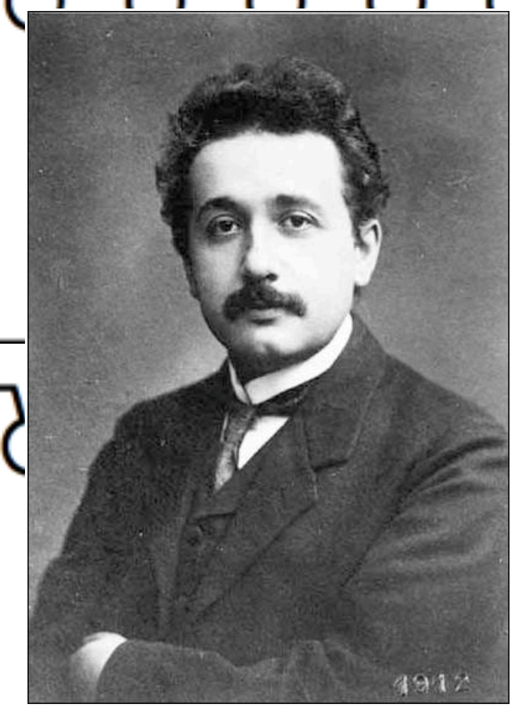
International School of Science 2016 August 6 (Sat)

Why & How we know there are black-holes?
Introduction to Einstein's Relativity

Hisaaki Shinkai

1 **Special Relativity: Theory of time**

2 **General Relativity: Theory of gravity**



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History of Physics (1)

1905

1600 1700 1800 1900

Astronomy

Kepler
Gelilei

Optics

Mechanics

Newton

Thermodynamics

Fluiddynamics

Electromagnetism

Faraday
Maxwell

Physics is completed!?

History of Physics (2)

1905

1915 | 1925 | 1960s | 1980s

General Relativity

Special Relativity

Brown motion

photo-electric effect

Statistical Mechanics

Quantum Mechanics

Physical Chemistry

Complex Systems

Econo-Physics

Material Physics

Biological Physics

Nuclear Theory

Field Theory

Particle Theory

0.1 Kepler's laws of planetary motion

Johannes Kepler discovered the following laws on the motion of planets.

Kepler's law (1609, 1618)

1st law	The orbit of a planet is an ellipse with the Sun at one of the two foci.
2nd law	A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3rd law	The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

Figure 1: Johannes Kepler (1571-1630)

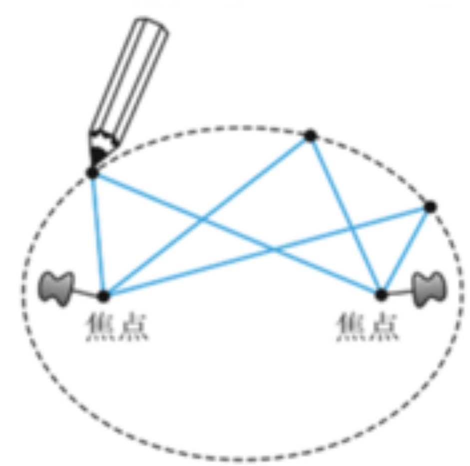


Figure 2: An ellipse and two foci.

