Visualizations of Sinusoidal Spirals, Limacons of Pascal, and Conic Curves using Equiangular Spirals of Secondary Metallic Ratios

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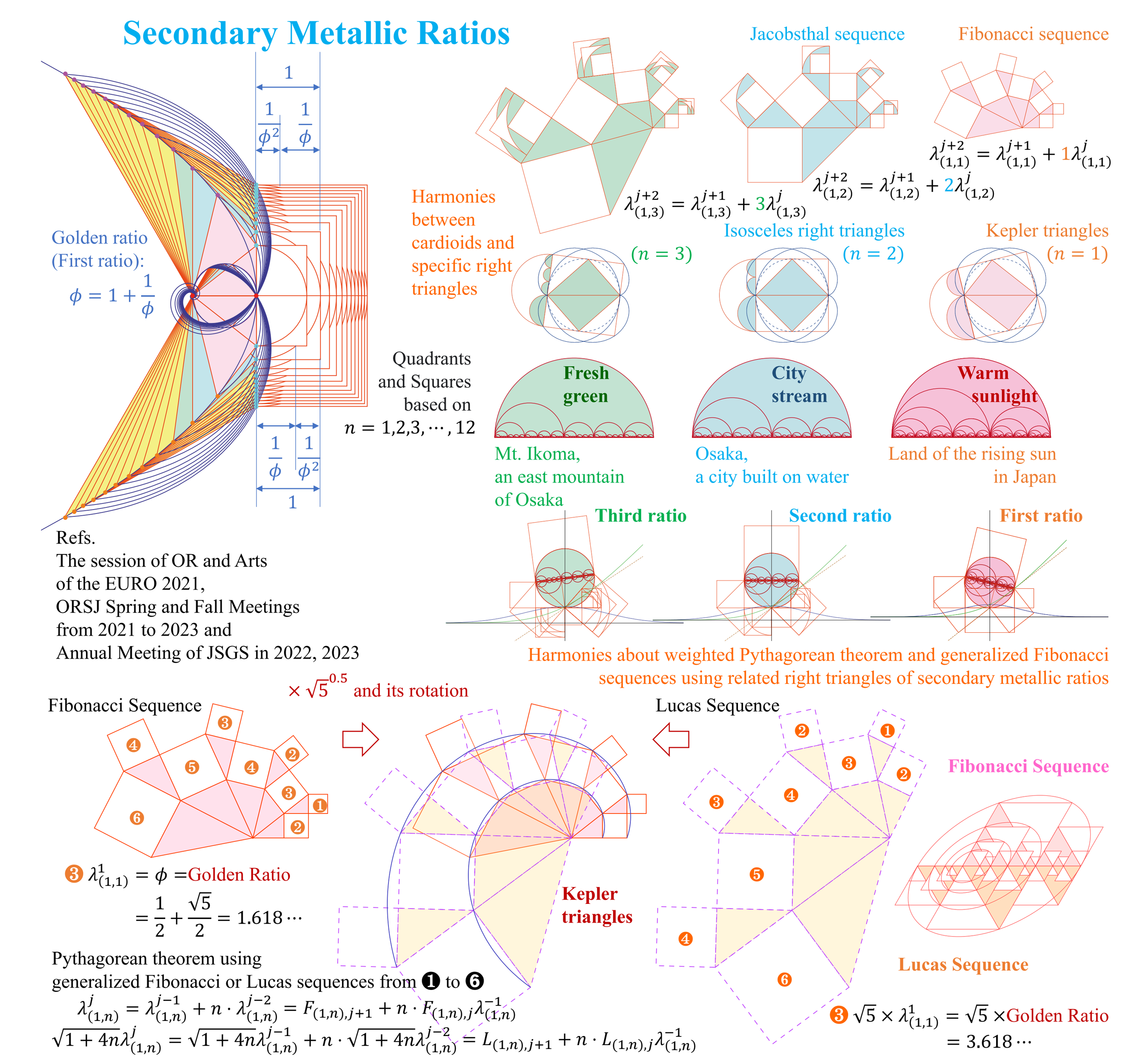
Acknowledgements:
The author, Shingo NAKANISHI, would like to show my grateful to many research members at above academic societies.

Original References:
Operations Research Society of Japan (ORSJ),
European Conference on Operational Research (EURO2021),
Japan Society for Graphical Science (JSGS),
Osaka Institute of Technology in Japan (OIT).

