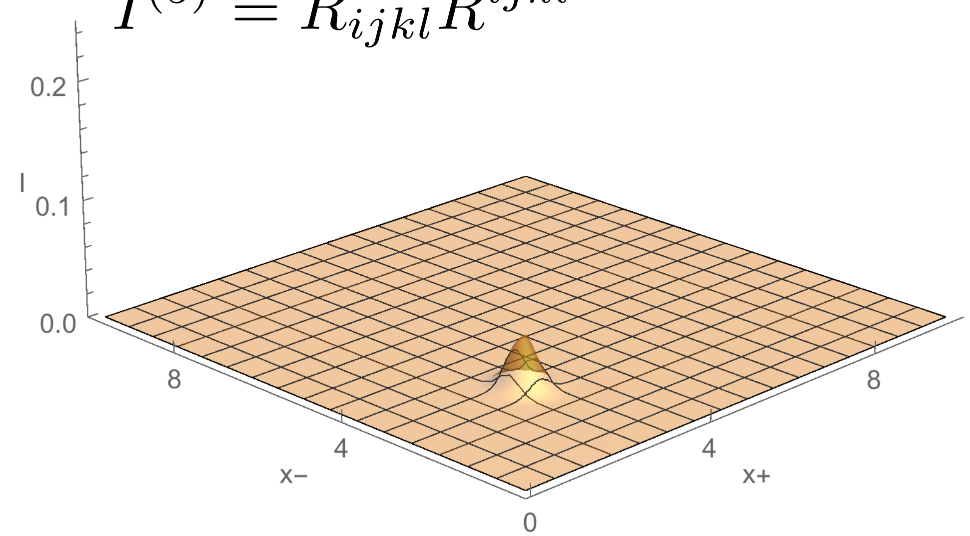
$\partial_+\psi$



ψ

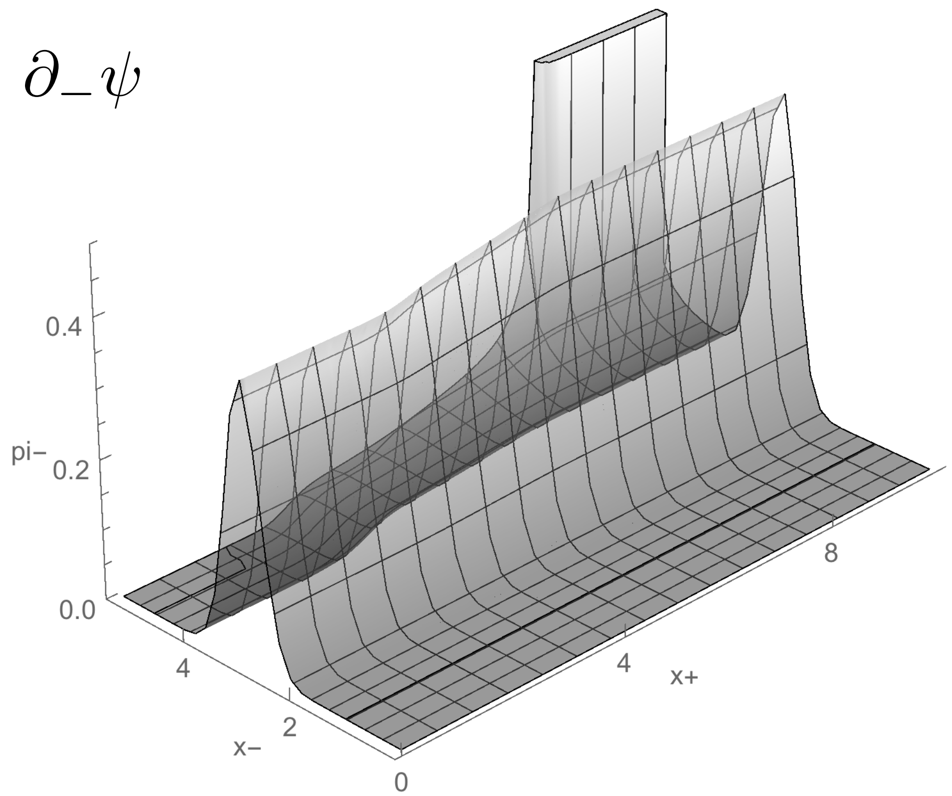


$I^{(5)} = R_{ijkl}R^{ijkl}$

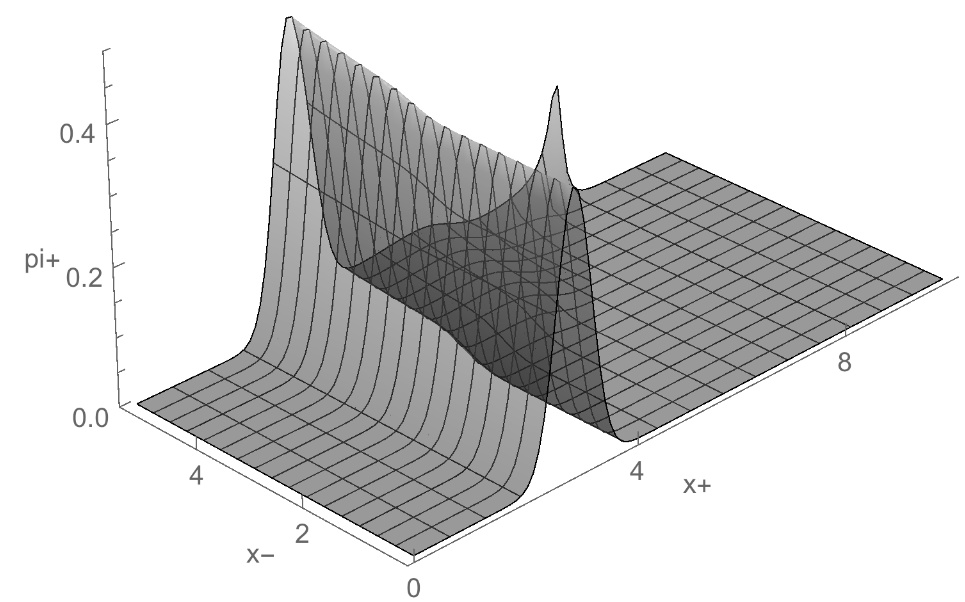


GR 5d: large amplitude waves

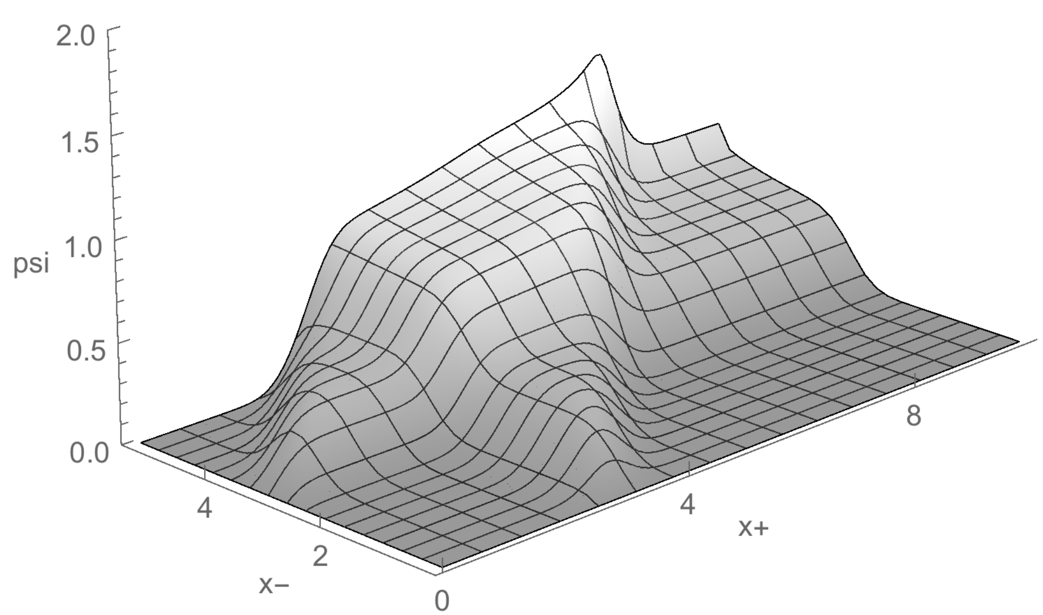
$\partial_- \psi$



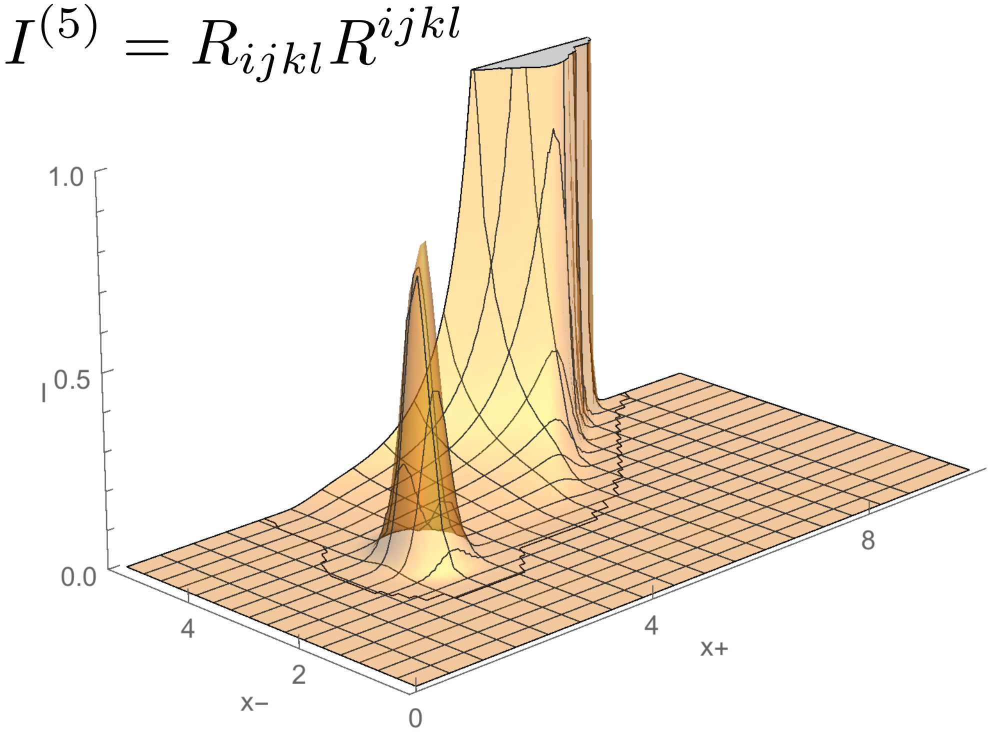
$\partial_+ \psi$

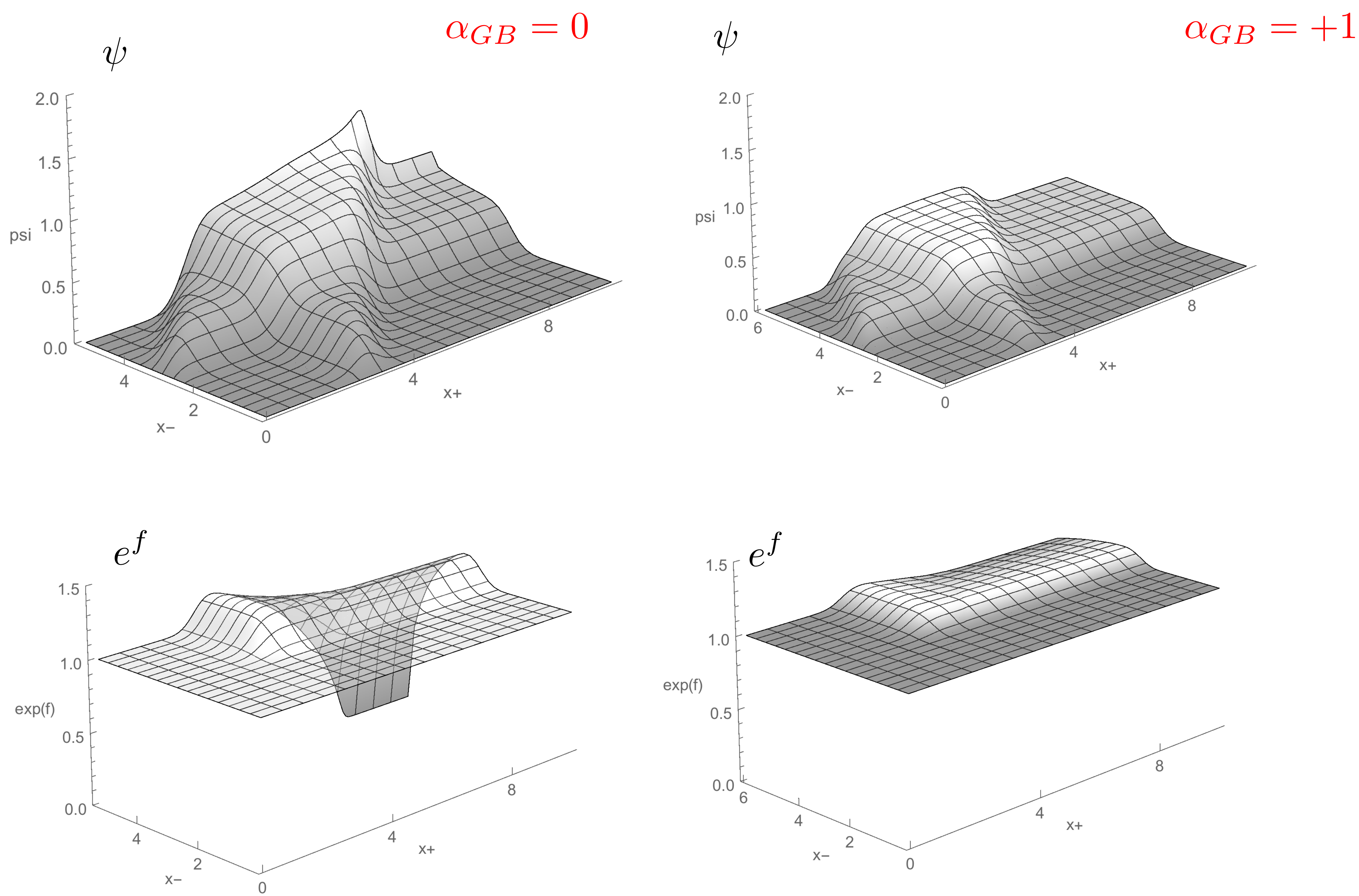


ψ



$I^{(5)} = R_{ijkl} R^{ijkl}$





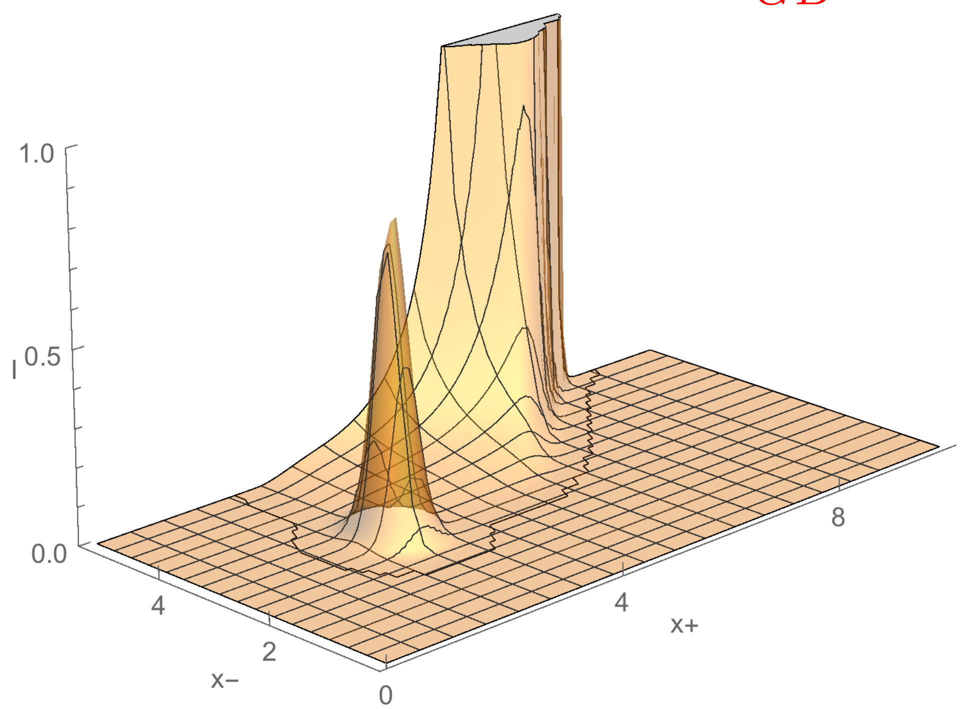
GR 5d

GaussBonnet 5d

large amplitude waves

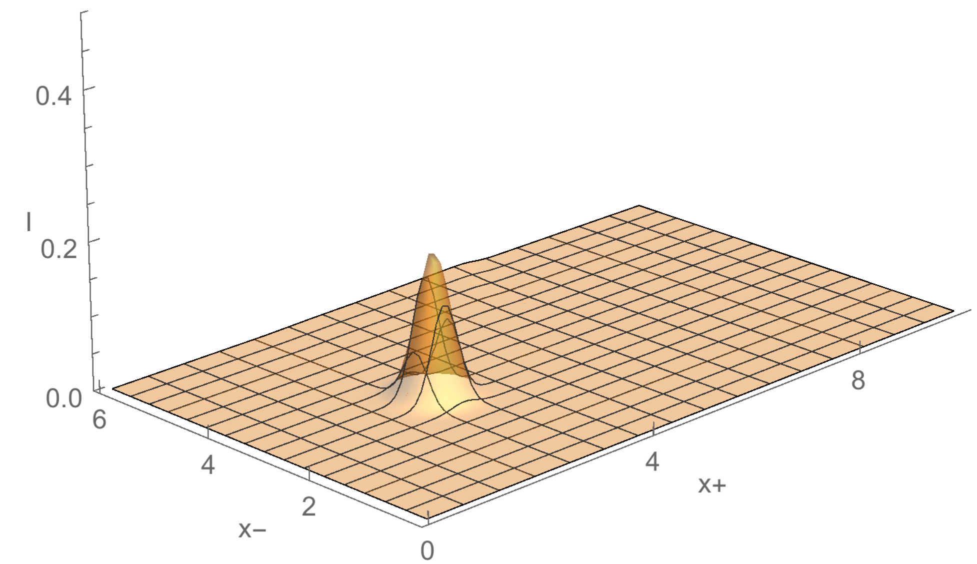