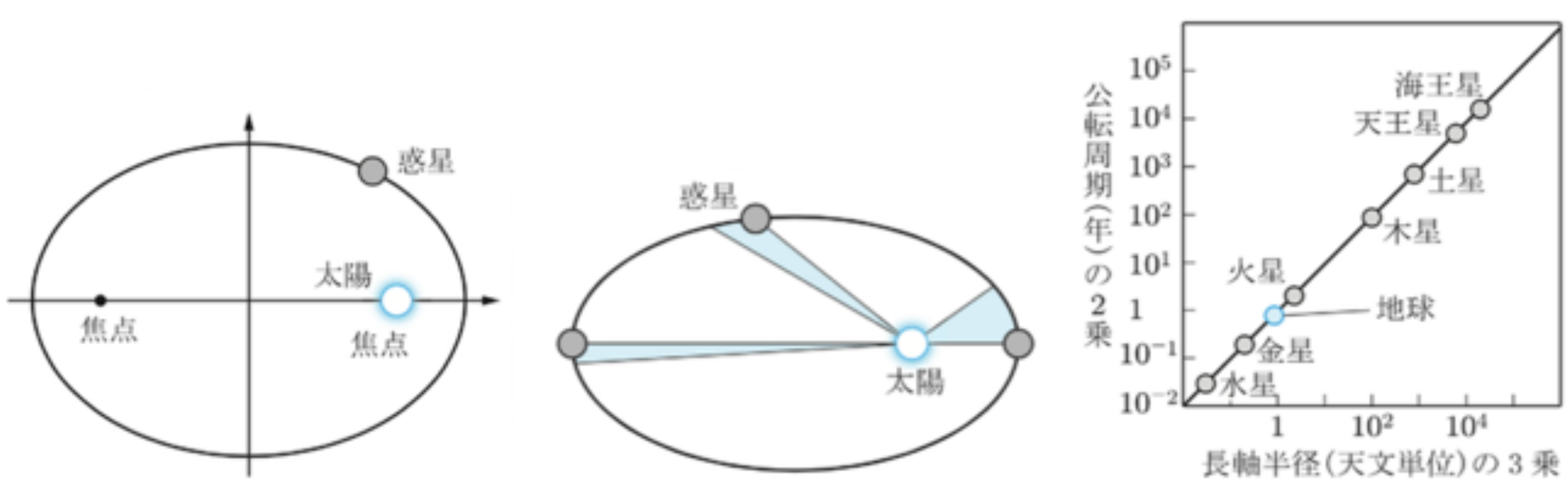


Figure 3: Kepler's first, second and third law of planetary motion.

Newton's law

0.2 Newton's laws of motion

Isaac Newton formulated the basic laws of the motion.

Figure 4: Isaac Newton (1642–1726)

Newton's law of motion (1687)

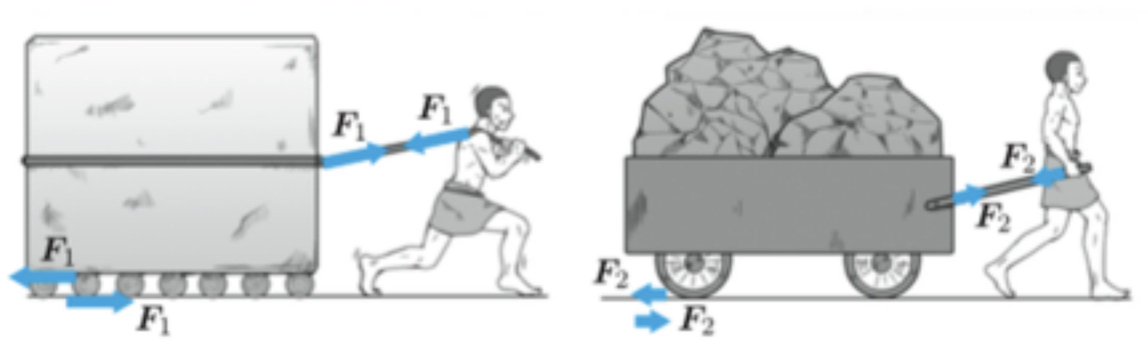
1st law — If there is no net force, an object either remains at rest or continues to move at a constant velocity.

2nd law — The force $\boldsymbol{F}$ produces the acceleration $\boldsymbol{a}$ to an object ([m/s^2]). The relation (“**equation of motion**”) is