URL: http://www.oit.ac.jp/center/~nakanishi/english/

Secondary Metallic Ratios



Refs.
The session of OR and Arts of the EURO 2021,
ORSJ Spring and Fall Meetings from 2021 to 2023 and
Annual Meeting of JSGS in 2022, 2023

$$\lambda^{1}_{(1,1)} = \phi = \text{Golden Ratio} = \frac{1}{2} + \frac{\sqrt{5}}{2} = 1.618\cdots$$

Pythagorean theorem using generalized Fibonacci or Lucas sequences from ❶ to ❻

$$\lambda^{j}_{(1,n)} = \lambda^{j-1}_{(1,n)} + n\cdot\lambda^{j-2}_{(1,n)} = F_{(1,n),j+1} + n\cdot F_{(1,n),j}\lambda^{-1}_{(1,n)}$$

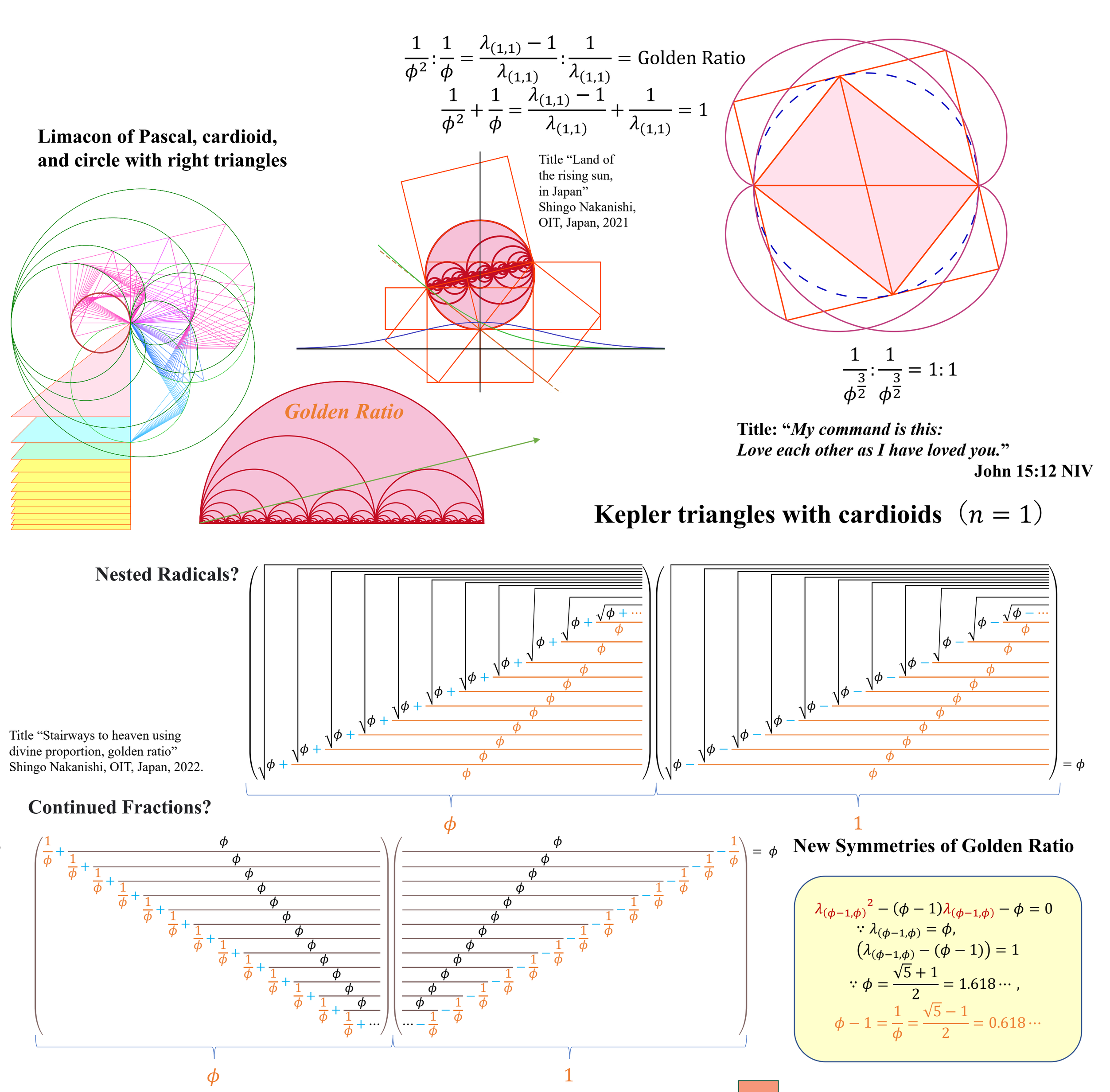
$$\sqrt{1+4n}\lambda^{j}_{(1,n)} = \sqrt{1+4n}\lambda^{j-1}_{(1,n)} + n\cdot\sqrt{1+4n}\lambda^{j-2}_{(1,n)} = L_{(1,n),j+1} + n\cdot L_{(1,n),j}\lambda^{-1}_{(1,n)}$$

$$\sqrt{5}\times\lambda^{1}_{(1,1)} = \sqrt{5}\times\text{Golden Ratio} = 3.618\cdots$$