$I^{(5)} = R_{ijkl}R^{ijkl}$

$\alpha_{GB} = 0$



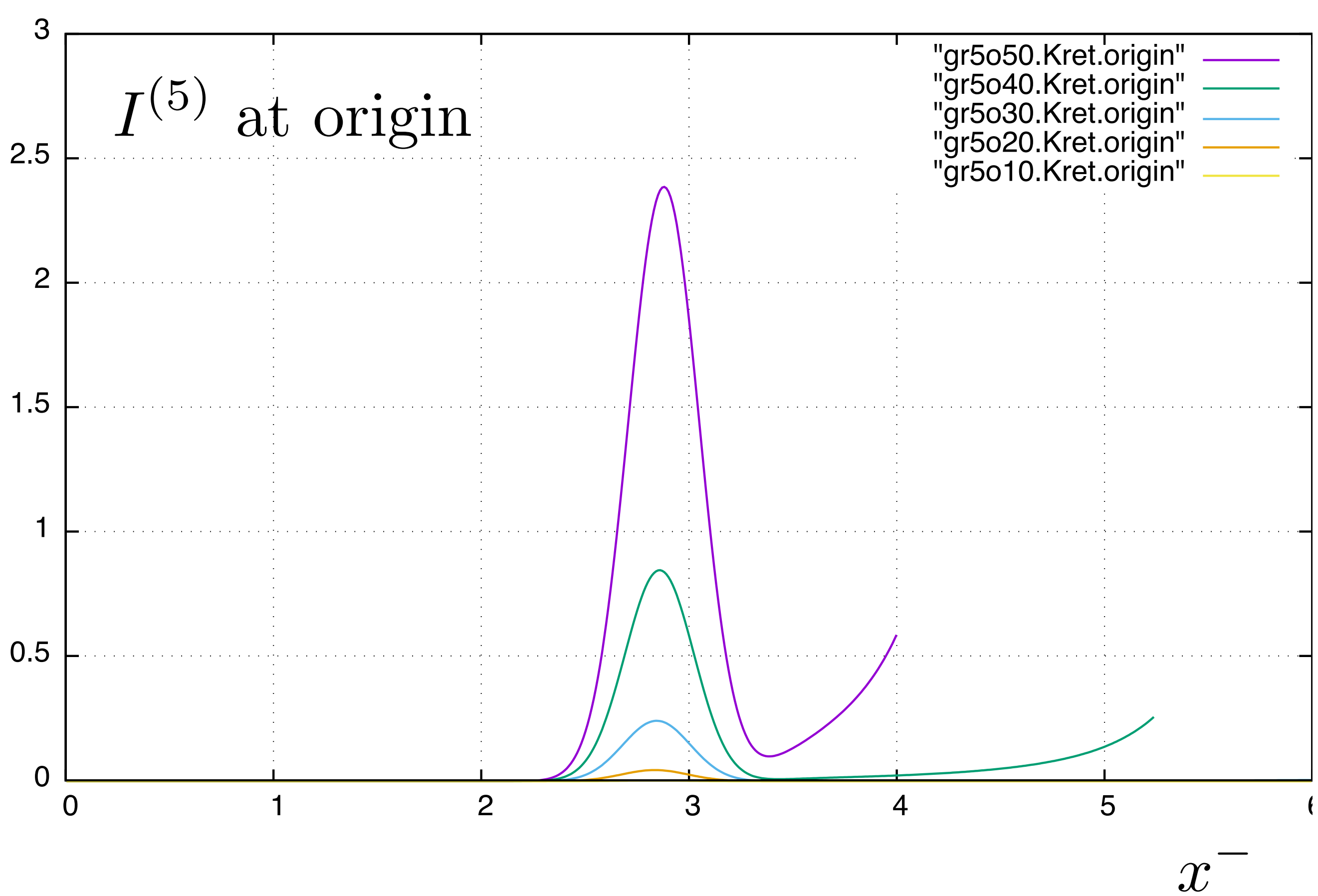
$I^{(5)} = R_{ijkl}R^{ijkl}$

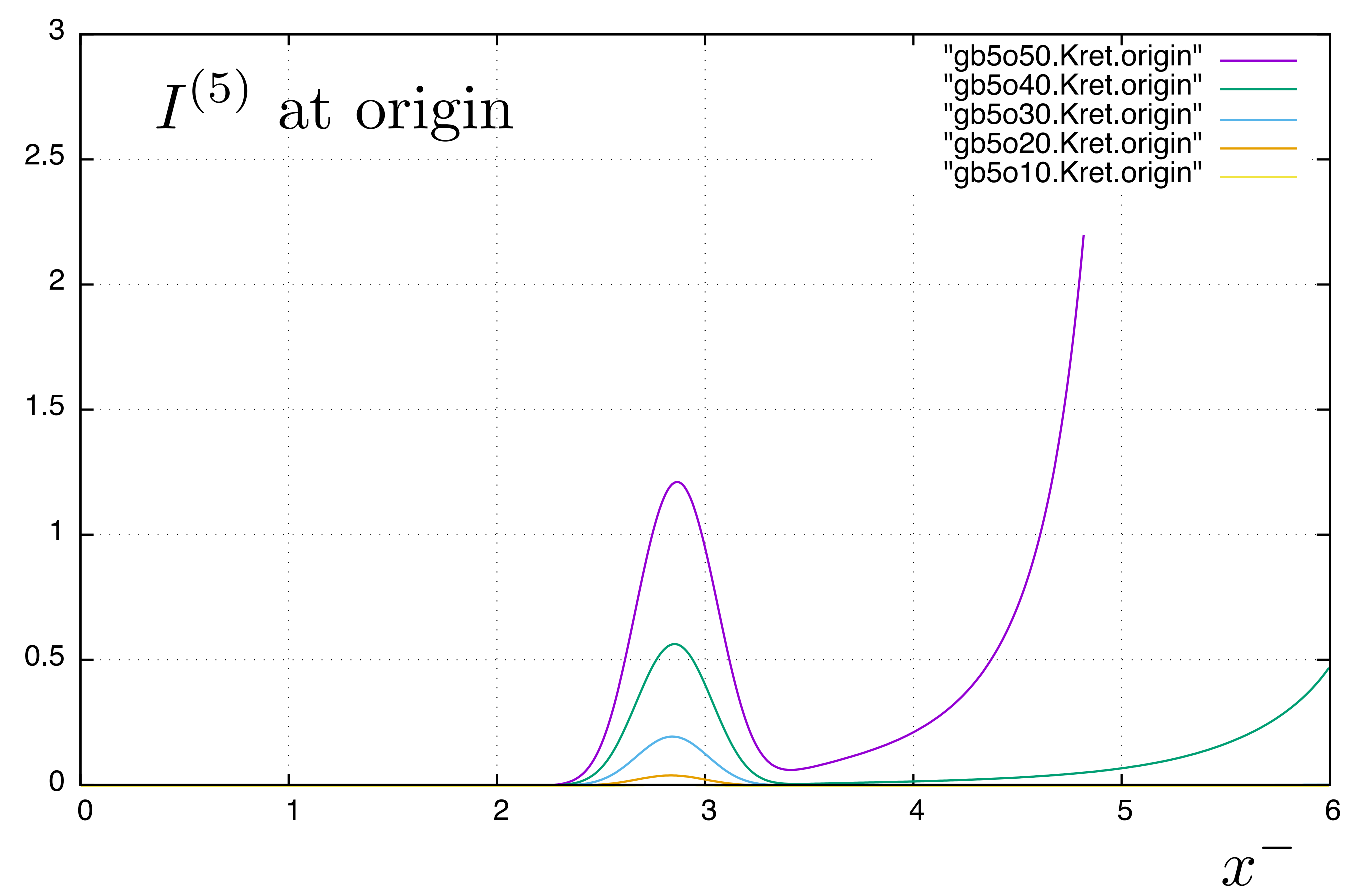
$\alpha_{GB} = +1$

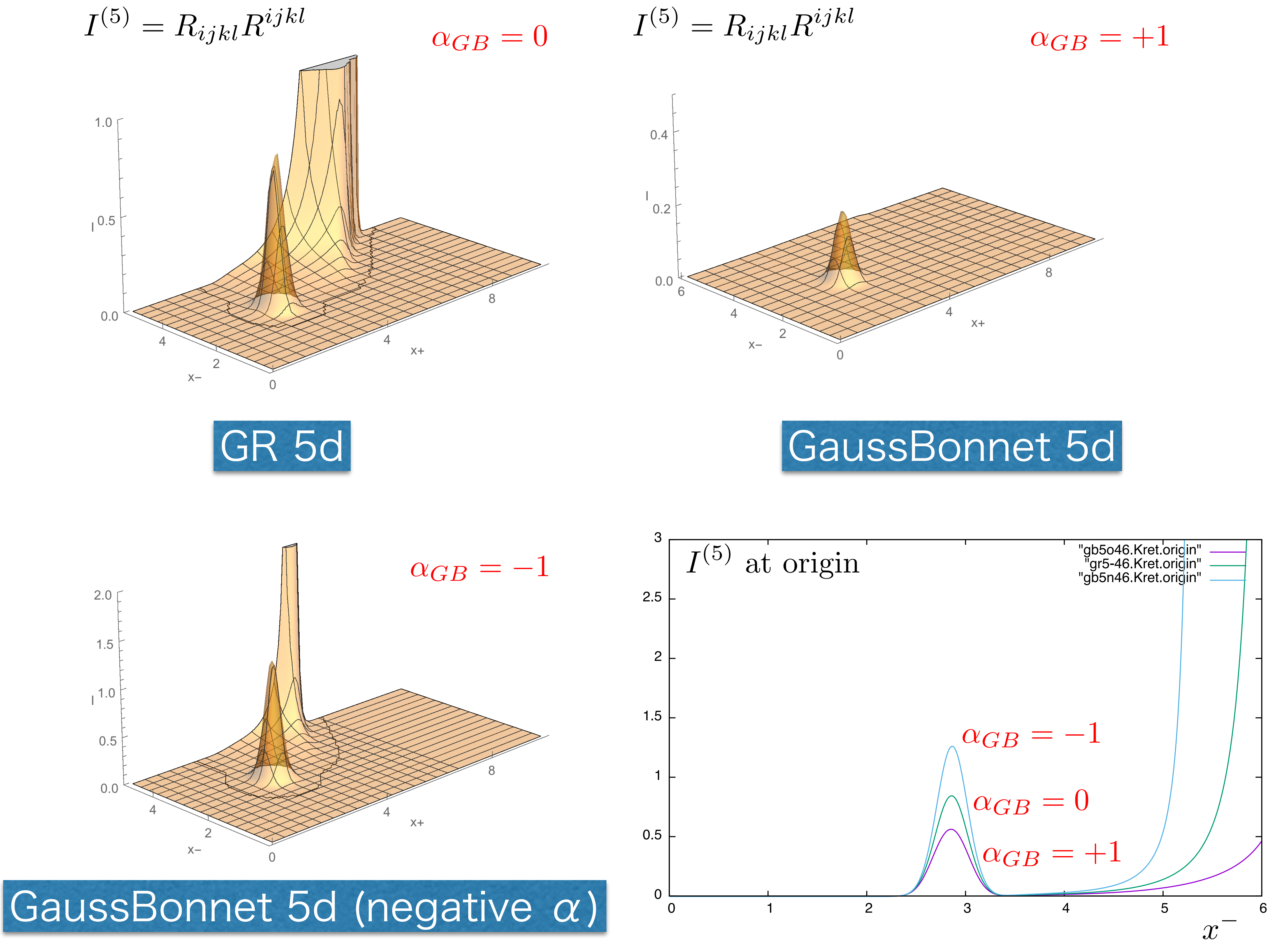


GR 5d

GaussBonnet 5d

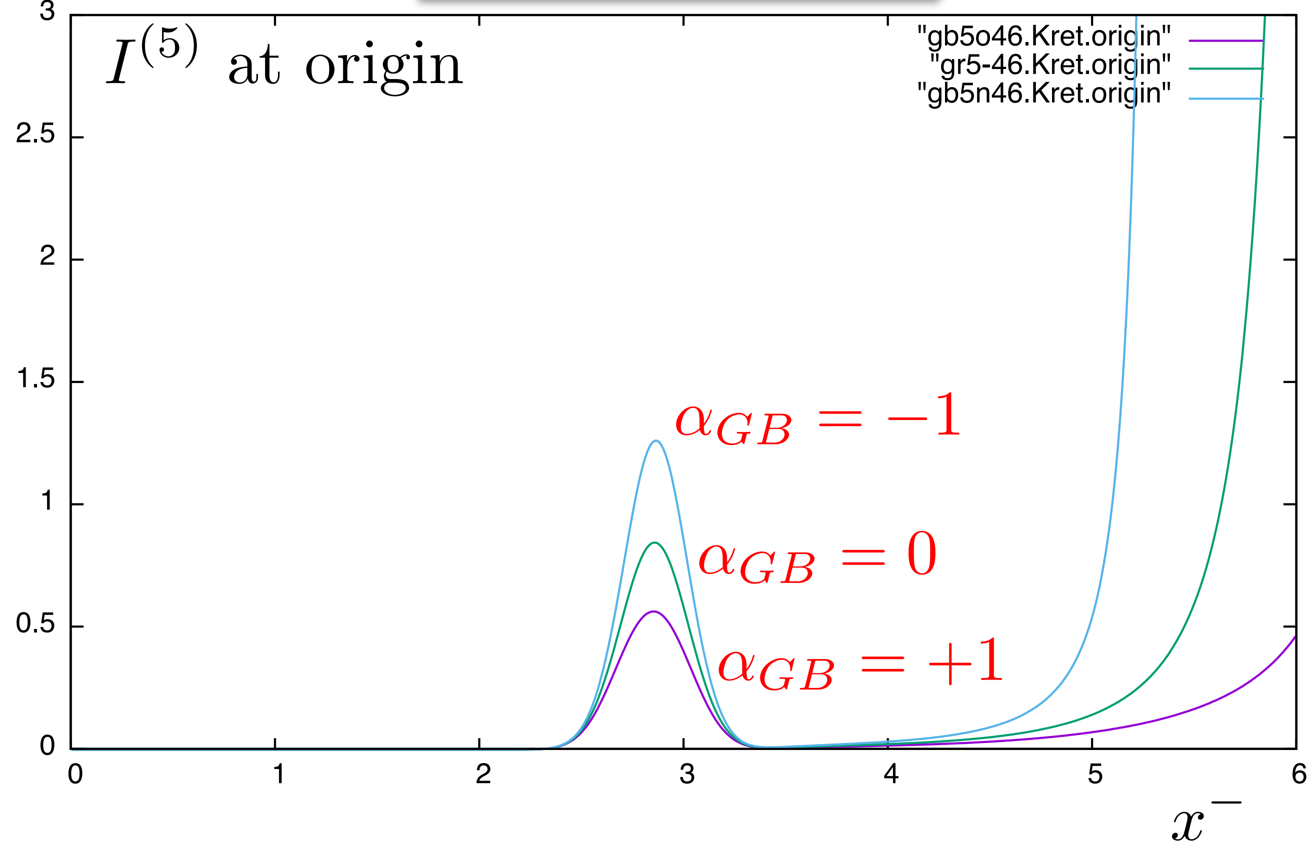




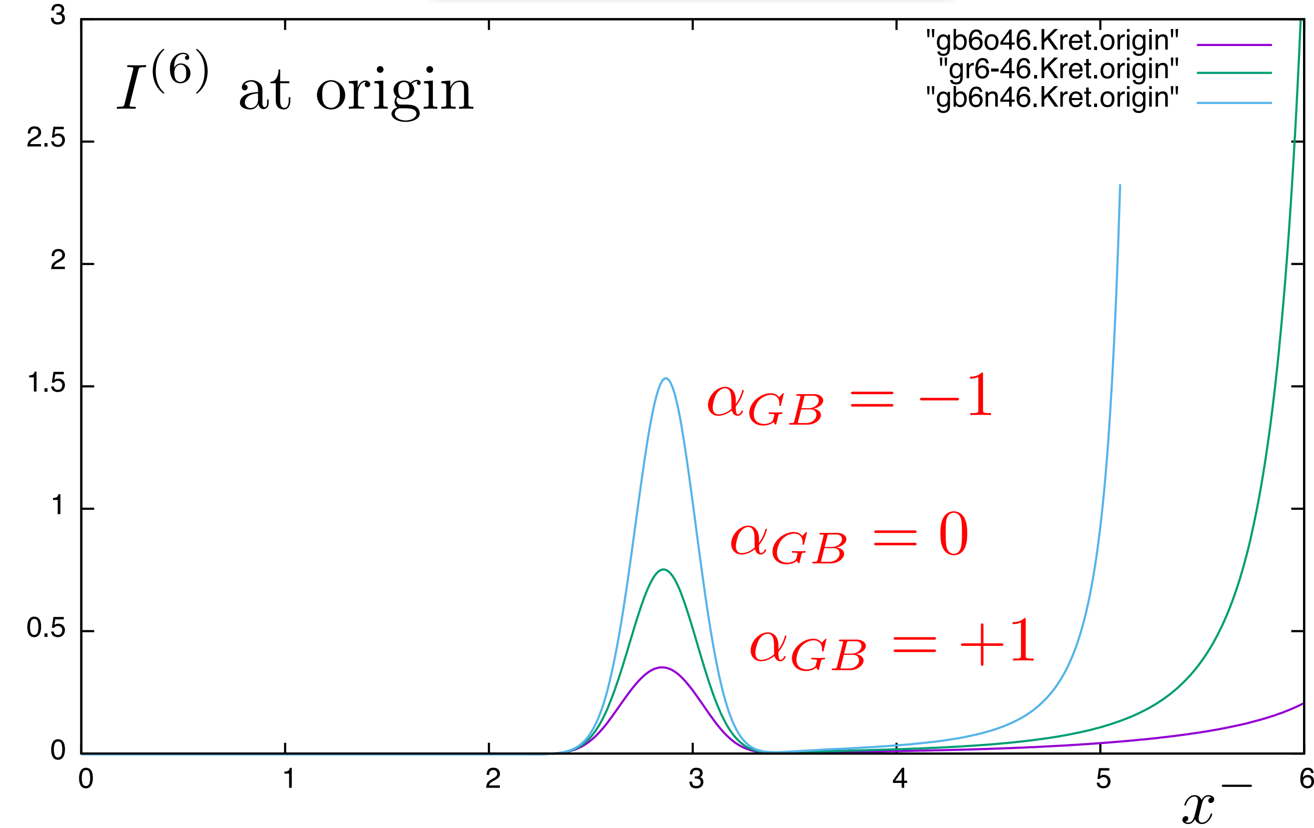


$I^{(5)} = R_{ijkl}R^{ijkl}$
$\alpha_{GB} = 0$
GR 5d
$I^{(5)} = R_{ijkl}R^{ijkl}$
$\alpha_{GB} = +1$
GaussBonnet 5d
$\alpha_{GB} = -1$
GaussBonnet 5d (negative α)
$I^{(5)}$ at origin
"gb5o46.Kret.origin"
"gr5-46.Kret.origin"