$$\boldsymbol{F} = m\boldsymbol{a}, \tag{0.1}$$

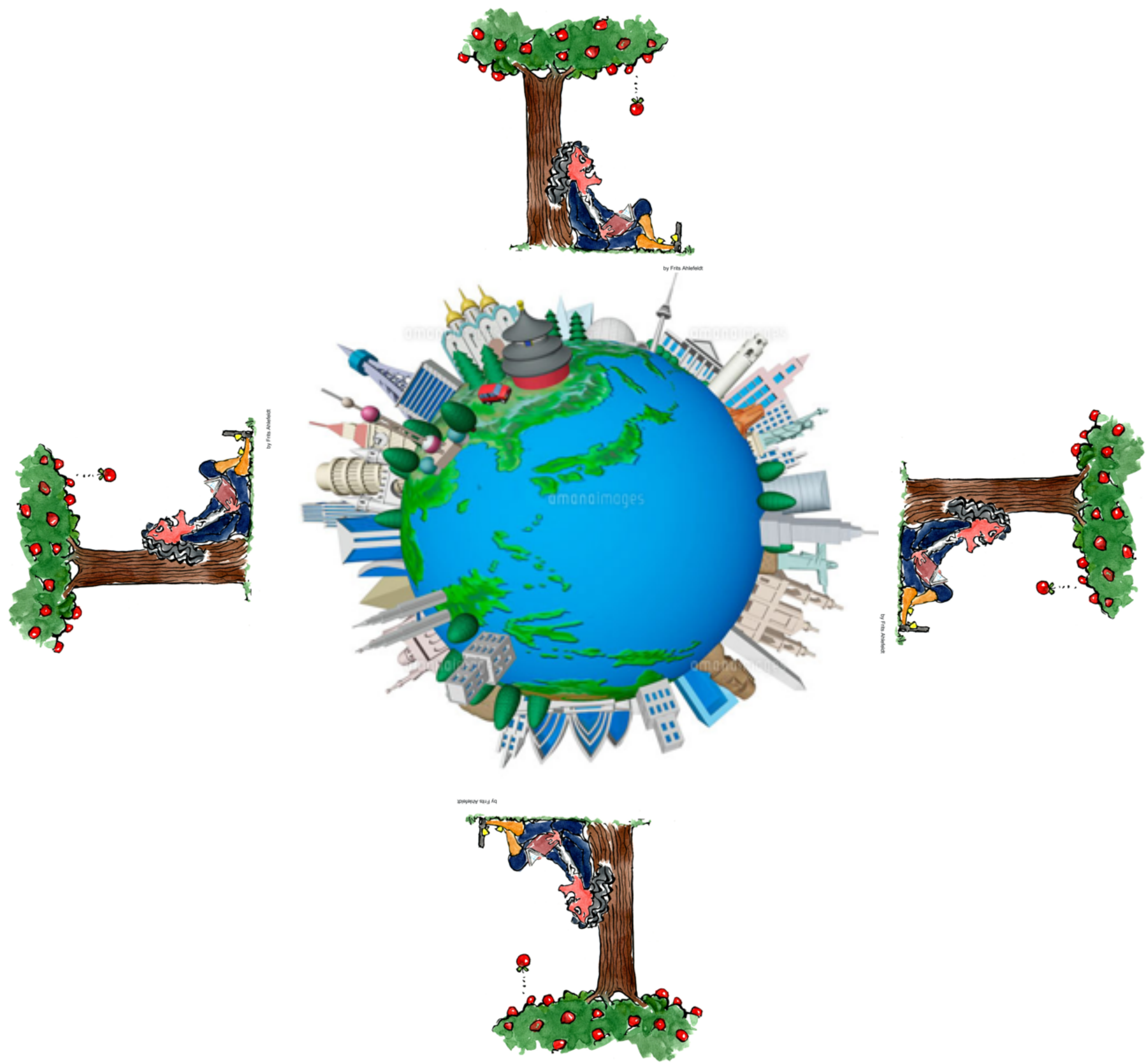
where m is the mass of the object ([kg]).

3rd law — When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.