This study aims to investigate the geometry of secondary metallic ratios proposed by de Spinadel in the poster of this conference. Equiangular spirals and related right triangles using the secondary metallic ratios indicate various fundamental shapes based on sinusoidal spirals coincidentally. The relation between the order of sinusoidal spirals and x-th power of the equiangular spirals should be verified as a simple unique equation precisely. Conic curves such as parabolas, hyperbolas, and ellipses with two types of equiangular spirals create geometrically beautiful harmonies with some sinusoidal spirals such as cardioids, lemniscates, Cayley's sextics, and Tschirnhausen cubics from artistic viewpoint respectively. Similarly, limacons of Pascal including cardioids can be illustrated concisely using the reverses of related right triangles based on equiangular spirals. Even though we explain that in the field of the plane geometry, we can also display the lemniscates with toruses and hyperbolas with cones as 3-dimensinal graphics to confirm the concepts using the equiangular spirals and related right triangles more theoretically.

$$\frac{1}{\phi^2}:\frac{1}{\phi} = \frac{\lambda_{(1,1)}-1}{\lambda_{(1,1)}}:\frac{1}{\lambda_{(1,1)}} = \text{Golden Ratio}$$

$$\frac{1}{\phi^2}+\frac{1}{\phi} = \frac{\lambda_{(1,1)}-1}{\lambda_{(1,1)}}+\frac{1}{\lambda_{(1,1)}} = 1$$



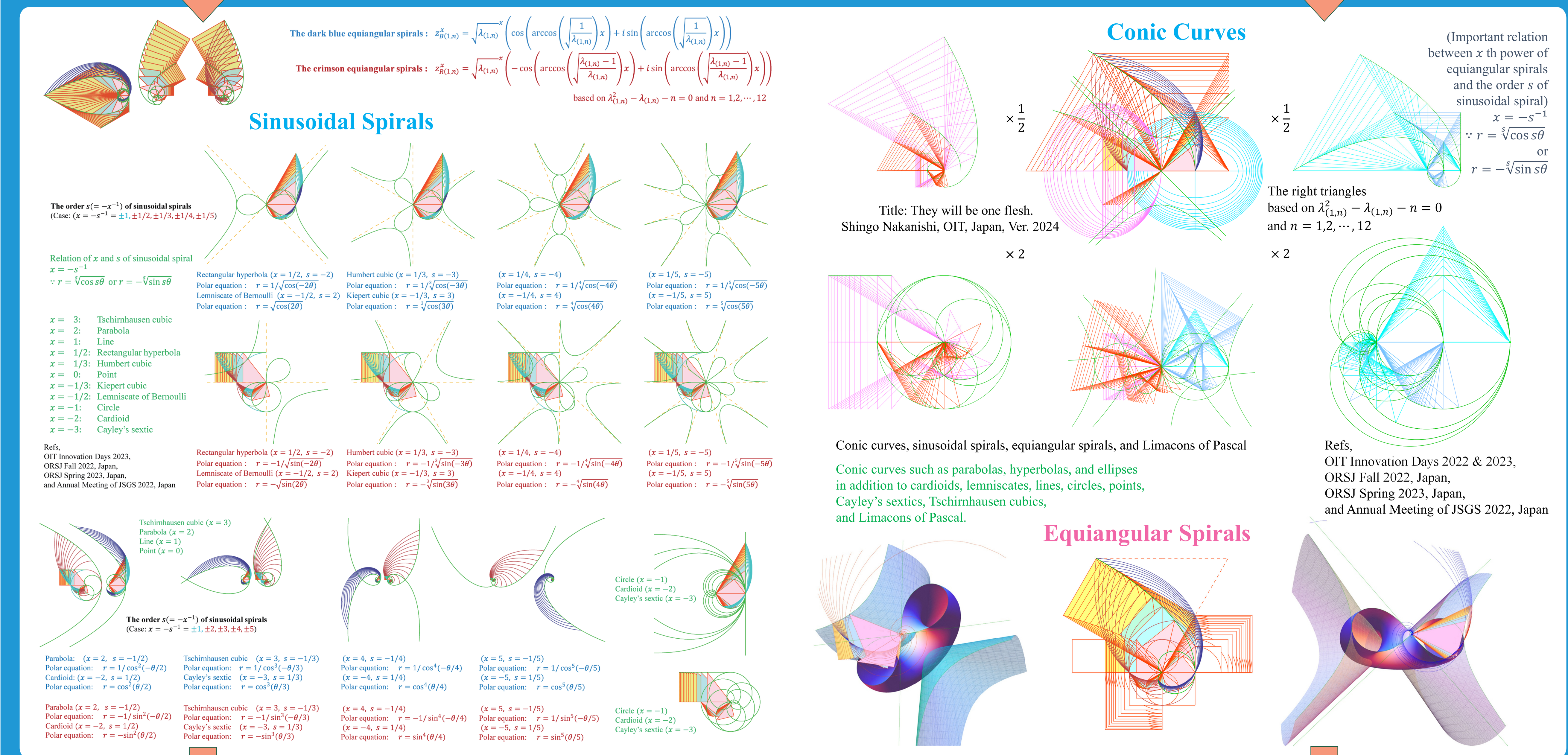
Sinusoidal Spirals

The dark blue equiangular spirals: $z^{x}_{B(1,n)} = \sqrt{\lambda_{(1,n)}}^{\,x}\left(\cos\left(\arccos\left(\frac{1}{\sqrt{\lambda_{(1,n)}}}\right)x\right) + i\sin\left(\arccos\left(\frac{1}{\sqrt{\lambda_{(1,n)}}}\right)x\right)\right)$