"gb5n46.Kret.origin"
$\alpha_{GB} = -1$
$\alpha_{GB} = 0$
$\alpha_{GB} = +1$
x^-
x+
x−
I

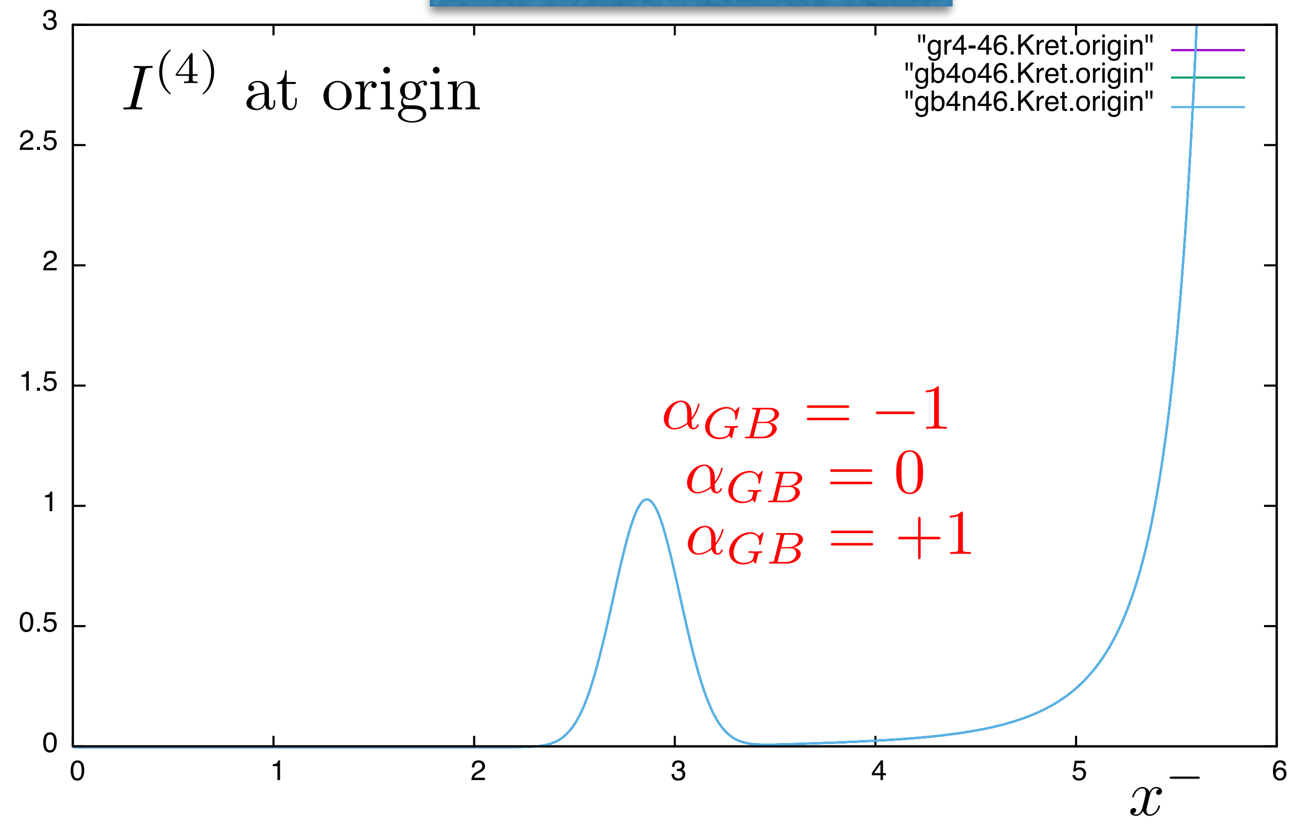
GR & GB 5d



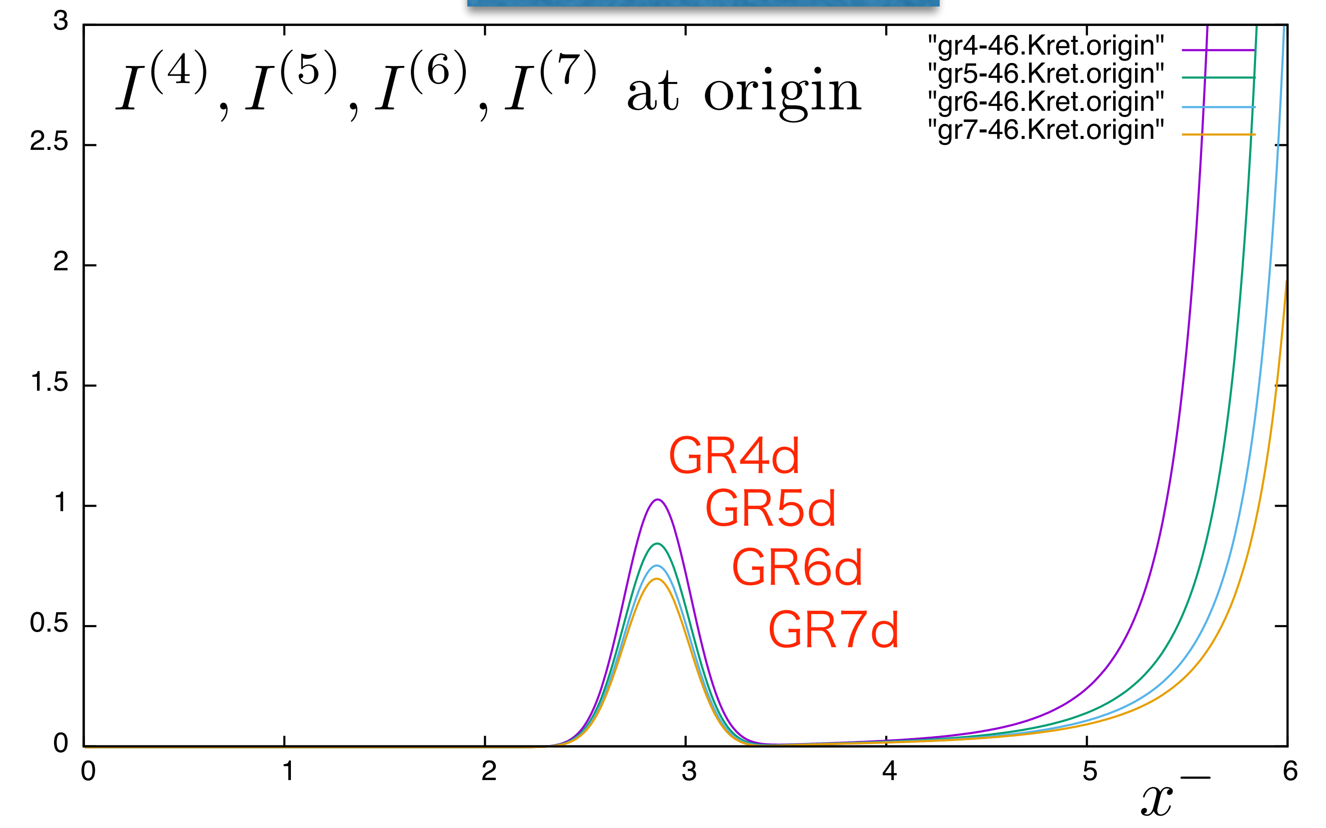
GR & GB 6d



GR & GB 4d



GR 4d—7d



Existence of re-scaling symmetry

When $\alpha_{GB} > 0$, then the set of equations remain same if we define re-scaled quantities:

$$\bar{x}_{\pm} = \frac{1}{\sqrt{\tilde{\alpha}}} x_{\pm}, \tag{1}$$

$$\bar{\Lambda} = \tilde{\alpha}\Lambda, \tag{2}$$

$$\bar{k} = k, \tag{3}$$

where $\tilde{\alpha} = (n-3)(n-4)\alpha_{GB}$.