Gravity？

http://hikingartist.com/



Gravity？

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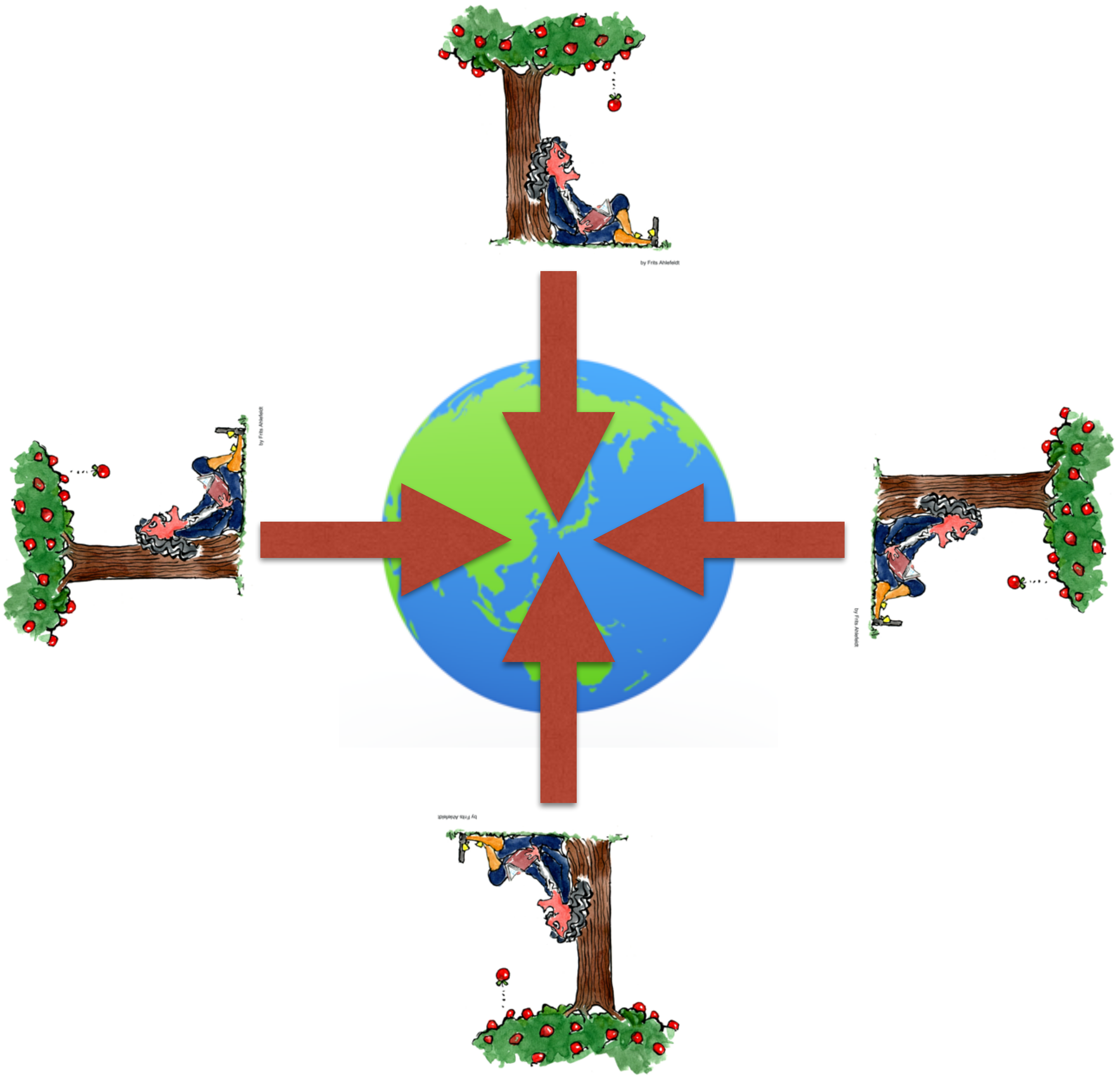
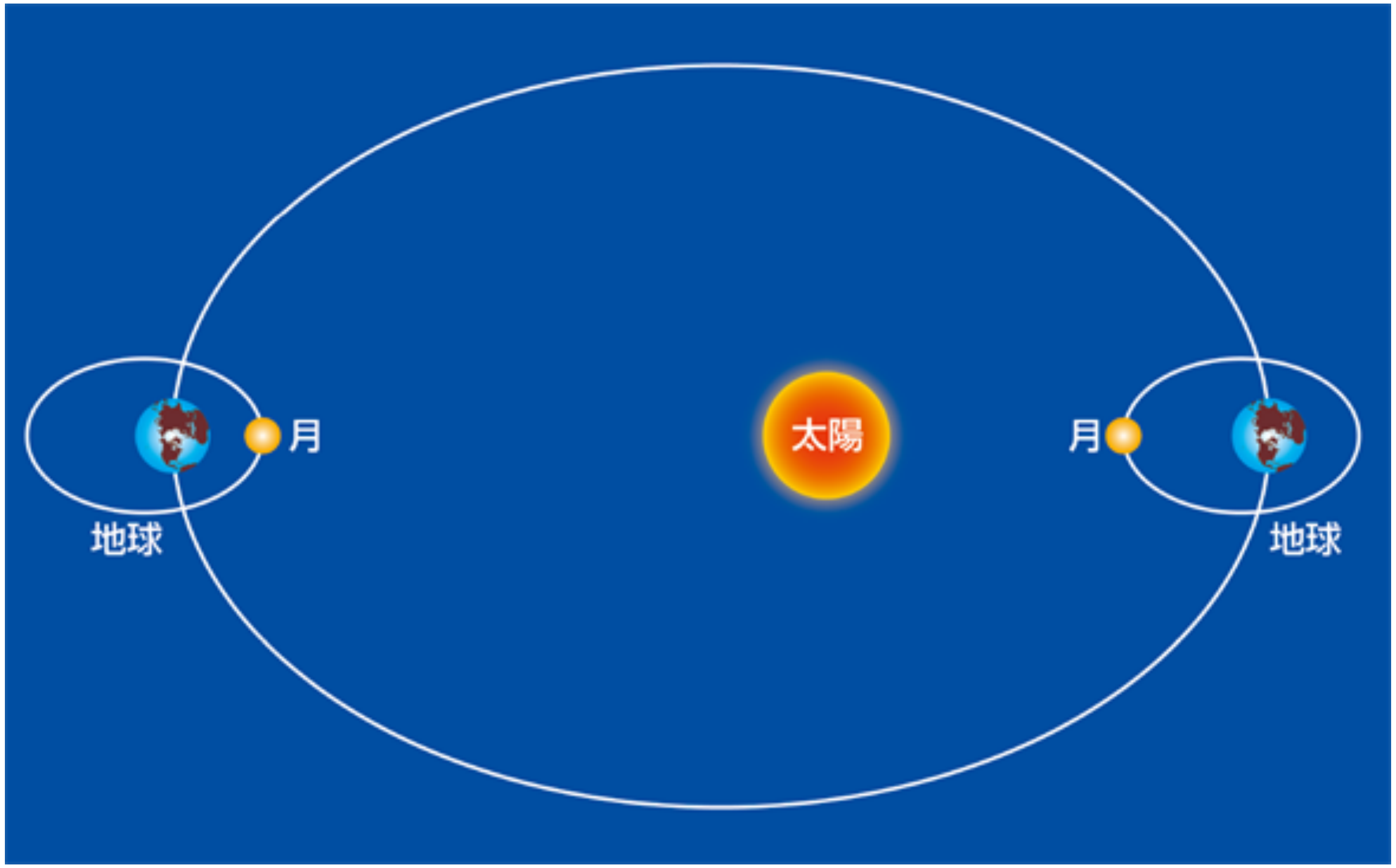
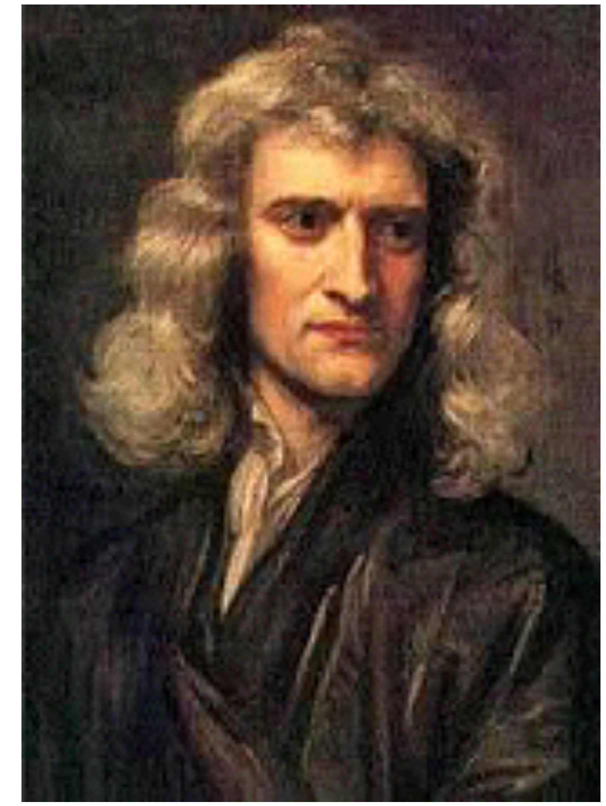