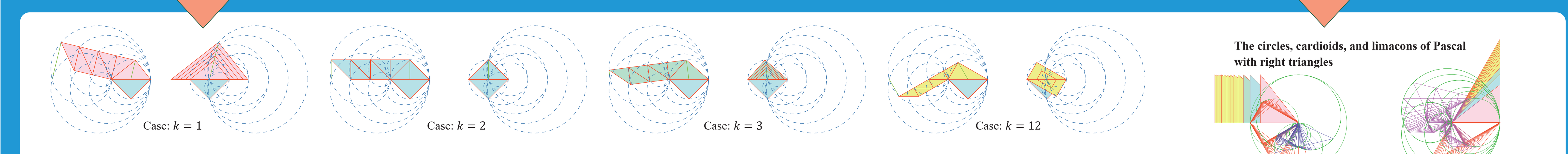
The crimson equiangular spirals: $z^{x}_{R(1,n)} = \sqrt{\lambda_{(1,n)}}^{\,x}\left(-\cos\left(\arccos\left(\sqrt{\frac{\lambda_{(1,n)}-1}{\lambda_{(1,n)}}}\right)x\right) + i\sin\left(\arccos\left(\sqrt{\frac{\lambda_{(1,n)}-1}{\lambda_{(1,n)}}}\right)x\right)\right)$

Conic Curves

Equiangular Spirals



Limacons of Pascal



Comparisons of drawing methods using right triangles of similar metallic ratios about $\left(\frac{\cos\theta(1+k\cos\theta)}{2} + \frac{1-k}{2}, \frac{\sin\theta(1+k\cos\theta)}{2}\right)$ and $\left(\frac{\cos\theta(1+k\cos\theta)}{2}, \frac{\sin\theta(1+k\cos\theta)}{2}\right)$

Geometric Characterizations and Symmetric Relations between Standard Normal Distribution and Inverse Mills Ratio based on Pythagorean Theorem

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Acknowledgements: The first author, Shingo NAKANISHI, would like to show my grateful to many research members at Operations Research Society of Japan, Kansai-tiku Koryukai of The Securities Analysts Association of Japan, Osaka Institute of Technology in Japan, and Osaka University in Japan.

Original References: Operations Research Society of Japan (ORSJ),
Research Institute for Mathematical Sciences, Kyoto University (RIMS Kôkyûroku, 2078-10),
The 15th International Symposium on Econometric Theory and Applications (SETA2019).

URL: http://www.oit.ac.jp/center/~nakanishi/english/

0. Background

We present the geometric characterizations and symmetric relations between standard normal distribution and inverse Mills ratio by circles and squares from the viewpoint with considering the height of densities such as ancient Egyptian drawing styles and using the Greek Pythagorean Theorem.

First, we can clarify the integral forms of various cumulative distribution functions including standard normal distribution based on the aspect ratio (=1.0, see 1).

Second, we reconsider what the several times of standard deviations multiplied by the square root of the time mean with their positive and negative expectations multiplied by the time under the condition the aspect ratio=1.0 (see 2). At this time, we can understand the following things.

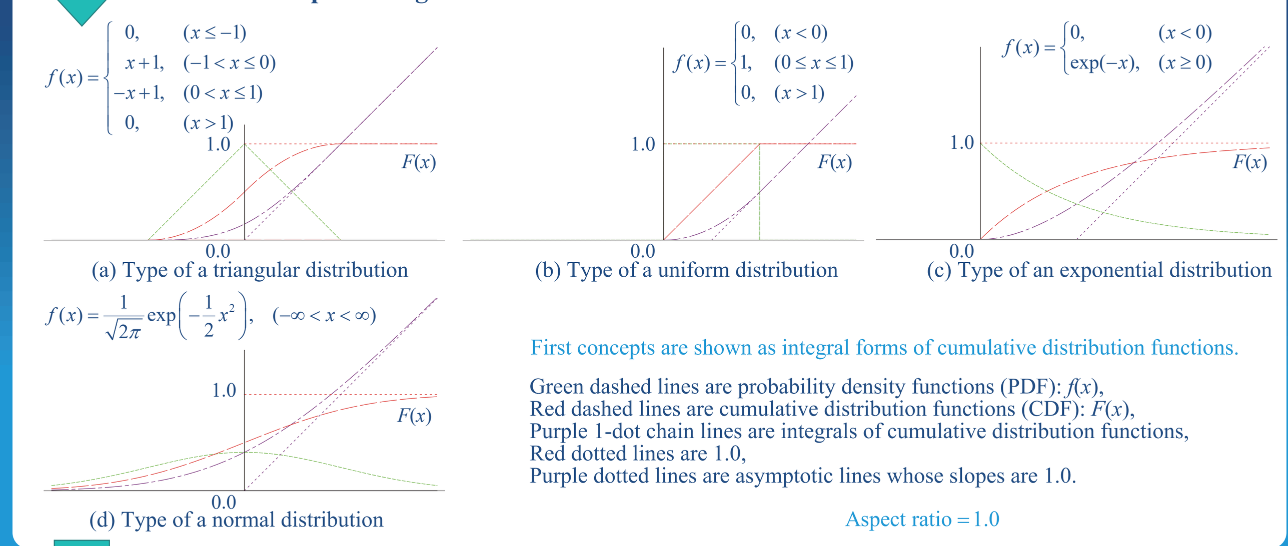
(1) We can find the equilibrium formulation at any real numbers of the time t (see 2).
(2) Its constant number, 0.612003, was found by Karl Pearson about 100 years ago.
(3) Sir David Roxbee Cox reconfirmed the value to cluster the normal distribution.
(4) Truman Lee Kelley also proposed the 27 percent rule formulation.