These rescalings correspond as

$$\bar{t} = t/\sqrt{\tilde{\alpha}} \tag{4}$$

$$\bar{\partial}_{\pm} = \sqrt{\tilde{\alpha}}\partial_{\pm} \tag{5}$$

$$\bar{r} = r/\sqrt{\tilde{\alpha}} \tag{6}$$

$$\bar{\Omega} = \sqrt{\tilde{\alpha}}\Omega \tag{7}$$

$$\bar{\vartheta}_{\pm} = \vartheta_{\pm} \tag{8}$$

$$\bar{f} = f \tag{9}$$

$$\bar{\nu}_{\pm} = \sqrt{\tilde{\alpha}}\nu_{\pm} \tag{10}$$

$$\bar{\psi} = \psi \tag{11}$$

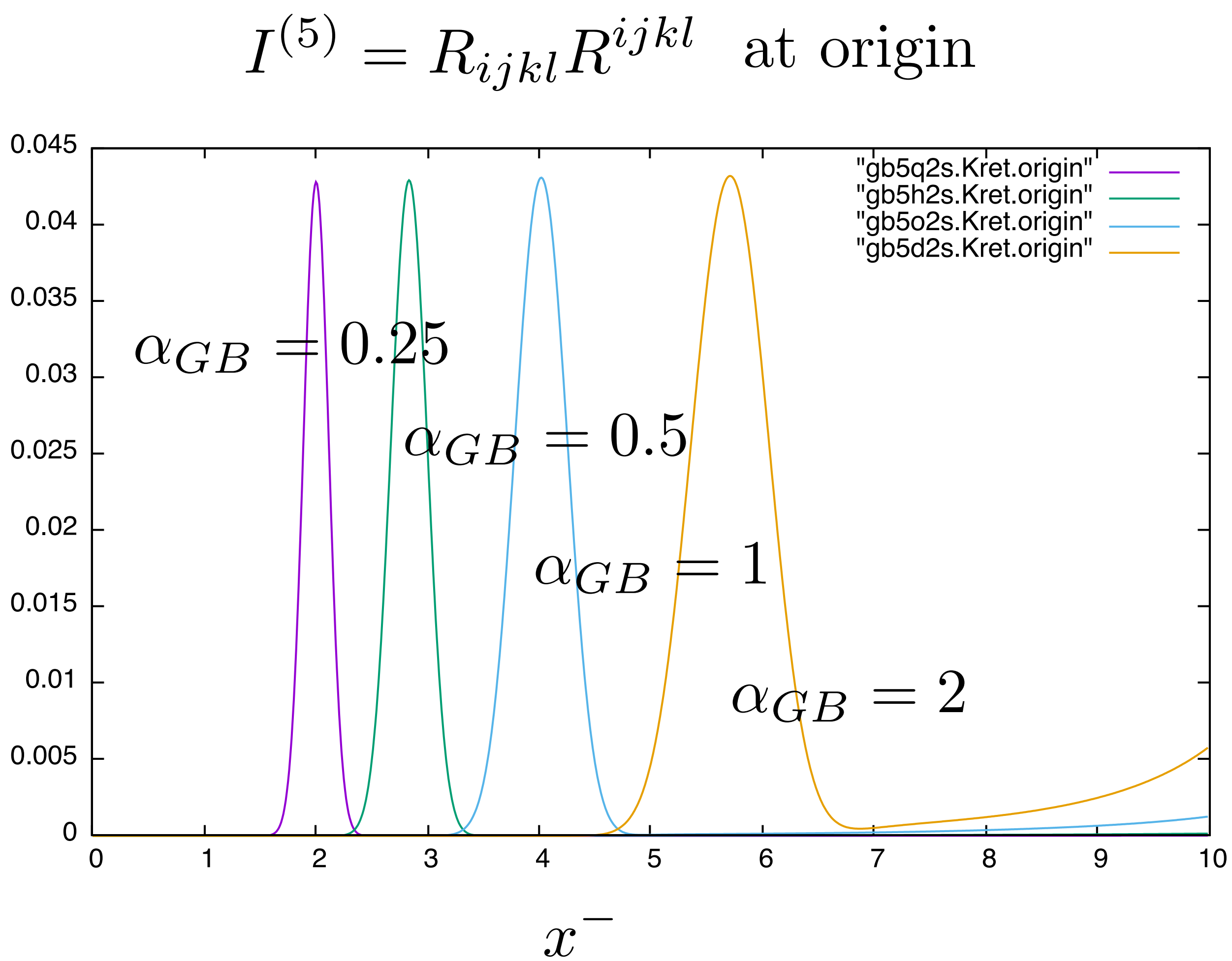
$$\bar{\phi} = \phi \tag{12}$$

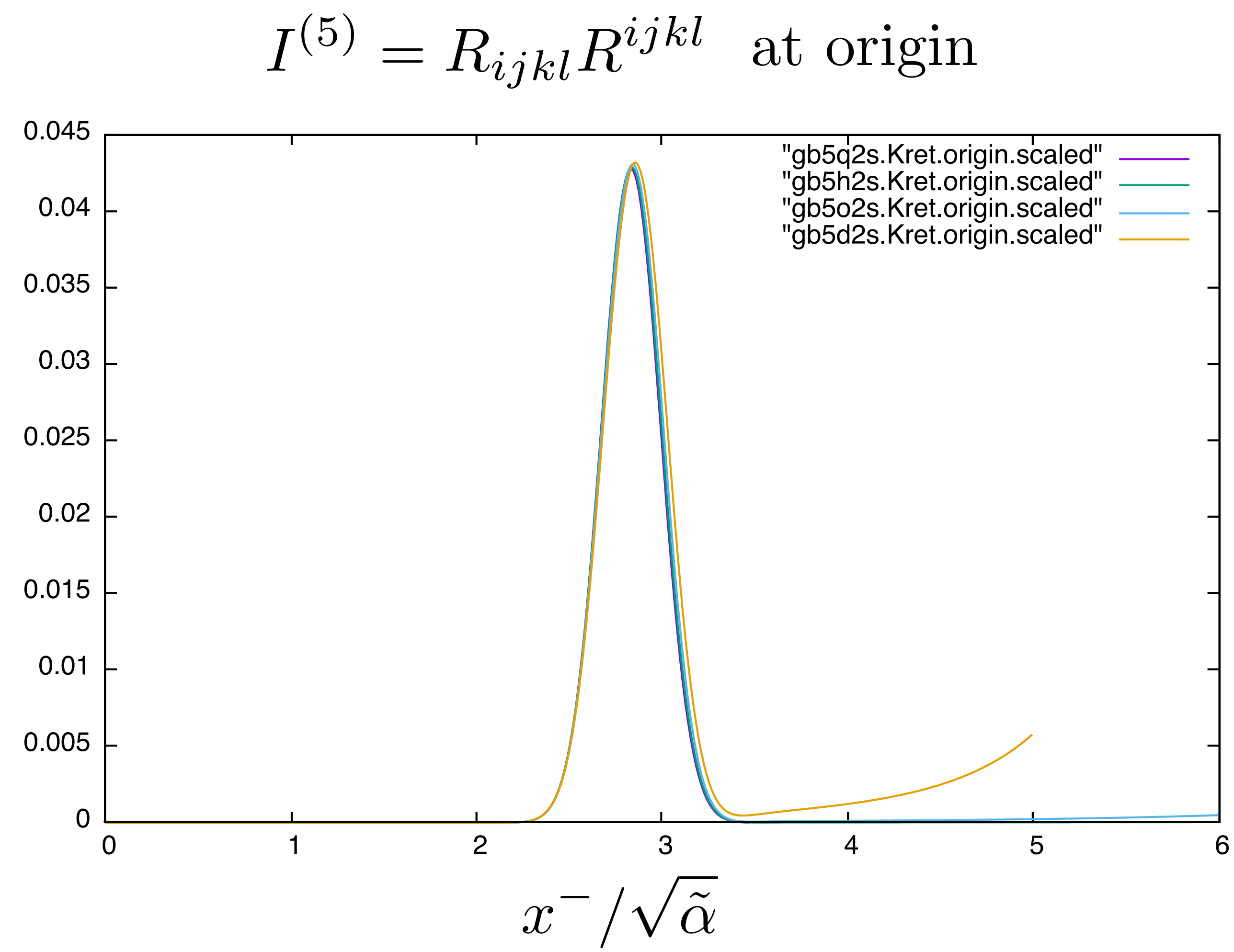
$$\bar{\pi}_{\pm} = \pi_{\pm} \tag{13}$$

$$\bar{p}_{\pm} = p_{\pm} \tag{14}$$

$$\overline{V_1 + V_2} = \tilde{\alpha}(V_1 + V_2) \tag{15}$$

and $\overline{A} = A, \overline{Z} = Z, \overline{W} = W$.





Initial data:

$\psi = 0$, $\pi_+ = a\exp(-b(z-c)^2)$ on $x_- = 0$ surface, where $z = x^+/\sqrt{2}$

$\psi = 0$, $\pi_- = a\exp(-b(z-c)^2)$ on $x_+ = 0$ surface, where $z = x^-/\sqrt{2}$

For Gauss-Bonnet gravity with $\tilde{\alpha} = (n-3)(n-4)\alpha_{GB}$, we confirmed that our code provides identical results for rescaled evolutions with the re-scaled initial data

$\psi = 0$, $\pi_+ = a\exp(-b(z-\sqrt{\tilde{\alpha}}c)^2/\tilde{\alpha}^2)$ on $x^- = 0$ surface, where $z = x^+/\sqrt{2}$

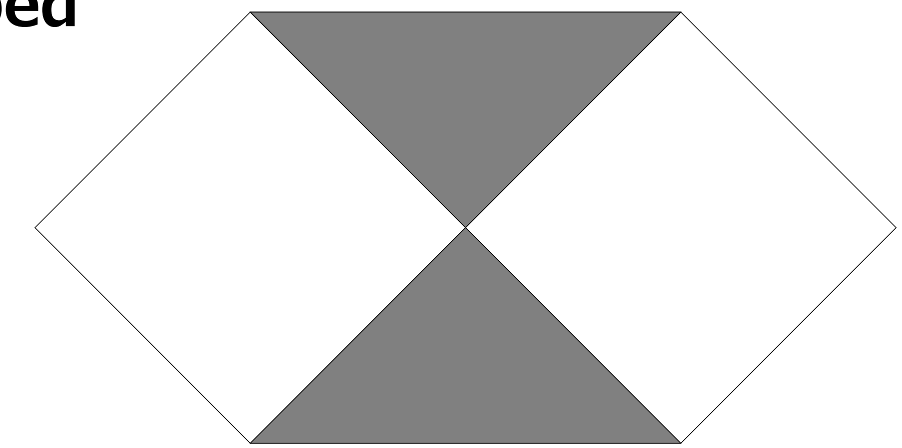
$\psi = 0$, $\pi_- = a\exp(-b(z-\sqrt{\tilde{\alpha}}c)^2/\tilde{\alpha}^2)$ on $x^+ = 0$ surface, where $z = x^-/\sqrt{2}$

BH & WH are interconvertible?

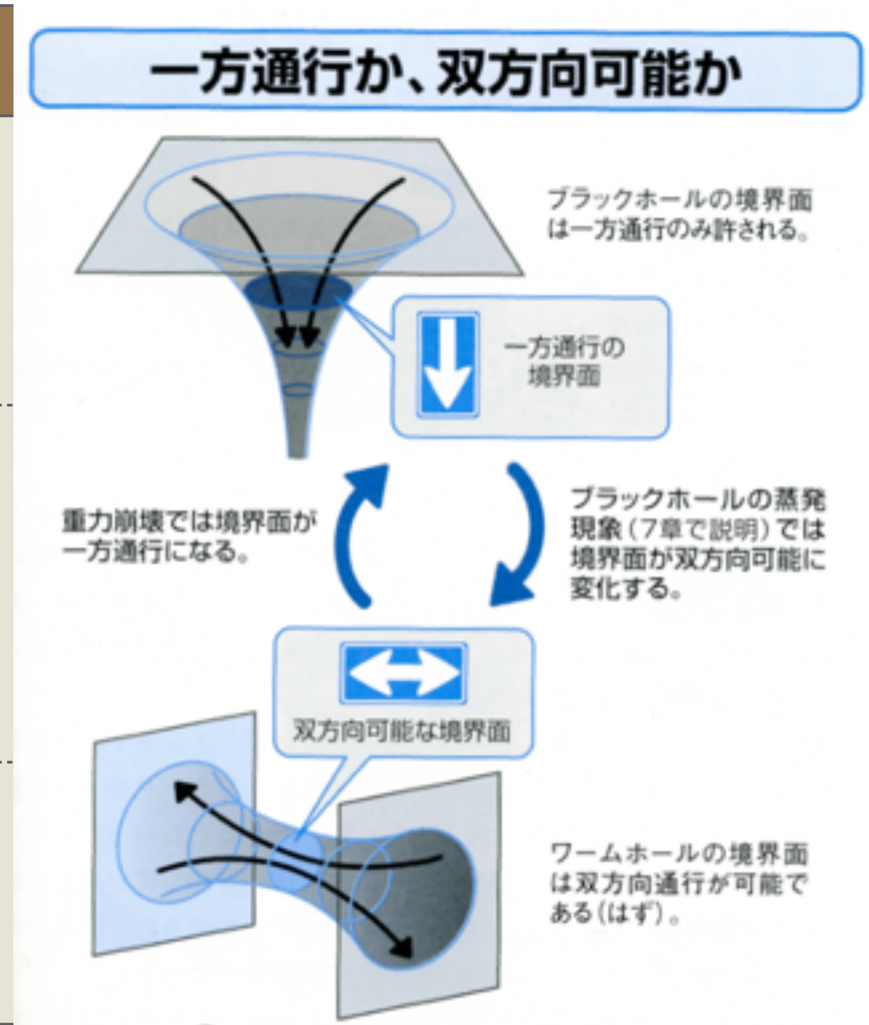
S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

They are very similar -- both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)

Only the causal nature of the THs differs, whether THs evolve in plus / minus density which is given locally.



	Black Hole	Wormhole
Locally defined by	Achronal (spatial/null) outer TH ⇒ 1-way traversable	Temporal (timelike) outer THs ⇒ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appear-ance	occur naturally	Unlikely to occur naturally. but constructible??



initial data: wormhole cases

- Static condition

$$(\partial_+ + \partial_-)\Omega = 0 \implies \vartheta_+ + \vartheta_- = 0$$
$$(\partial_+ + \partial_-)\psi = 0 \implies \pi_+ + \pi_- = 0$$
$$(\partial_+ + \partial_-)\phi = 0 \implies p_+ + p_- = 0$$
$$\left.\begin{array}{l}(\partial_+ + \partial_-)\vartheta_+ = 0 \\ (\partial_+ + \partial_-)\vartheta_- = 0\end{array}\right\} \implies \vartheta_+\nu_+ + \frac{1}{A}\Omega\kappa^2(\pi_+^2 - p_+^2) = \vartheta_-\nu_- + \frac{1}{A}\Omega\kappa^2(\pi_-^2 - p_-^2)$$

- Solve x^+ and x^- equations with the starting condition at the throat

$$\vartheta_+ = \vartheta_-(=0)$$
$$\nu_+ = \nu_-(=0)$$
$$-\kappa^2\Omega(\pi_+^2 - p_+^2)e^f = -\frac{1}{\Omega}\left[-\alpha_1\Omega^2\frac{(n-2)(n-3)}{2}k + \Lambda + \kappa^2(V_1 + V_2)\right] + \tilde{\alpha}\Omega^3\frac{(n-2)(n-5)}{2}k^2$$

If we assume only ghost field ϕ, then

$$p_+ = -p_- = \sqrt{\frac{1}{\kappa^2 e^f}\left[\alpha_1\frac{(n-2)(n-3)}{2}k - \frac{1}{\Omega^2}(\Lambda + \kappa^2 V_2) + \tilde{\alpha}\Omega^2\frac{(n-2)(n-5)}{2}k^2\right]}$$

- add perturbation

$$p_+(x^+ = x, x^- = 0) = p_+(\text{solution}) + a\exp[-100(x-0.5)^2]$$

Ghost pulse input -- Bifurcation of the horizons（4d)