Gravity is universal





Newton

Universal Force

=Every objects have attractive force

$$F = G\frac{Mm}{r^2} \qquad m\frac{d^2x}{dt^2} = F$$

Conservation laws of Mechanics (1)

These laws derive some conservation laws.

(A) conservation of linear momentum

When two objects (with mass m_A, m_B) interact each other (such as collision, merger, separations, penetration), then the total linear momentum is conserved:

$$m_A \boldsymbol{v}_A + m_B \boldsymbol{v}_B = m_A \boldsymbol{v}'_A + m_B \boldsymbol{v}'_B \tag{0.2}$$

where $\boldsymbol{v}_A, \boldsymbol{v}_B$ are velocities of each object before interaction ([m/s] with its directional information), $\boldsymbol{v}'_A, \boldsymbol{v}'_B$ are those of after interaction ([m/s] with its directional information).

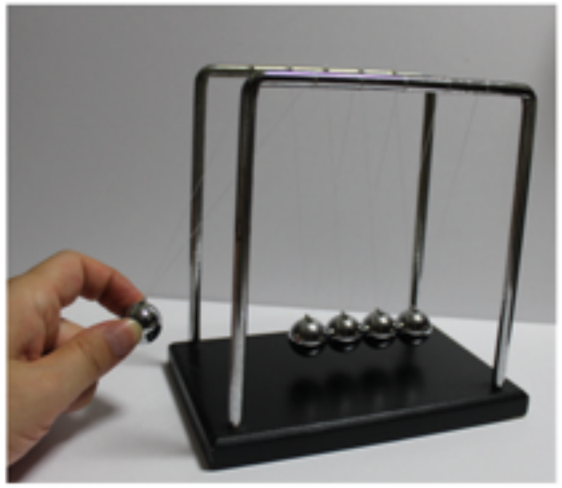


Figure 6: Newton's cradle.

Figure 7: On the ice, two standing couple begin pushing each other. How will they move?

Conservation laws of Mechanics (2)

(B) conservation of energy

If there is no friction, the total energy of the system E (sum of the potential energy and kinetic energy) is conserved:

$$E = mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_2^2 \qquad (0.3)$$

where m, g, h, v are the mass of an object ([kg]), gravitational acceleration (=9.8 [m/s^2]), height location of the object ([m]), and the speed of the object ([m/s]), respectively.

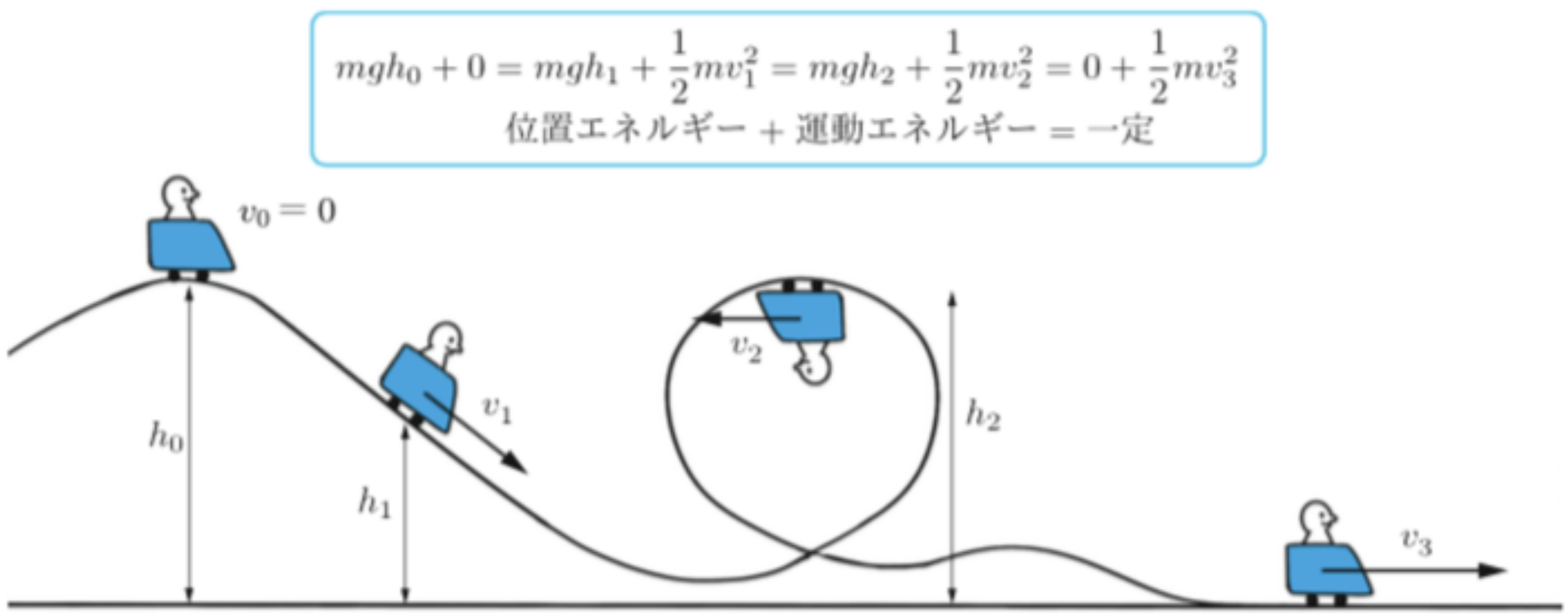


Figure 8: Jet coaster does not have its engine. The initial potential energy decides its later velocity.