Third, we can show the parabola for maximal profits including fees. And that is equal to its fee based on the 27 percent probability. In addition to these tendencies, we can clarify the relations between winners, losers, and their banker (see 3).

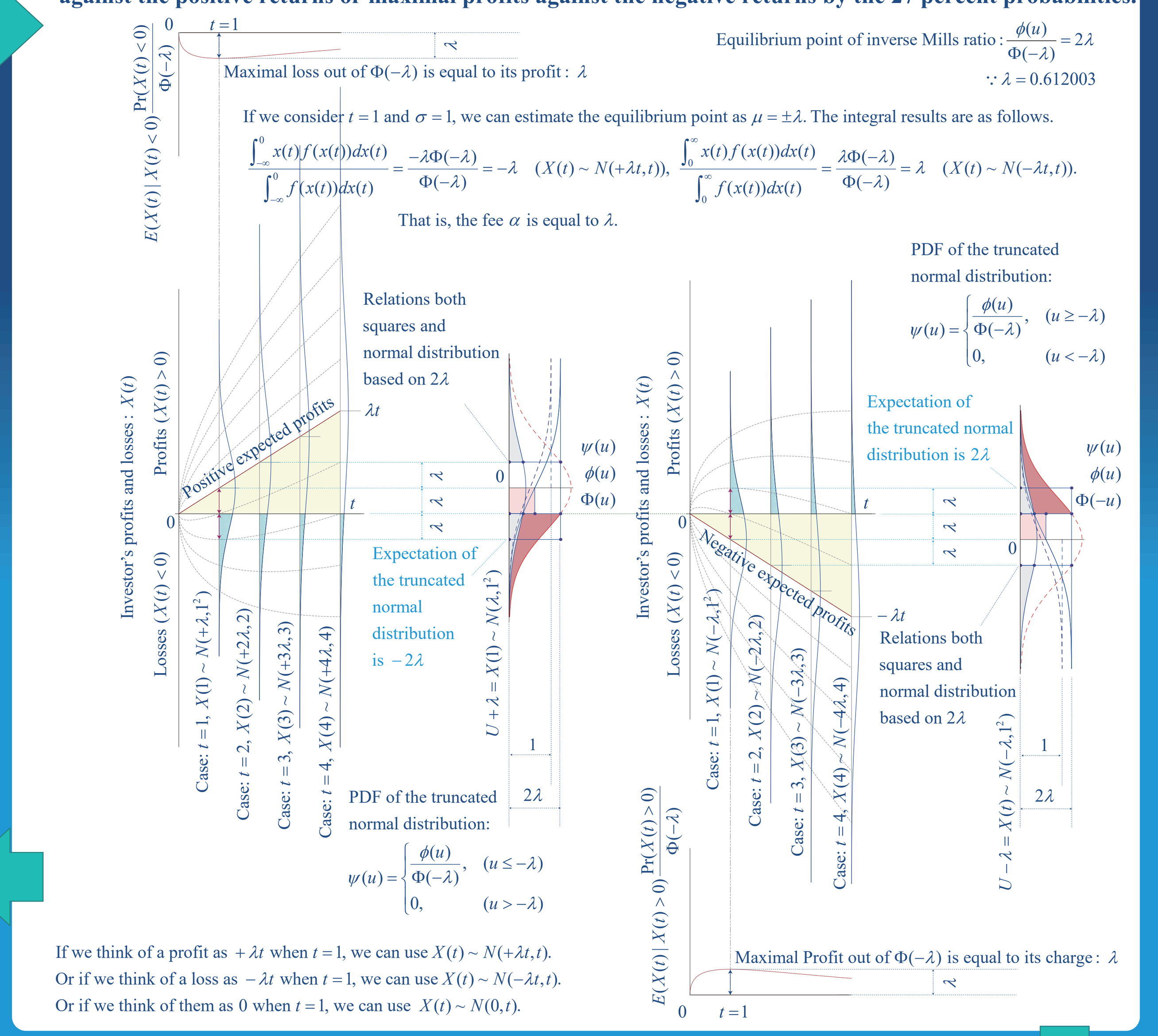
Fourth, we can understand two types of ordinary differential equations to explain that with circles and squares (see 4). One is the Bernoulli differential equation of inverse Mills ratio. The other is the second order linear differential equation of the integral of cumulative normal distribution function (see 1). From these tendencies, we can also get the modified intercept forms geometrically and symmetrically for maximal profits of winners, these losses of losers, and their banker's fee. We can understand that these equations should be changing the probability points and these probabilities based on Pythagorean Theorem correctly. If the probability point is that found by Karl Pearson, we can show you that it is the special point of standard normal distribution such as Sir Cox's proposal. Other point on the right triangle with 3:4:5 is also the third quantile of standard normal distribution.