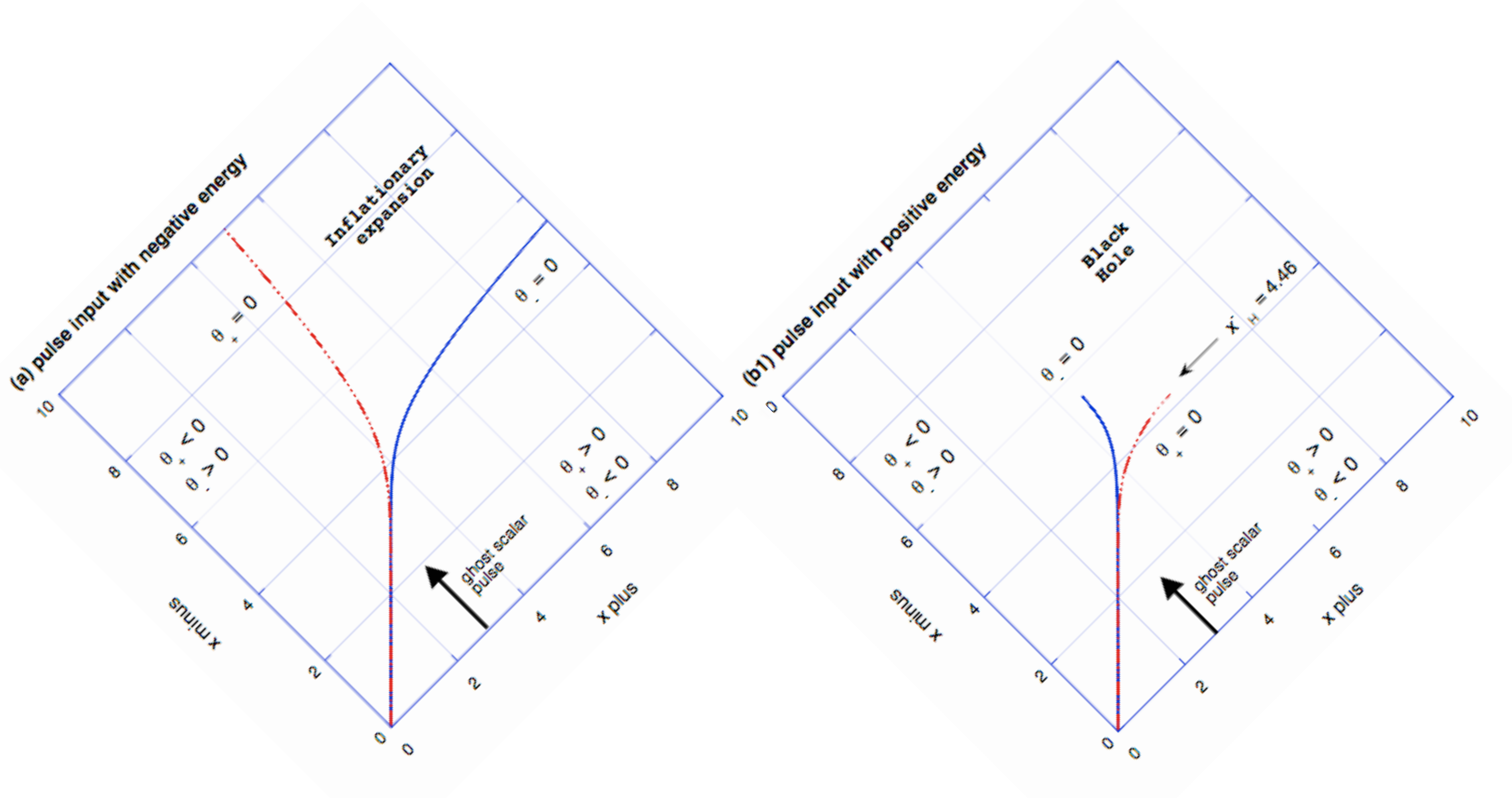
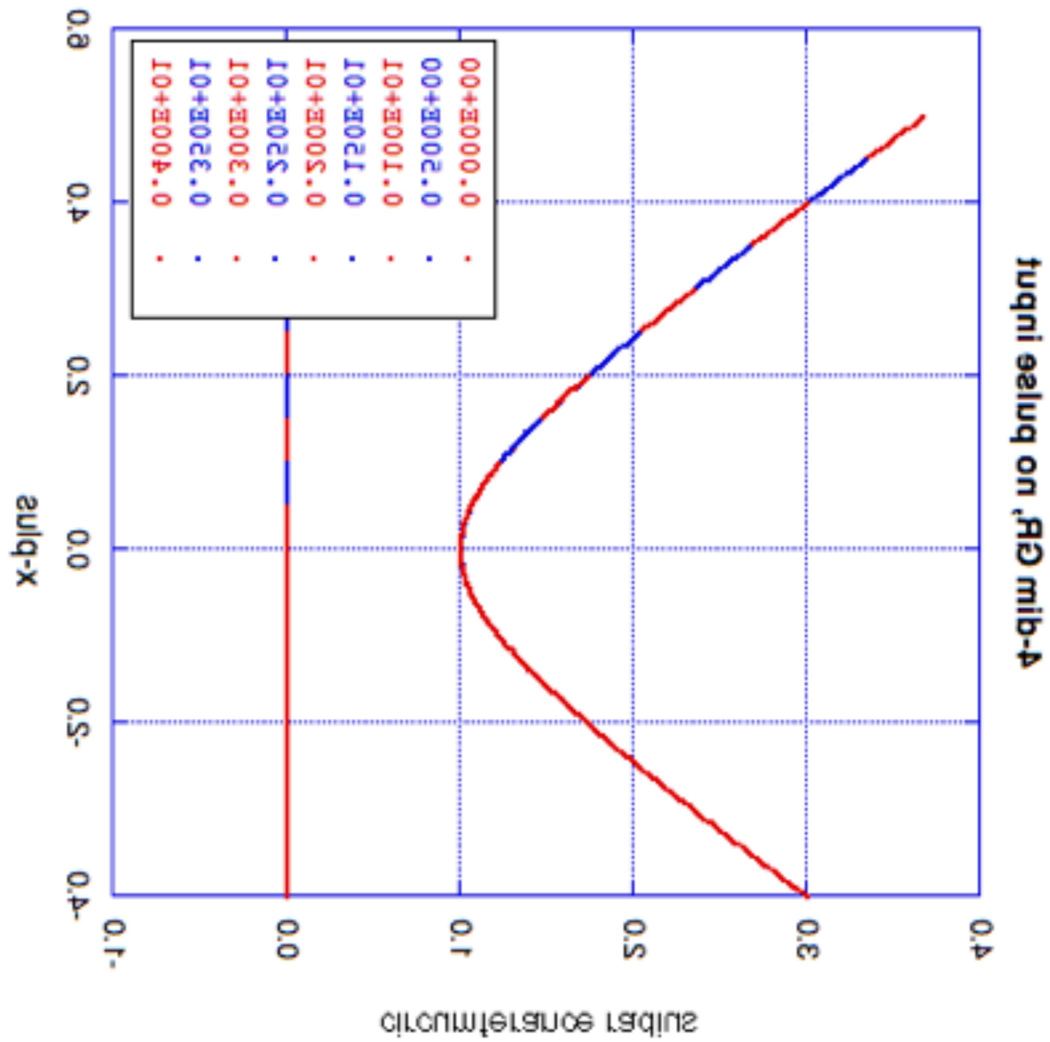
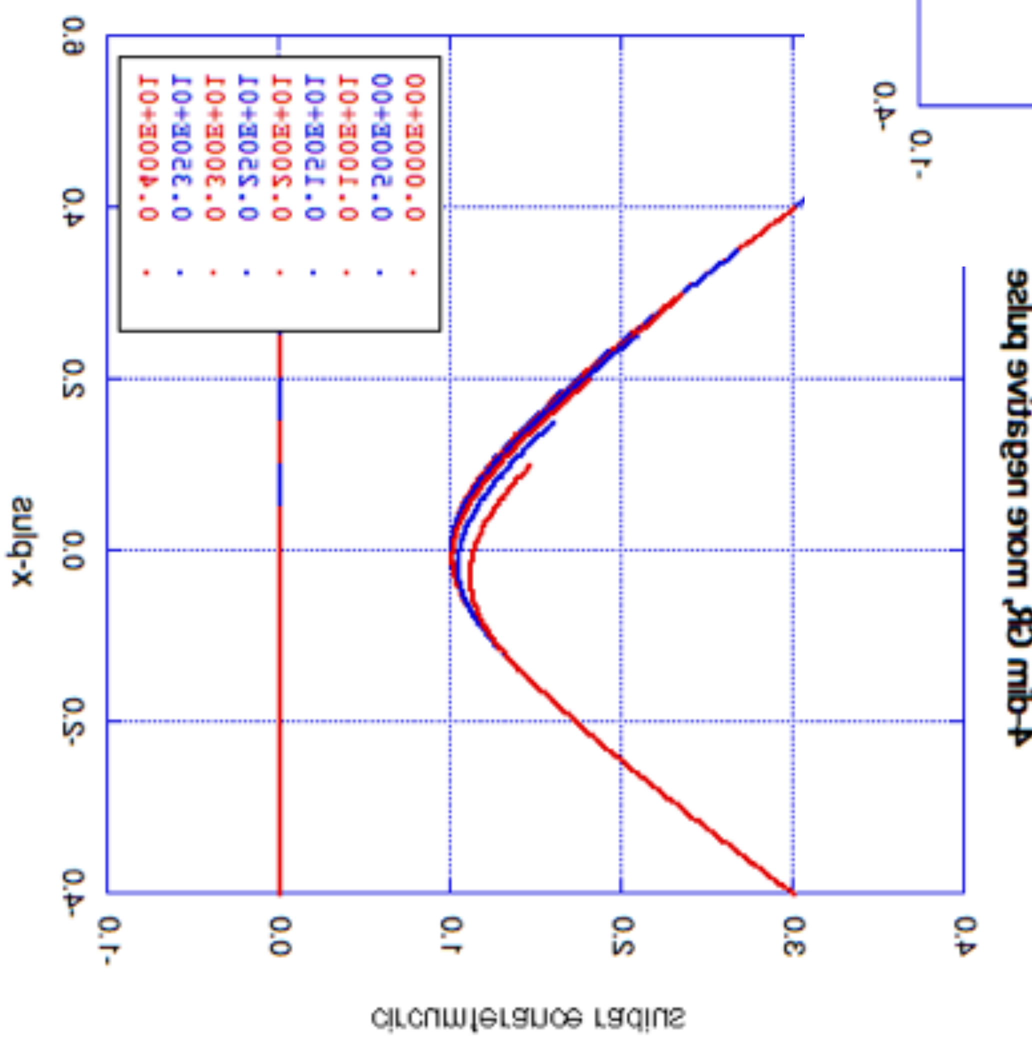


Figure 3: Horizon locations, $\vartheta_\pm = 0$, for perturbed wormhole. Fig.(a) is the case we supplement the ghost field, $c_a = 0.1$, and (b1) and (b2) are where we reduce the field, $c_a = -0.1$ and -0.01. Dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively. In all cases, the pulse hits the wormhole throat at $(x^+, x^-) = (3, 3)$. A 45° counterclockwise rotation of the figure corresponds to a partial Penrose diagram.

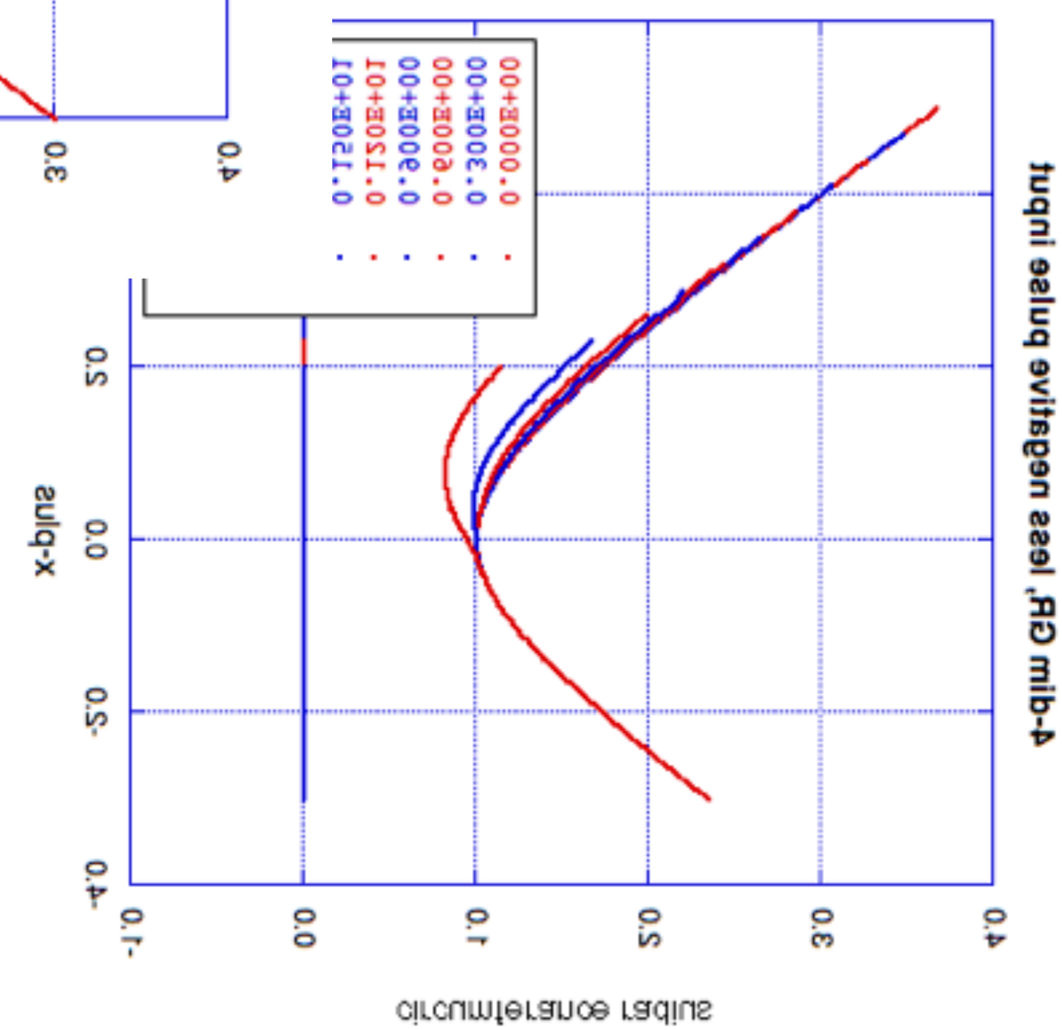
wormhole configurations (4dim. GR)



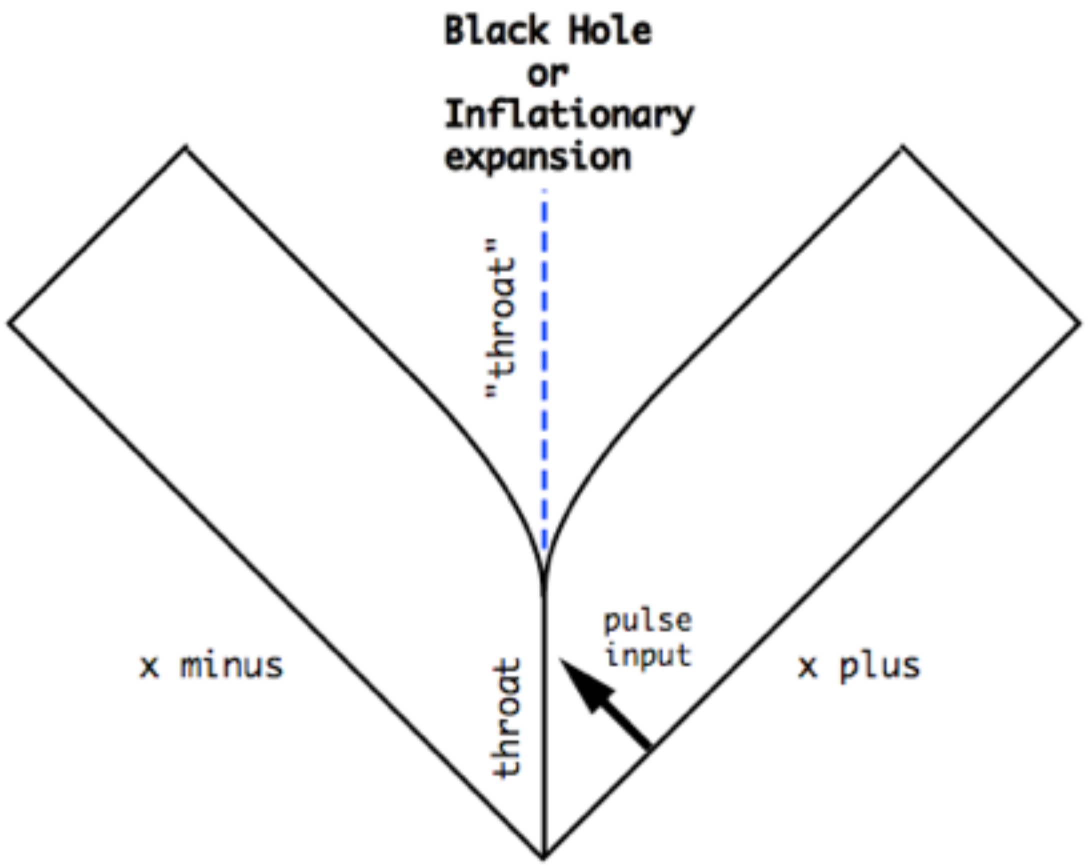
more negative field
-> throat expansion



less negative field
-> throat shrink



Bifurcation of the horizons
-- go to a Black Hole or Inflationary expansion



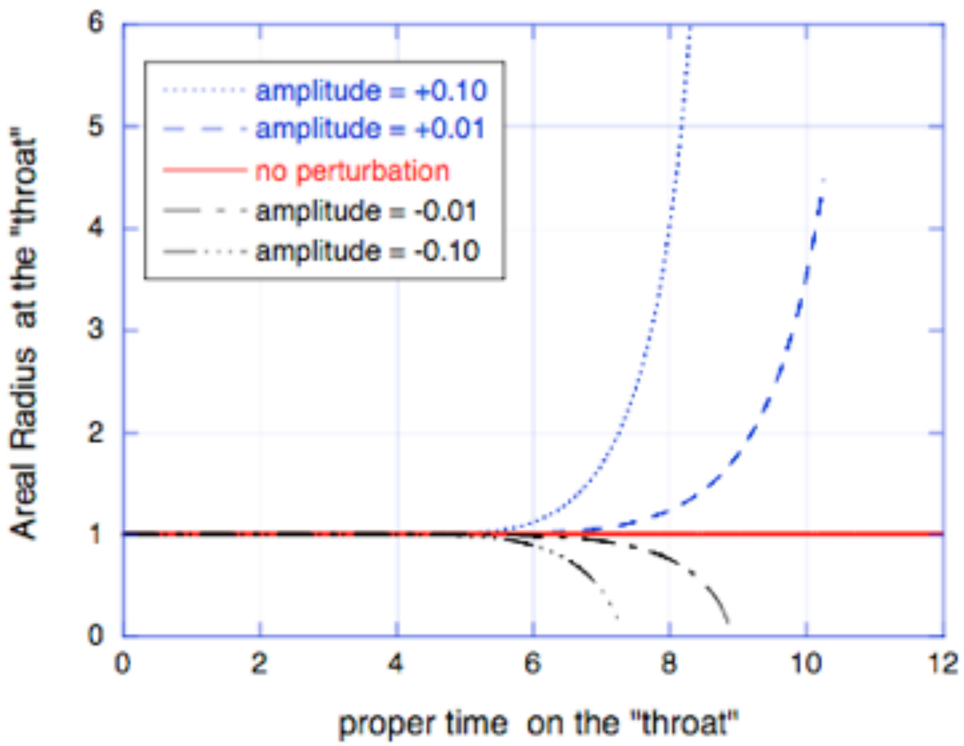


Figure 4: Partial Penrose diagram of the evolved space-time.
Figure 6: Areal radius r of the "throat" $x^+ = x^-$, plotted as a function of proper time. Additional negative energy causes inflationary expansion, while reduced negative energy causes collapse to a black hole and central singularity.