Conservation laws of Mechanics (3)

(C) conservation of angular momentum

If an object begins rotation, the angular momemtum,

$$\boldsymbol{r} \times m\boldsymbol{v} \tag{0.4}$$

is conserved, where $\boldsymbol{r}, m, \boldsymbol{v}$ are directional vector from rotating axis ([m]), mass of an object ([kg]), and the velocity of an object ([m/s] with directional information), respectively.

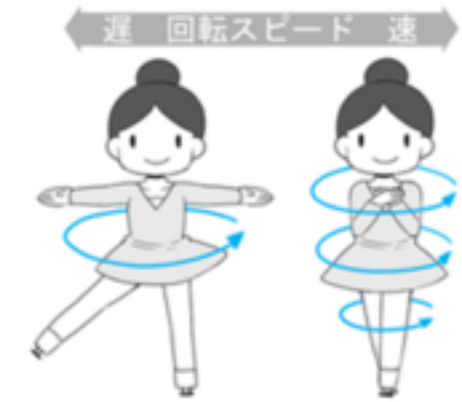


Figure 9: Figure skater rotates faster by shrinking her arms.

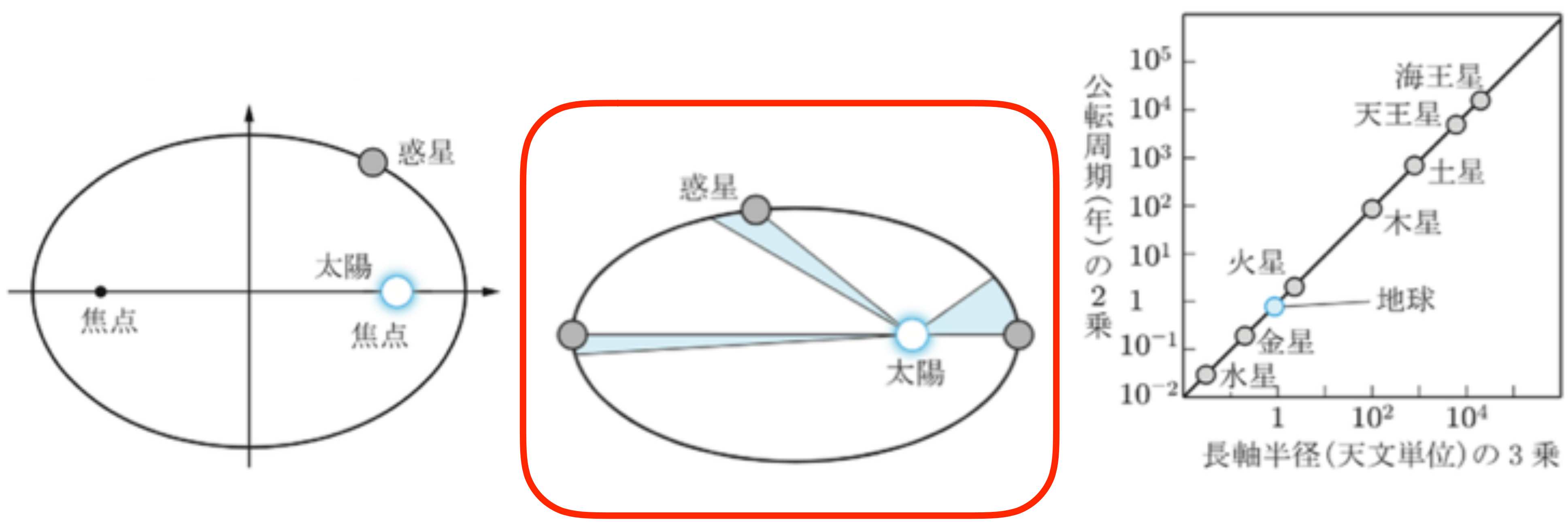


Figure 3: Kepler's first, second and third law of planetary motion.