Finally, we can also realize there are many similar tendencies close to the relations between circles and squares such as Vitruvian man by Da Vinci and various Mandalas although there might not be related to normal distribution directly and historically. The ancient Egyptian drawing styles enable us to illustrate the geometric characterizations and symmetric relations between standard normal distribution and inverse Mills ratio with circle and square based on the Pythagorean theorem in the ancient Greece. We think that our ideas shall be contributed in the statistical modelling and these evaluation fields since our suggested figures should be much more easily understood and powerful than we thought.

1. First concept as integrals of various cumulative distributions

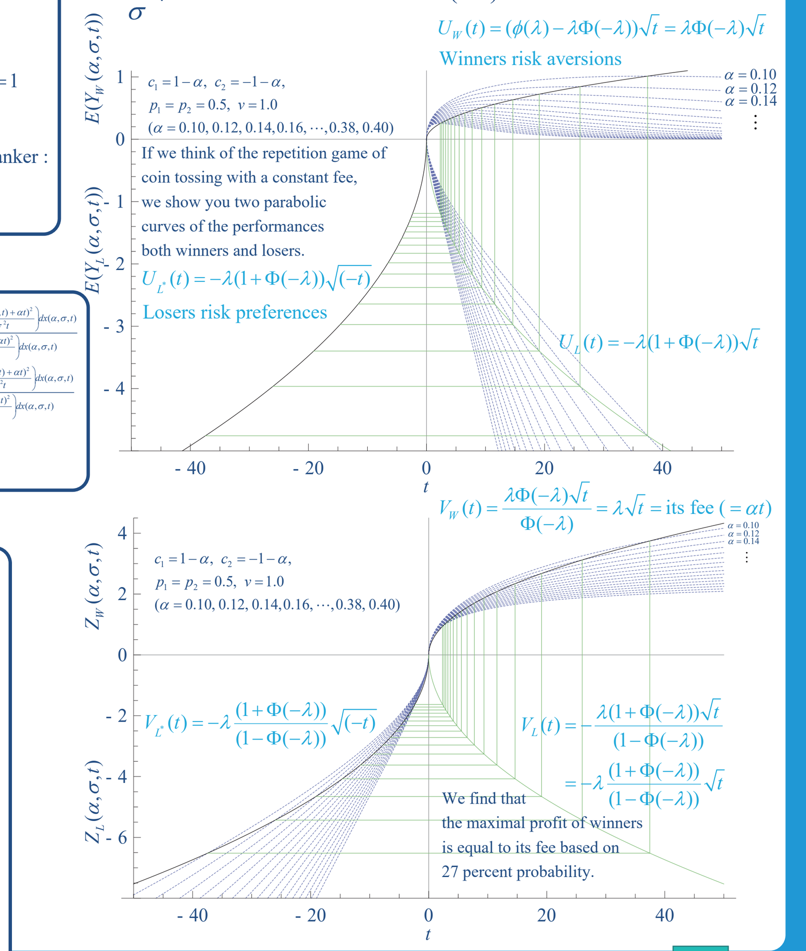


2. Second concept as distances of probability points based on the time t. There should be maximal losses against the positive returns or maximal profits against the negative returns by the 27 percent probabilities.