Normal pulse (a traveller) input -- Forming a Black Hole

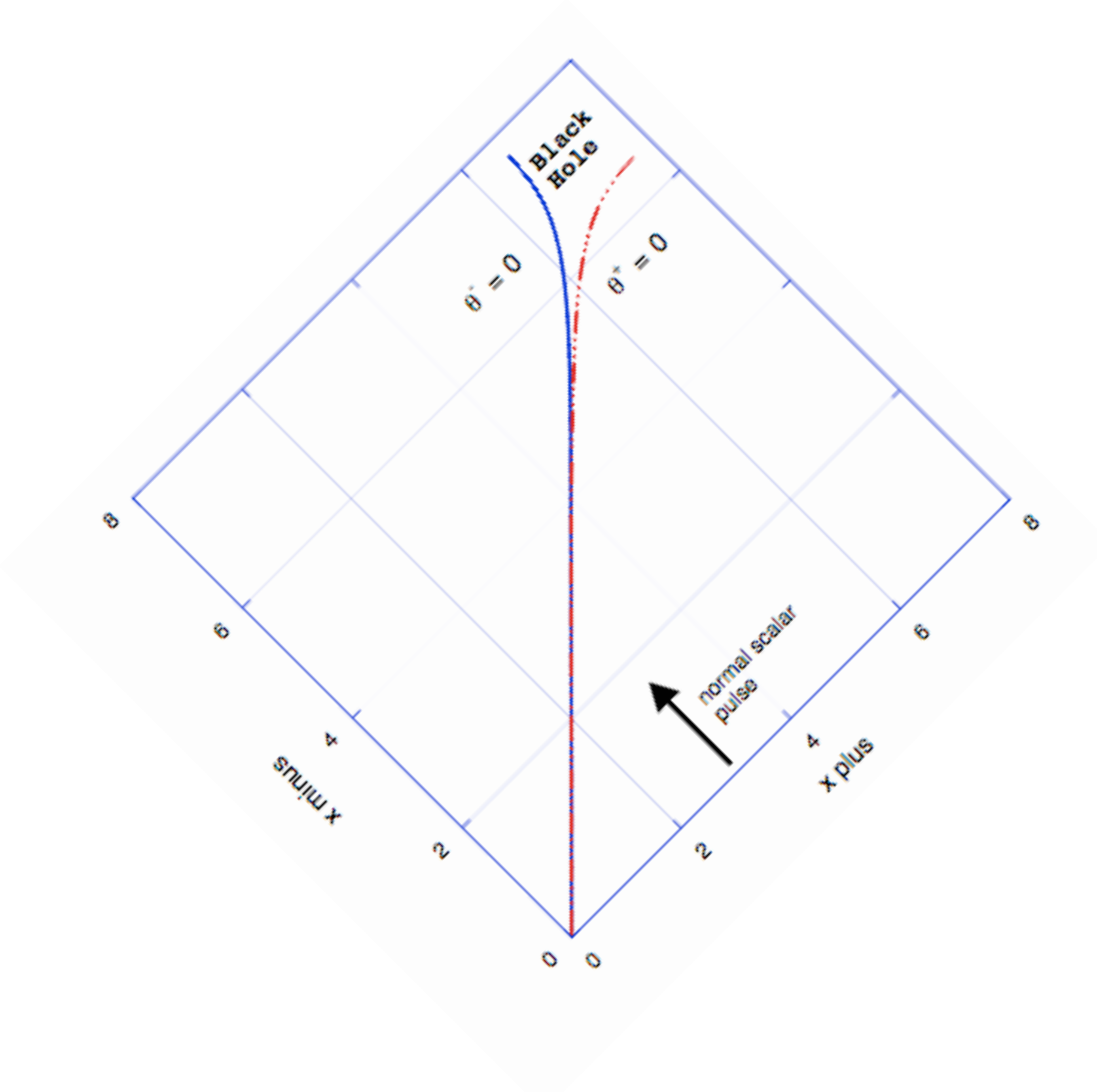
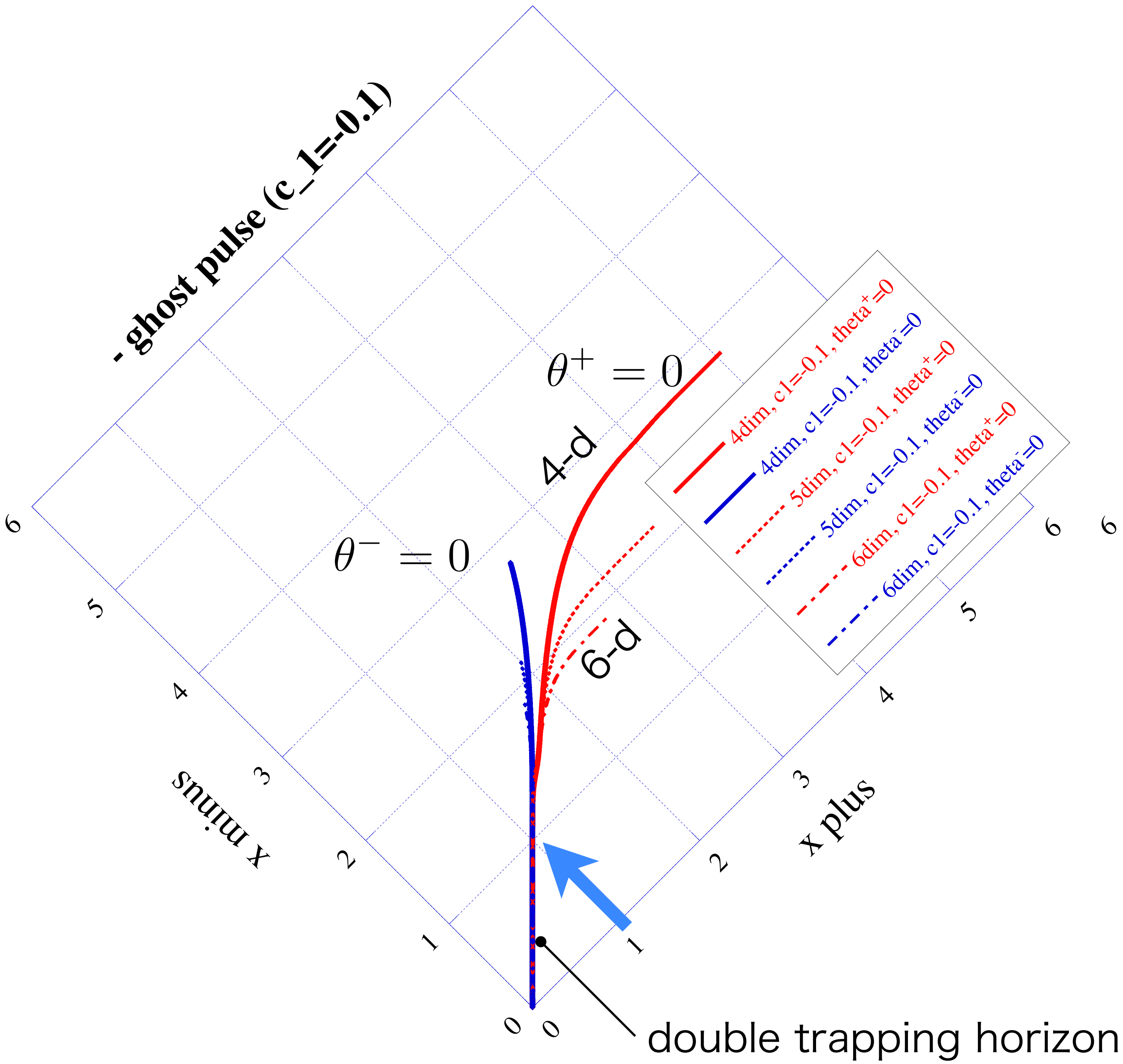
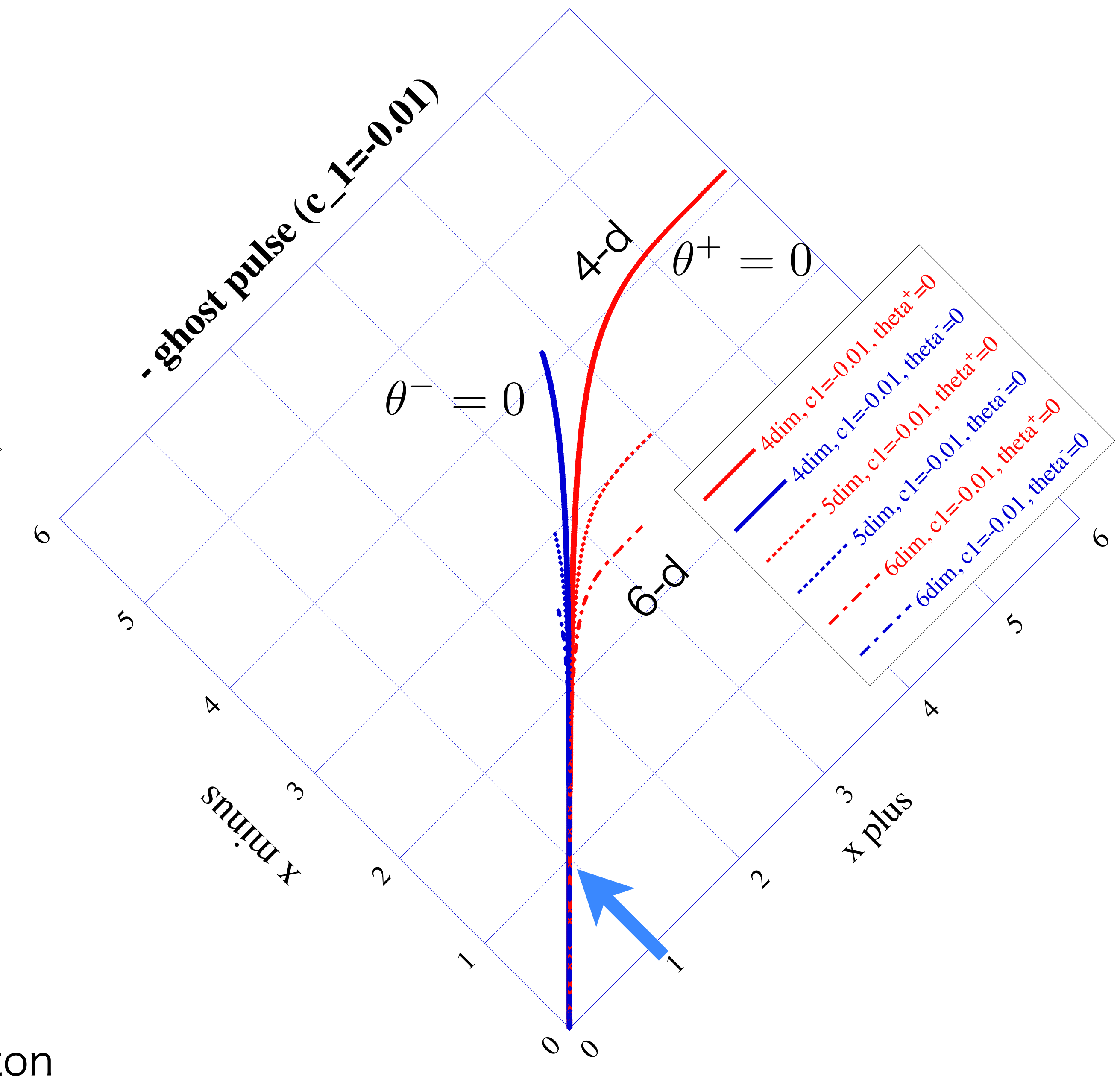


Figure 9: Evolution of a wormhole perturbed by a normal scalar field. Horizon locations: dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively.

