About the title

"Geometric Characterizations of Standard Normal Distribution

- Two Types of Differential Equations, Relationships with Square and Circle,

and Their Similar Characterizations -",

we will modify some things as follows to contribute mathematics and sciences precisely on November 14, 2018.

Important modification:

Present modified version: $h_2(u) = h_P(u)$,

$$\phi(u)^{-1}(h_P''(u) + uh_P'(u) - h_P(u)) = 0.$$

Previous misspelled version: $h_2(u) = m_P(u)$,

$$\phi(u)\big(m_P''(u) - um_P'(u) - m_P(u)\big) = 0.$$

The other equation in this paper: $g(u) = g_P(u)$,

$$g_P'(u) + ug_P(u) + g(u)^2 = 0.$$

If we think of three equations as follows

$$h_P(u) = \phi(u) + u\Phi(u),$$

$$m_P(u) = \frac{\Phi(u)}{\phi(u)},$$

$$g_P(u) = \frac{\phi(u)}{\Phi(u)},$$

these differential equations are expressed as

$$\phi(u)^{-1}(h_P''(u) + uh_P'(u) - h_P(u)) = 0,$$

$$\phi(u)\big(m_P''(u) - um_P'(u) - m_P(u)\big) = 0,$$

$$g_P'(u) + ug_P(u) + g(u)^2 = 0.$$

The upper two of the equations are self-adjoint differential equations.

And we find that there is the following relationship between above three equations $g_P(u)$, $h_P(u)$ and $m_P(u)$. That is,

$$-\frac{g_P'(u)}{g_P(u)} = \frac{h_P(u)}{h_P'(u)} = \frac{m_P'(u)}{m_P(u)}.$$

I have a plan to speak about them on November 22, 2018 to be modified.

Self-adjoint differential equation: $\phi(u)^{-1}(h_2''(u)+uh_2'(u)-h_2(u))=0$

$$\left(\phi(u)^{-1}h_2'(u)\right)'-\left(\phi(u)^{-1}h_2(u)\right)=0$$

Variable coefficient second order linear homogeneous differential equation for a standard normal distribution : $\dfrac{d^2h_2(u)}{du^2}+u\dfrac{dh_2(u)}{du}-h_2(u)=0,$

$$h_2(0)=\frac{1}{\sqrt{2\pi}}(=\phi(0)),$$

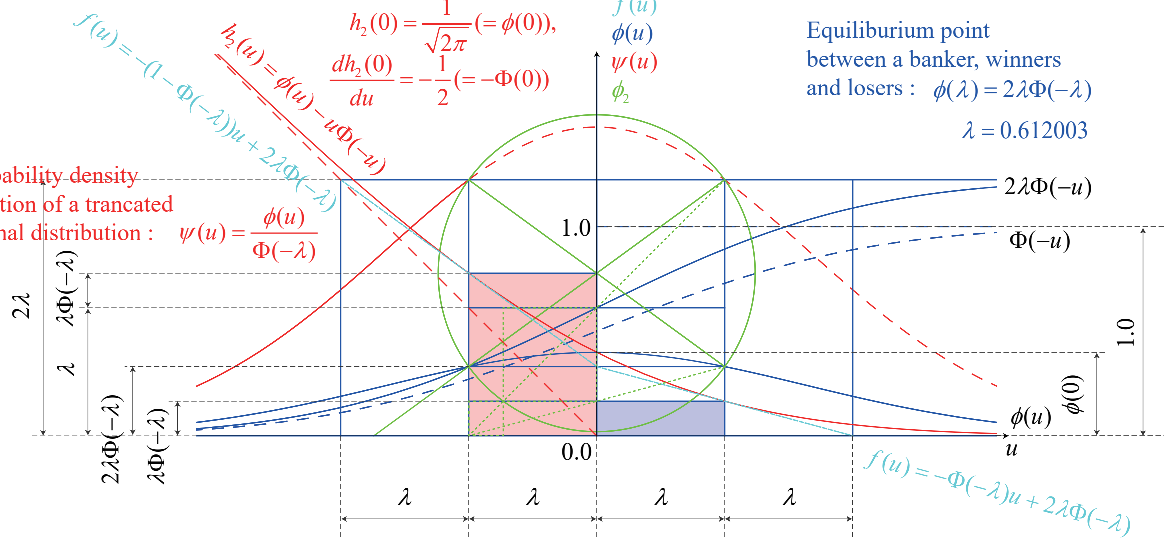
$$\frac{dh_2(0)}{du}=-\frac{1}{2}(=-\Phi(0))$$

$$\frac{dh_2(u)}{du}=-\Phi(-u) \qquad \frac{d^2h_2(u)}{du^2}=\phi(u)$$

Equilibrium point between a banker, winners and losers : $\phi(\lambda)=2\lambda\Phi(-\lambda)$

$\lambda=0.612003$

Probability density function of a trancated normal distribution : $\psi(u)=\dfrac{\phi(u)}{\Phi(-\lambda)}$



Tangential equation : $f(u)=\begin{cases}-(1-\Phi(-\lambda))u+2\lambda\Phi(\lambda) & (-2\lambda\le u\le 0)\\ -\Phi(-\lambda)u+2\lambda\Phi(\lambda) & (0\le u\le 2\lambda)\end{cases}$

Probability density function of a standard normal distribution : $\phi(u)$

Cumulative distribution function of a standard normal distribution : $\Phi(-u)$

Equilibrium point of an inverse Mills ratio : $\dfrac{\phi(\lambda)}{\Phi(-\lambda)}=2\lambda$

Intercept form of a linear equation for winners : $-\dfrac{1}{\lambda}u+\dfrac{1}{\lambda\Phi(-\lambda)}\phi_2=1$

Intercept form of a linear equation for losers : $-\dfrac{1}{\lambda\left(\dfrac{1+\Phi(-\lambda)}{1-\Phi(-\lambda)}\right)}u+\dfrac{1}{\lambda(1+\Phi(-\lambda))}\phi_2=1$

Intercept form of a linear equation for a banker : $-\dfrac{1}{\lambda}u+\dfrac{1}{\lambda}\phi_2=1$

Utility function for winners :

$U_W(t)=(\phi(\lambda)-\lambda\Phi(-\lambda))\sqrt{t}=\lambda\Phi(-\lambda)\sqrt{t}$

Modified Version of our study,

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and Their Similar Characterizations -” ,

http://www.kurims.kyoto-u.ac.jp/~kyodo/kokyuroku/contents/pdf/2078-10.pdf