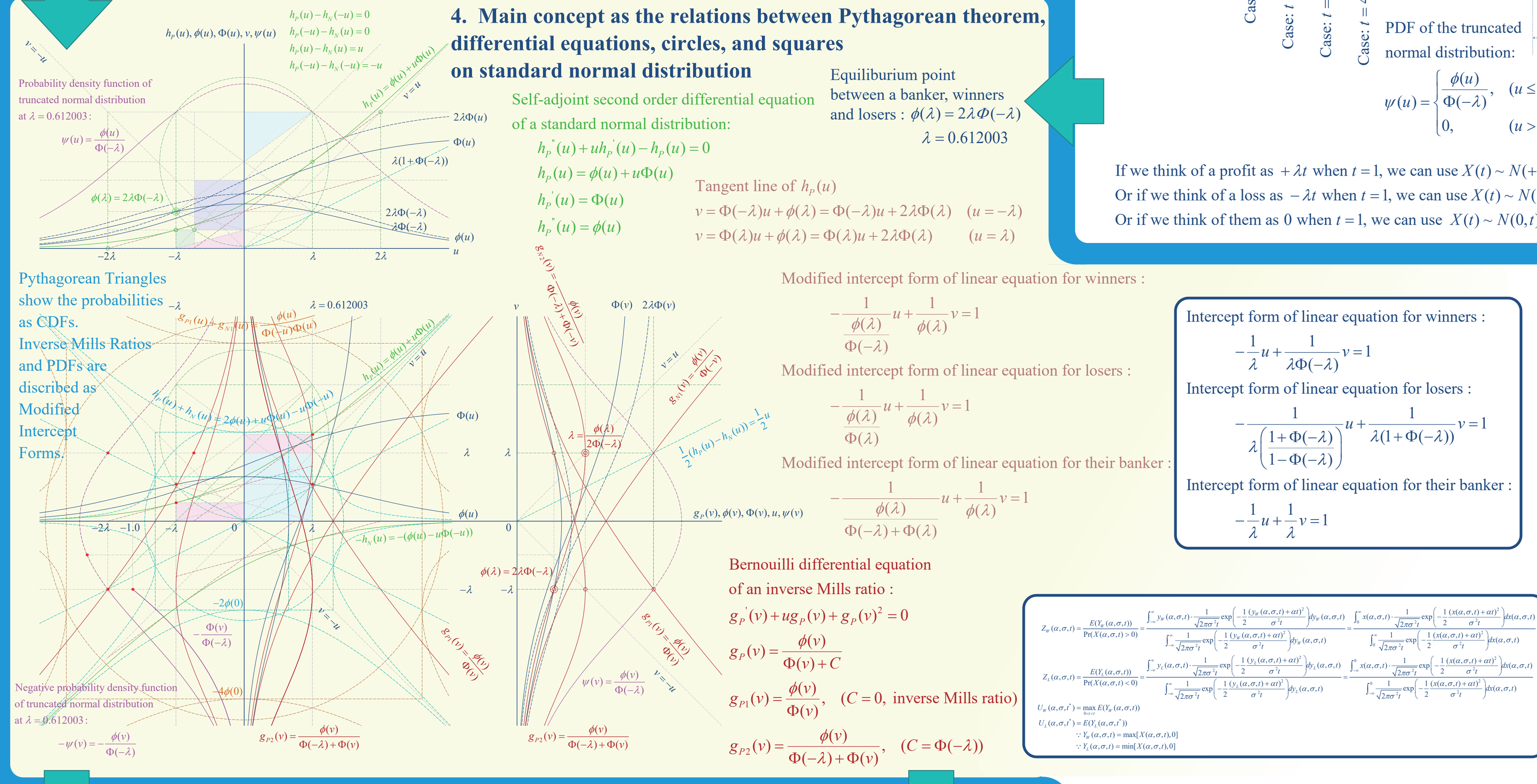


3. Relations between winners, losers, and their banker by the equilibrium formulation

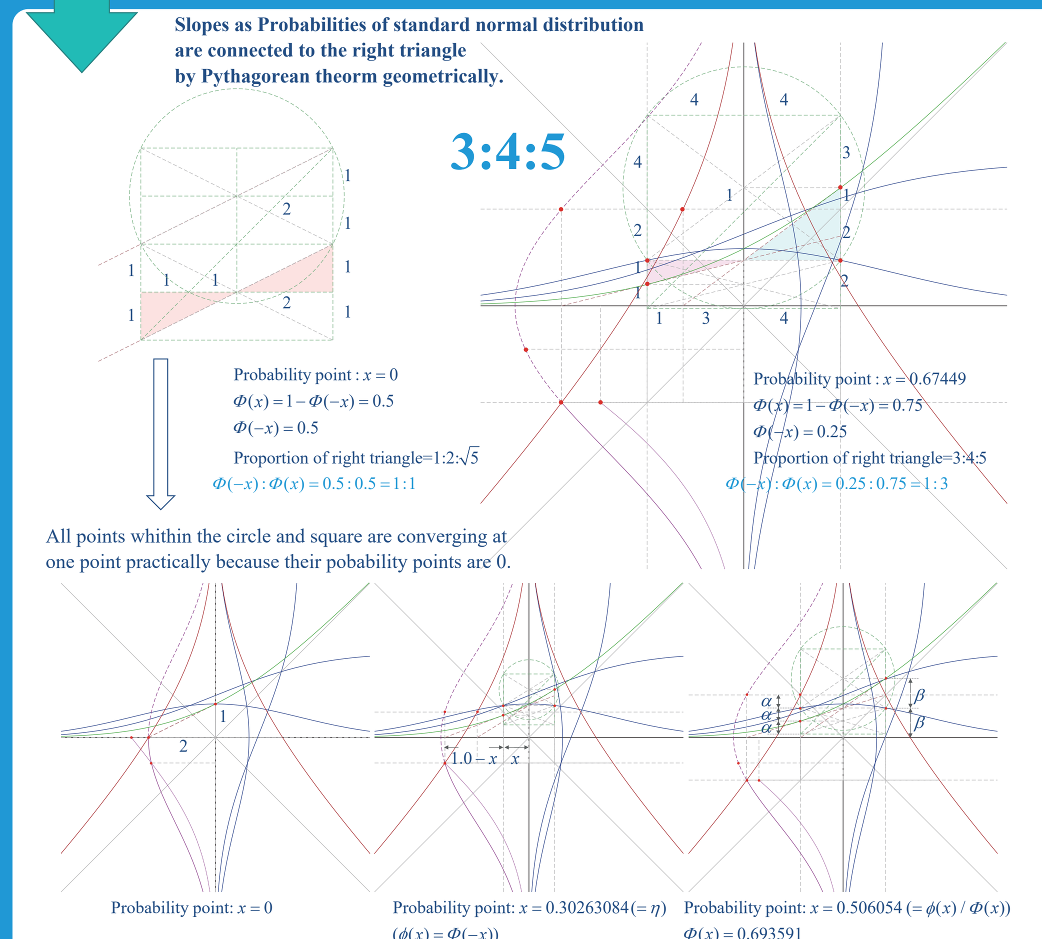
$\frac{\alpha}{\sigma}\sqrt{t} = \lambda = 0.612003$ and $\Phi(-\lambda) = 0.2702678$