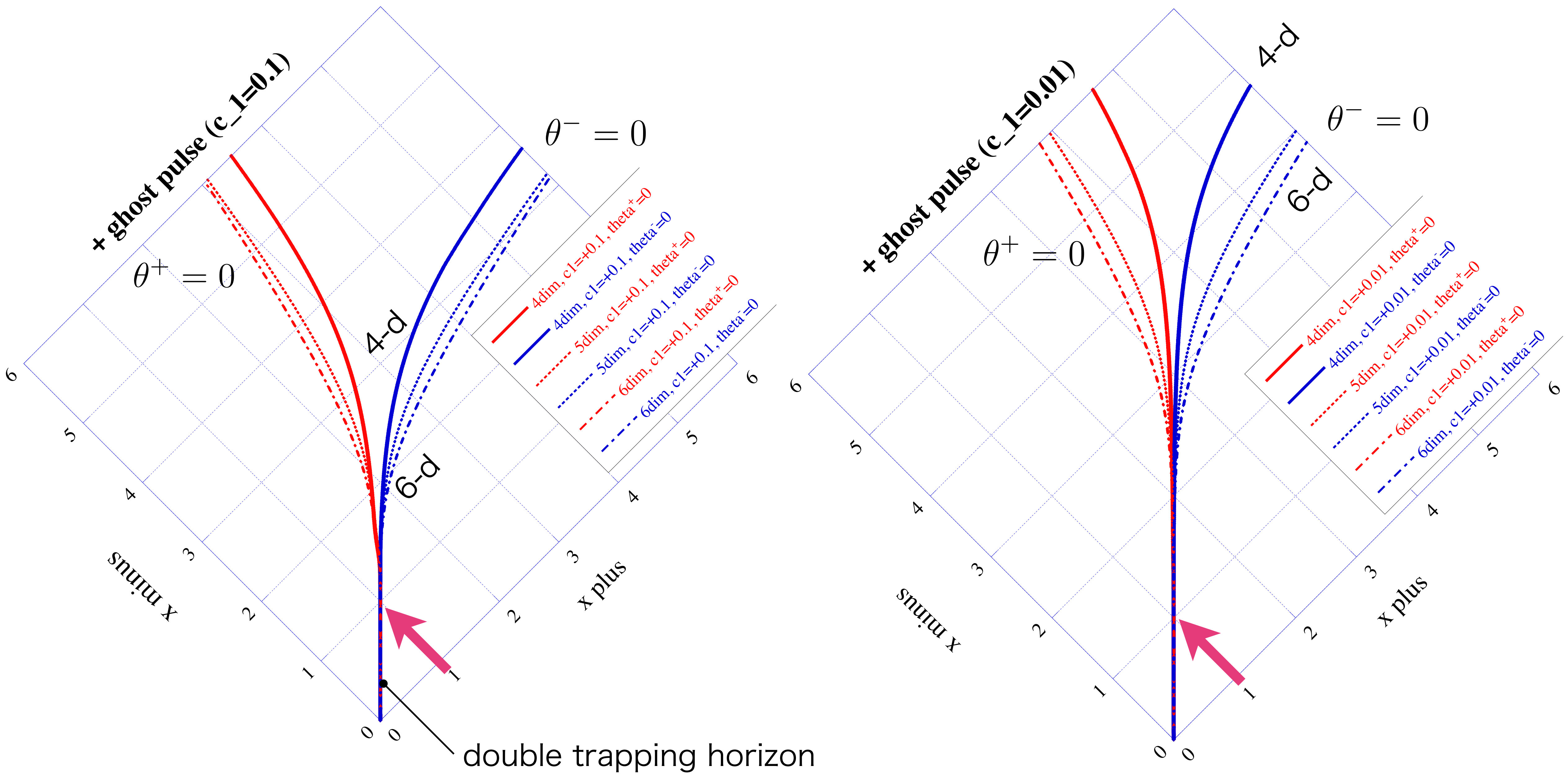
ghost pulse (negative amp.) input





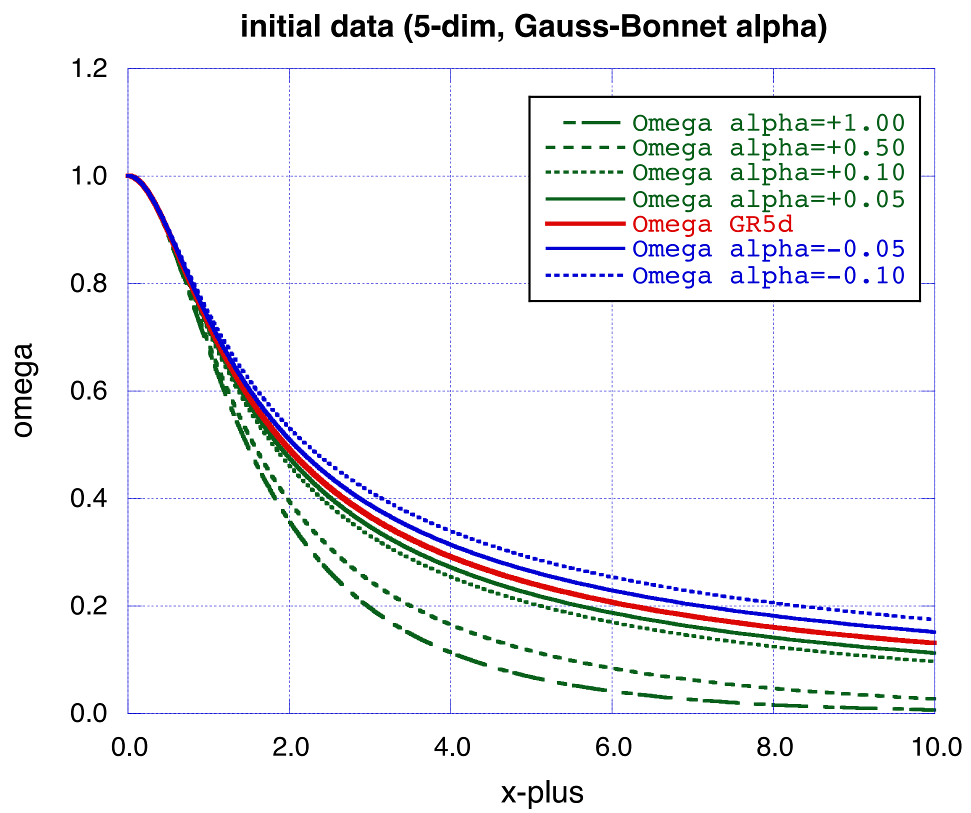
positive energy input --> BH formation

ghost pulse (positive amp.) input

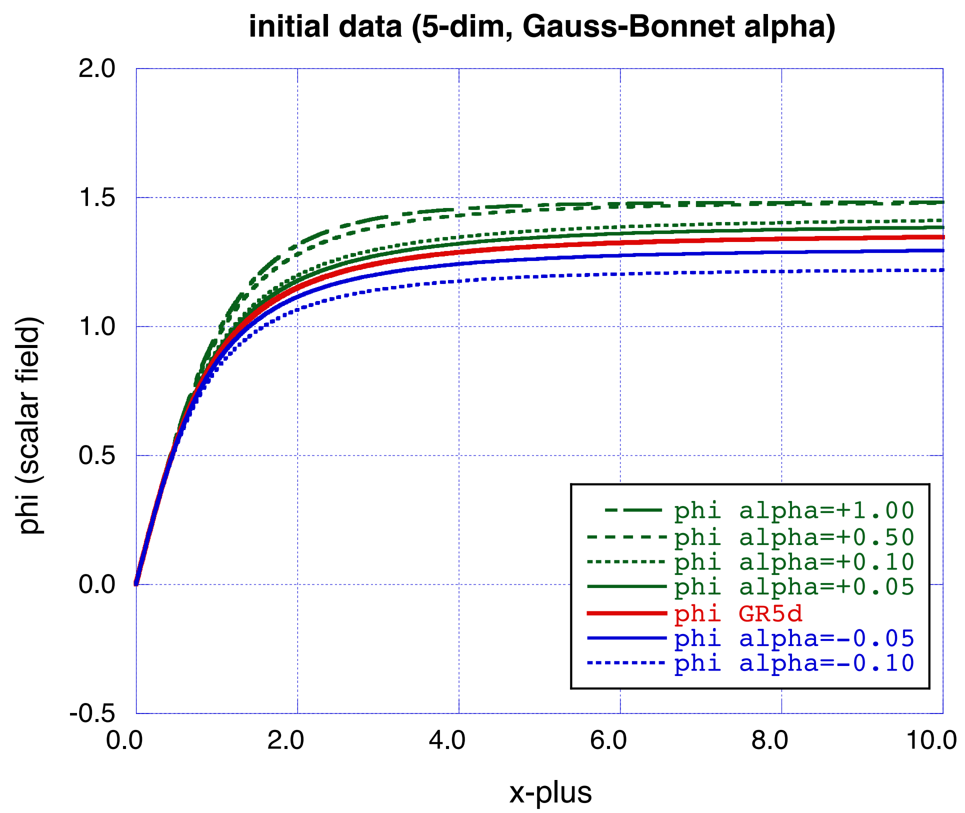


negative energy input --> throat inflates

Initial Data



conformal factor



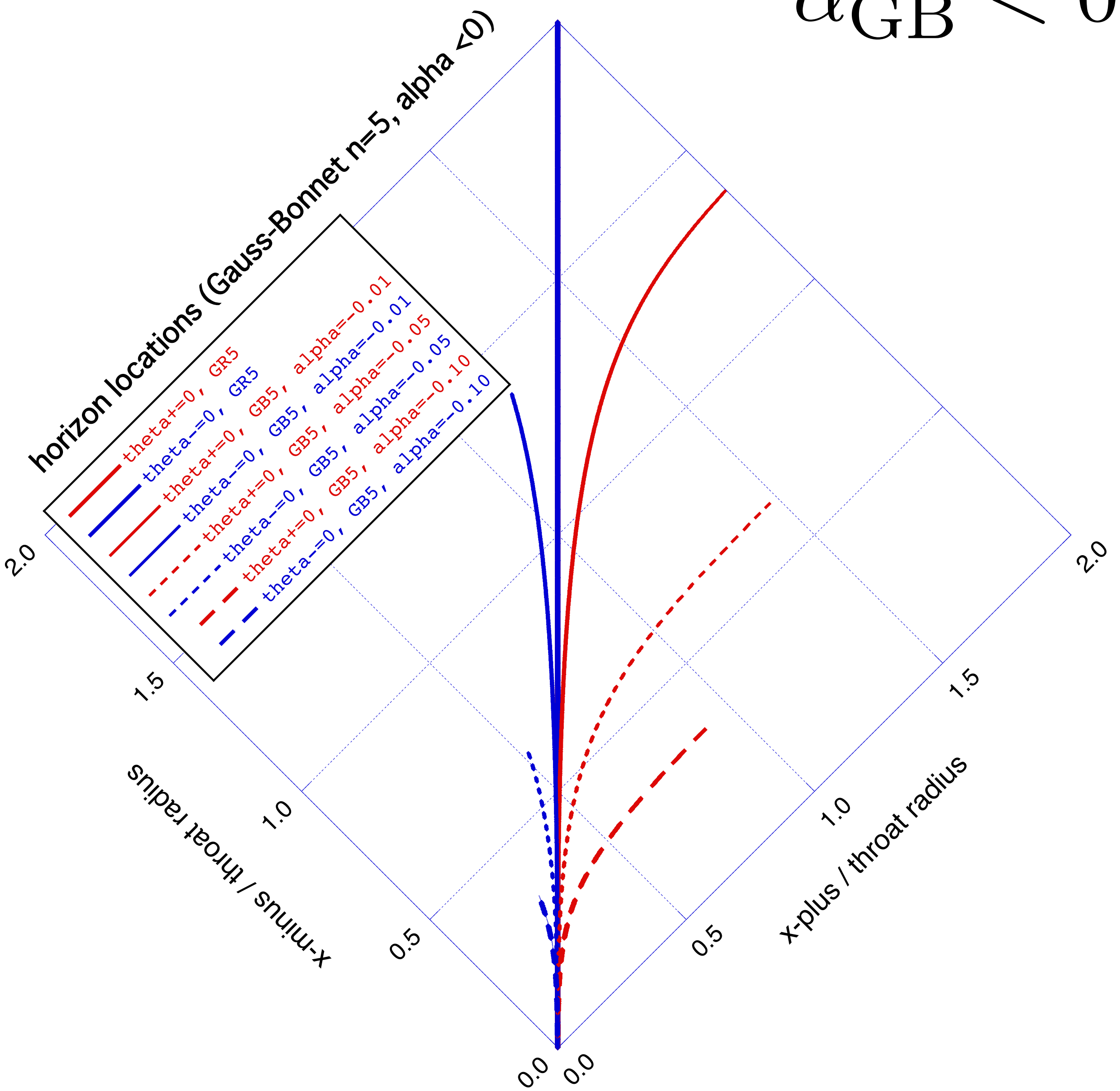
scalar field

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R} + \alpha_2\mathcal{L}_{\rm GB}\}+\mathcal{L}_{\rm matter}\Big]$$

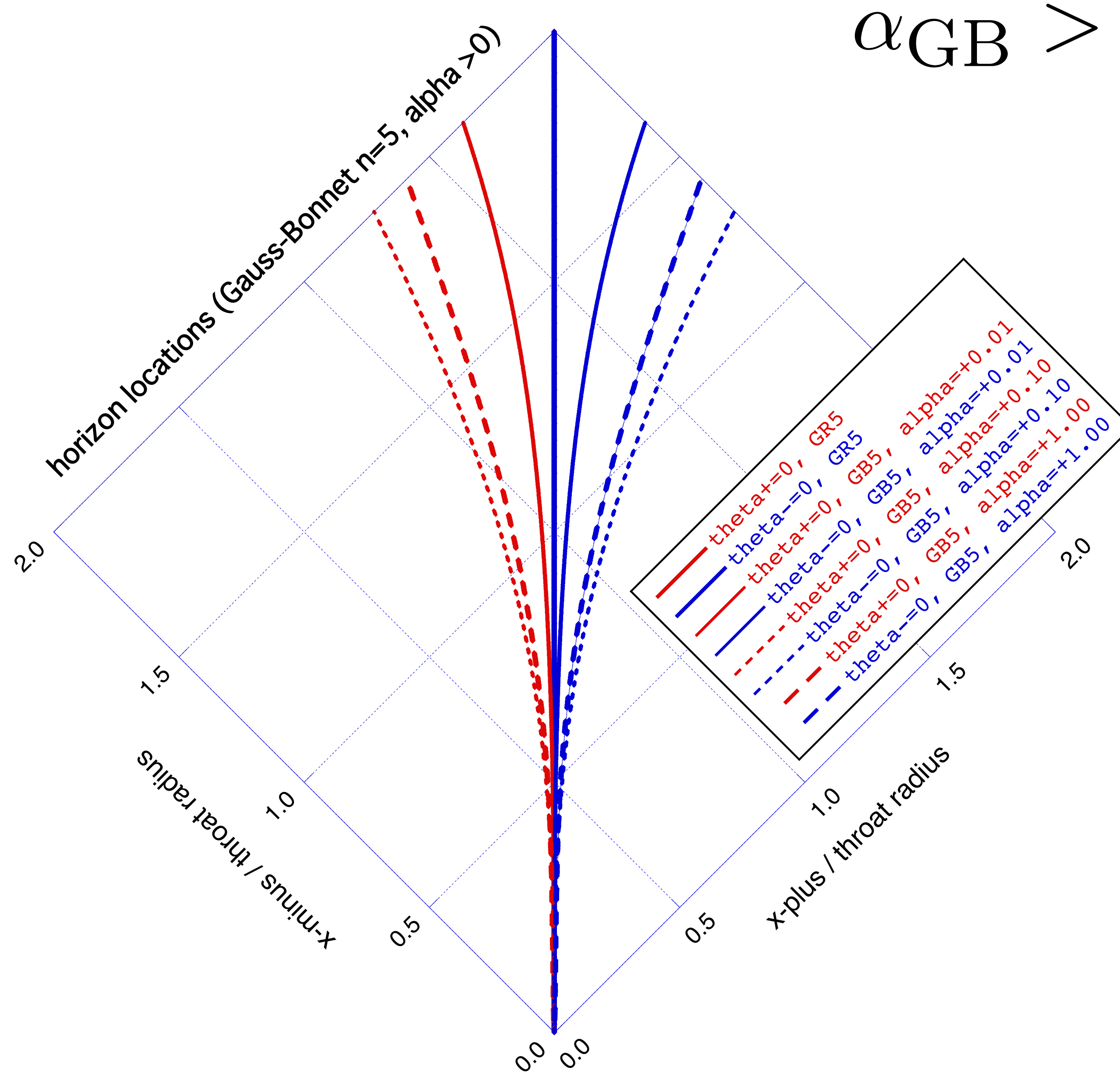
where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

5d GR vs Gauss-Bonnet   instability appears

$\alpha_{\rm GB} < 0$



$\alpha_{\rm GB} > 0$



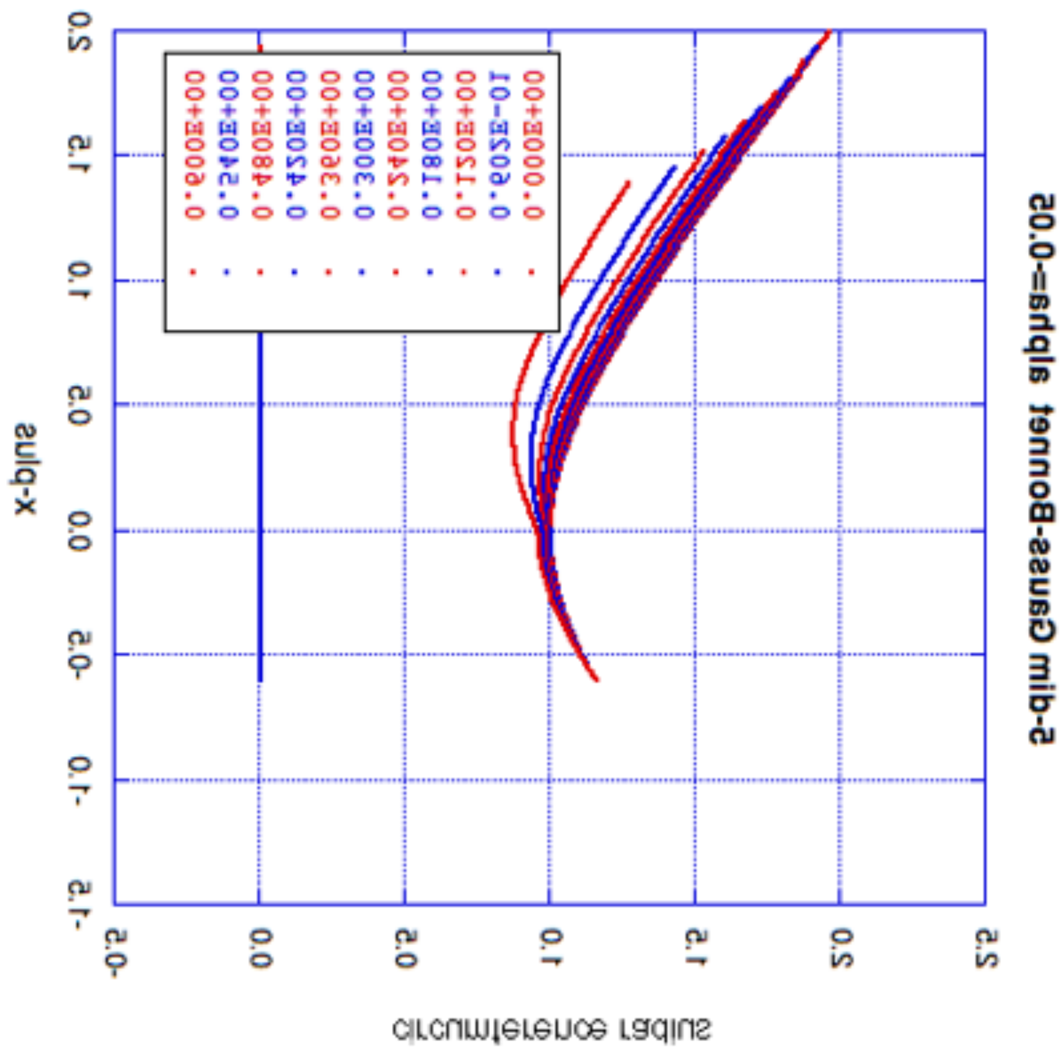
BH formation   throat inflates

$$S = \int_{\mathcal{M}} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1 \mathcal{R} + \alpha_2 \mathcal{L}_{\rm GB}\} + \mathcal{L}_{\rm matter}\Big]$$