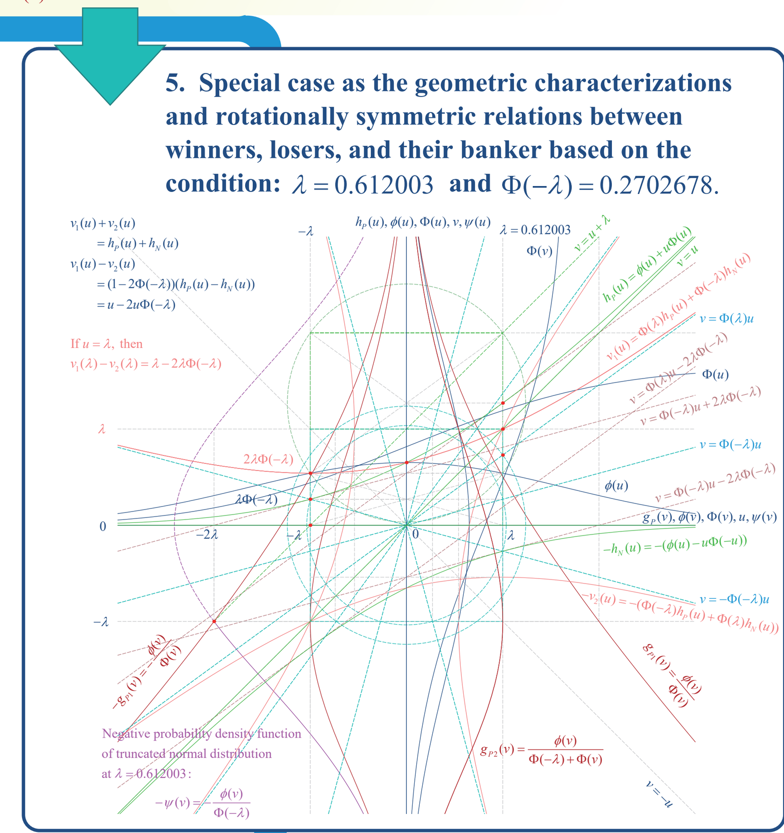


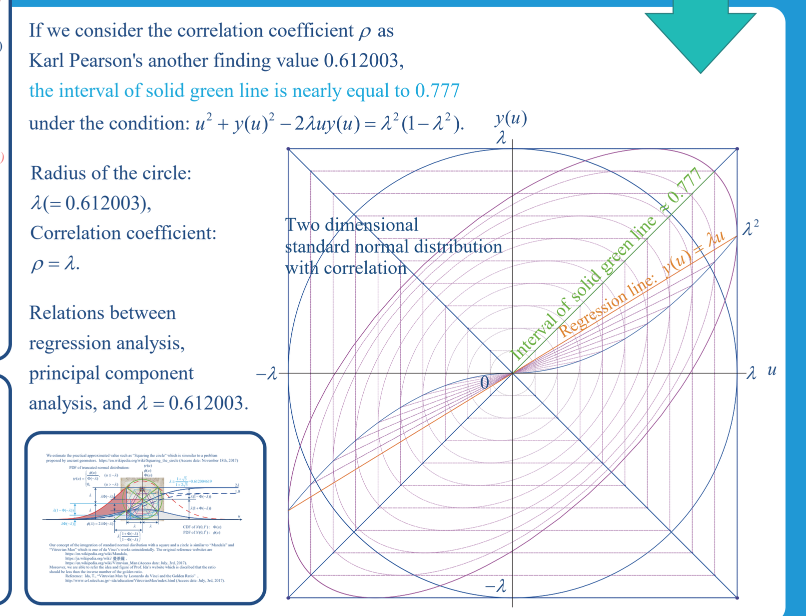
4. Main concept as the relations between Pythagorean theorem, differential equations, circles, and squares on standard normal distribution



5. Special case as the geometric characterizations and rotationally symmetric relations between winners, losers, and their banker based on the condition: $\lambda = 0.612003$ and $\Phi(-\lambda) = 0.2702678$.







Concluding Remarks

We can understand that the slopes should be the probabilites of standard normal distribution based on the probability points. And above numbers are also the special probability points.

MP4 Visual Animation

PowerPoint Visual Animation