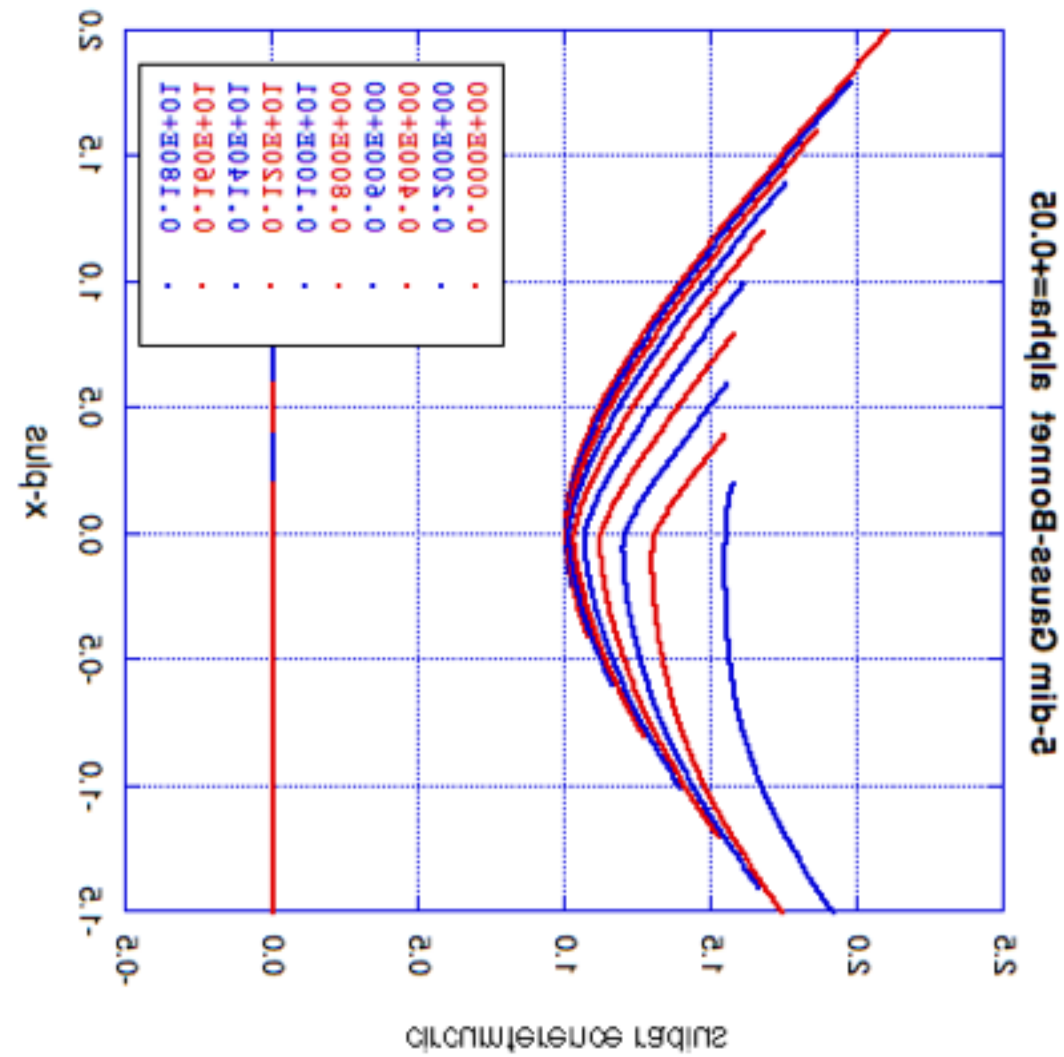
where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

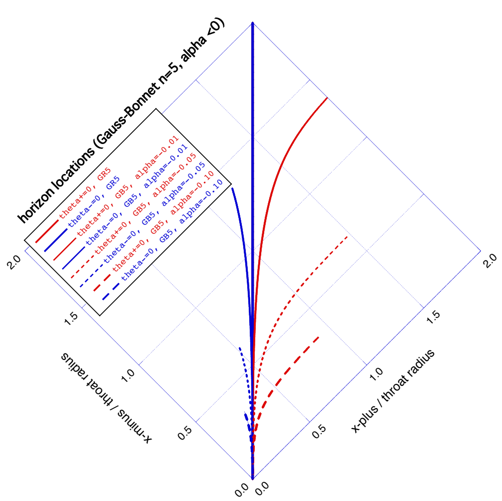
wormhole configurations (5dim. GaussBonnet)

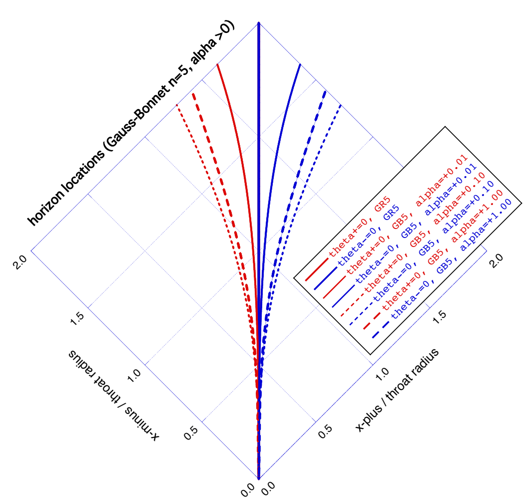
$\alpha_{\mathrm{GB}} < 0$



$\alpha_{\mathrm{GB}} > 0$

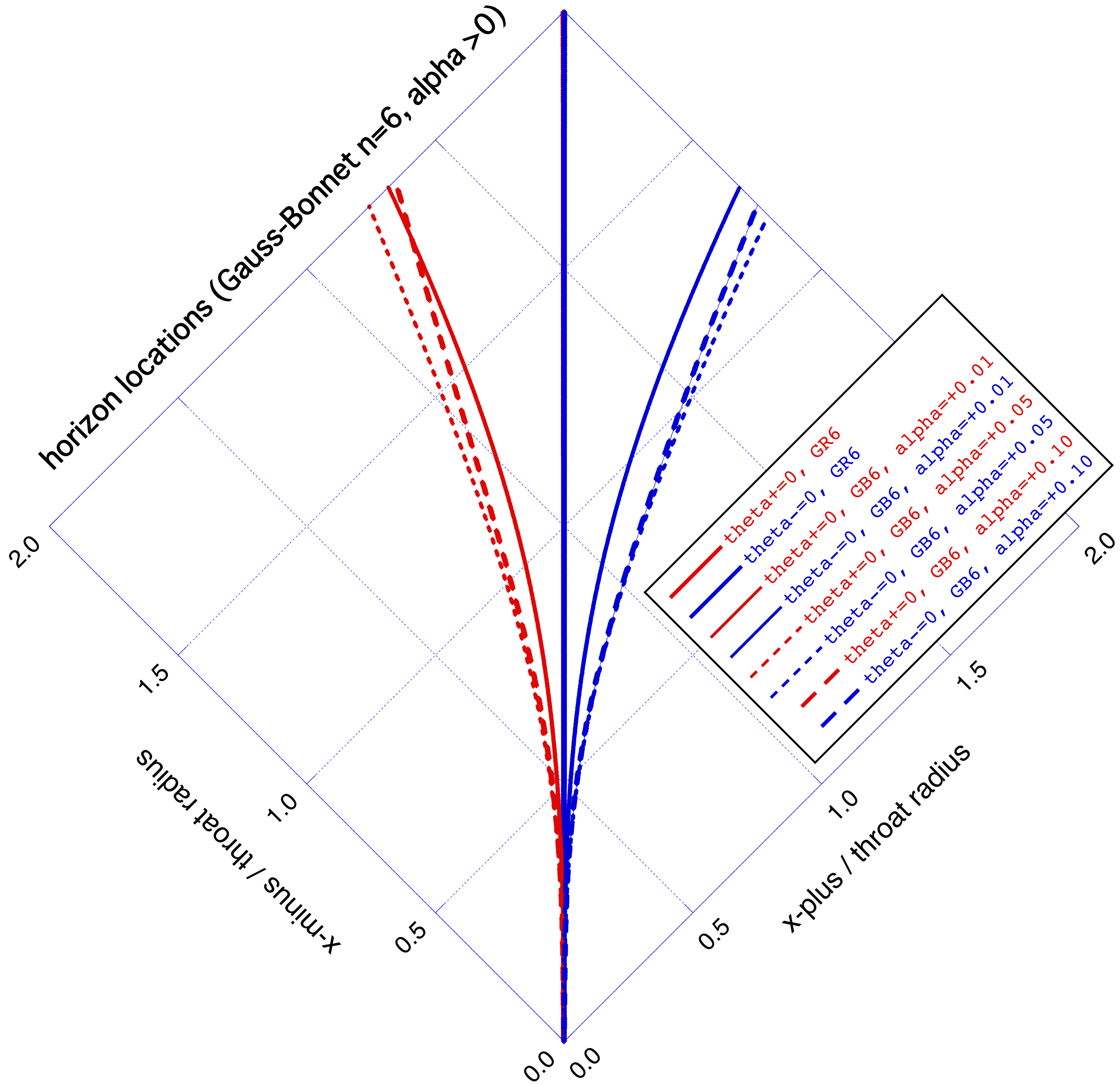






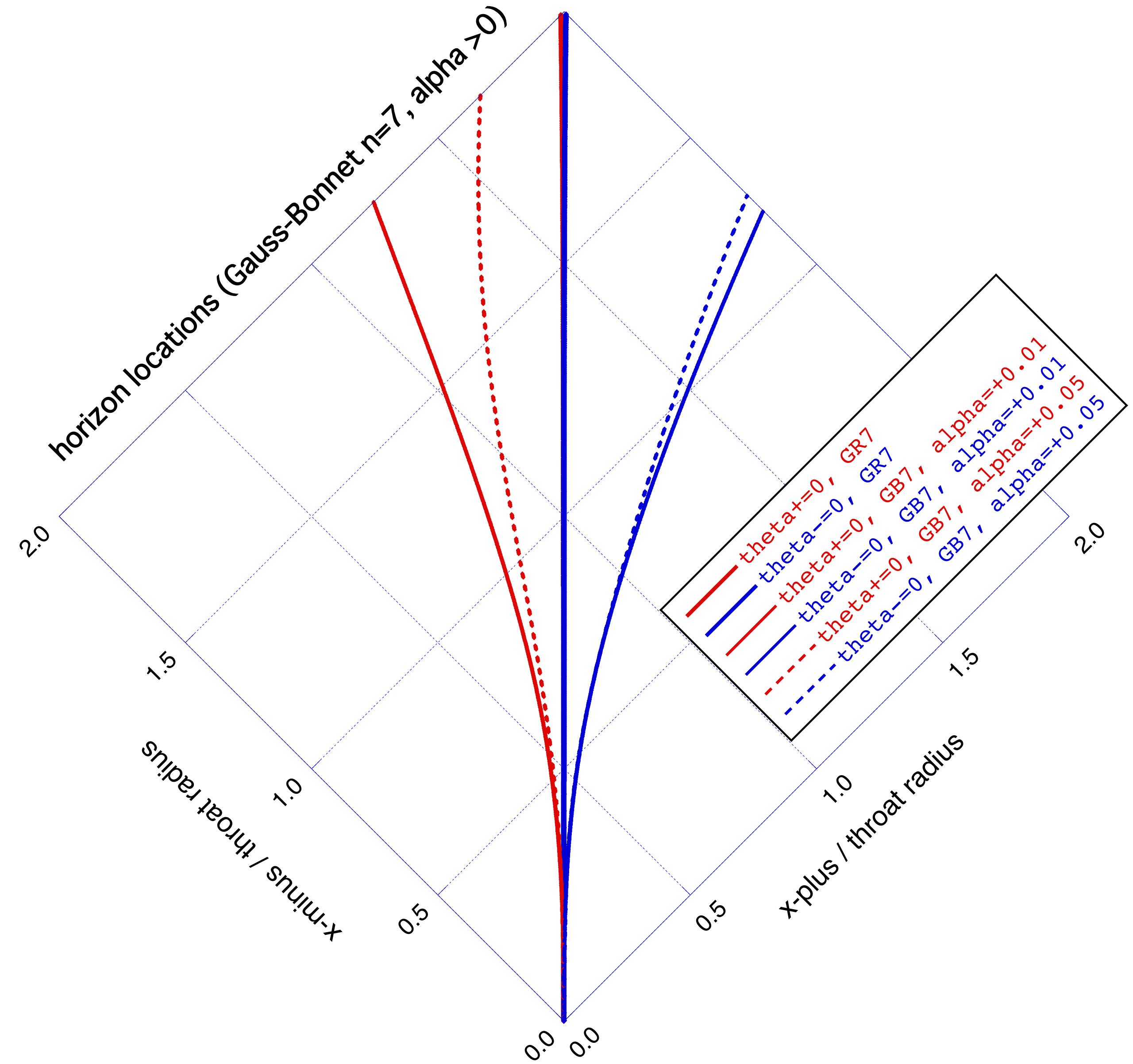
6d 7d Gauss-Bonnet instability appears easily

$\alpha_{\rm GB} > 0$



6 dim.

$\alpha_{\rm GB} > 0$



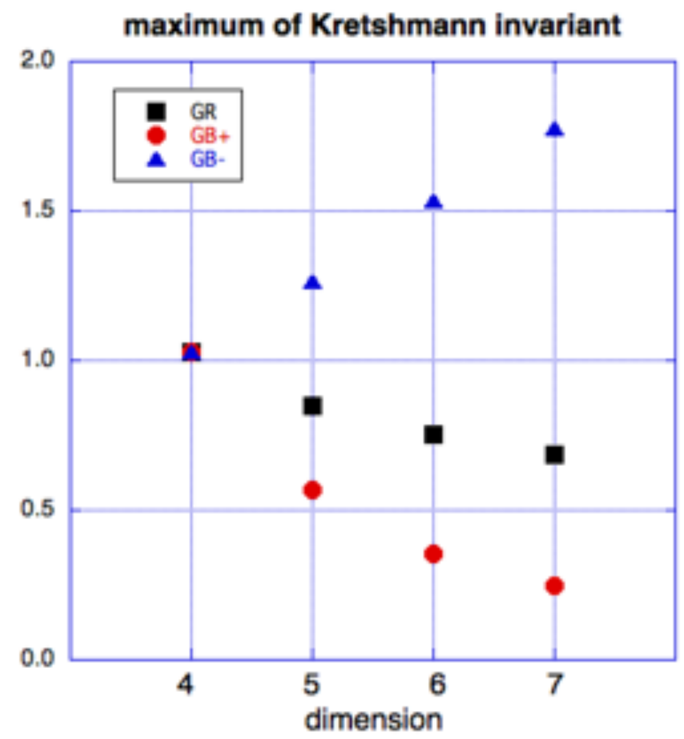
7 dim.

Summary

Colliding Scalar Waves

massless scalar waveの衝突による特異点形成

$\max\ (R_{ijkl}R^{ijkl})$



5,6,7次元 Gauss-Bonnet

＊4dim, 5dim, 6dim,… 高次元化

＊Gauss-Bonnet項（正αの項）

は，どちらも特異点形成条件を緩くさせる

Wormhole Evolution

wormhole解(ghost scalar)に摂動を加える

5,6,7次元 Gauss-Bonnet

負αの GB coupling --> BH collapse

正αの GB coupling --> Inflationary expansion