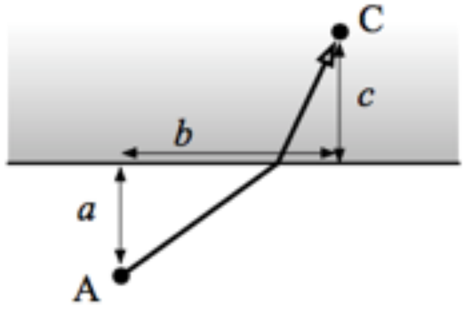
Least Approaching time of Life Saver

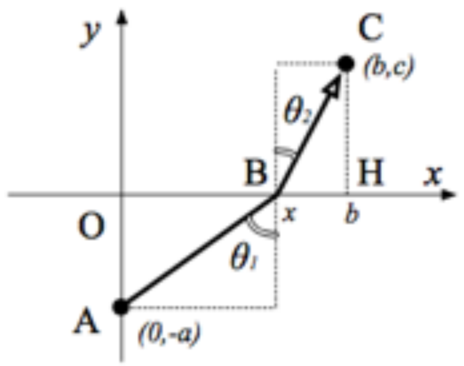
0.3 Least approaching time of life saver

The next problem is independent from the previous issues.

(Prob.) Least approaching time of life saver

A beautiful girl has fallen out of a boat, and she is screaming for help in the water (at C in the figure). We are at point A on land, and we see the accident, and we can run and can also swim. We can run 3 times faster than swim. If we plan to reach C as quick as we can, we should run a little greater distance on land. Where, then, is the point B on the shoreline at which we start to swim? Let the coordinate $A(0, -50)$ and $C(100, 50)$, and find $B(x, 0)$ using a software.





(4)

	A	B	C	D	E	F	G	H	I
1	distance a [m]	distance b [m]	distance c [m]	v1 [m/s]	v2 [m/s]	distance x [m]	t1 [s]	t2 [s]	t1+t2 [s]
2	50	100	50	3	1	50	23.570	70.711	94.281
3	50	100	50	3	1	51	23.807	70.007	93.814
4	50	100	50	3	1	52	24.046	69.311	93.357
5	50	100	50	3	1	53	24.288	68.622	92.910

=G2+H2

=G3+H3

(1)

(2)

=SQRT(A2*A2+F2*F2)/D2

=SQRT(A3*A3+F3*F3)/D3

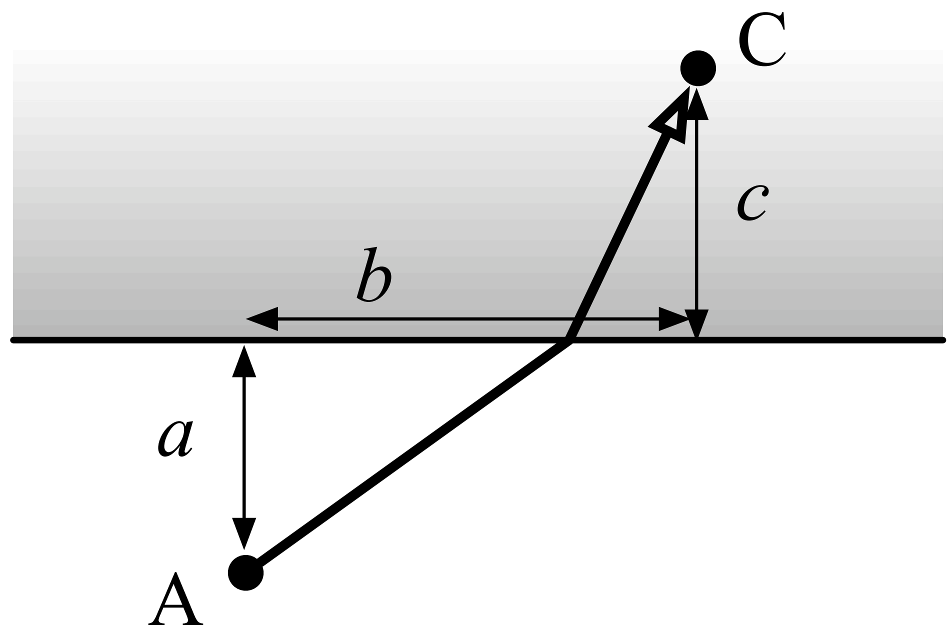
=SQRT(A4*A4+F4*F4)/D4

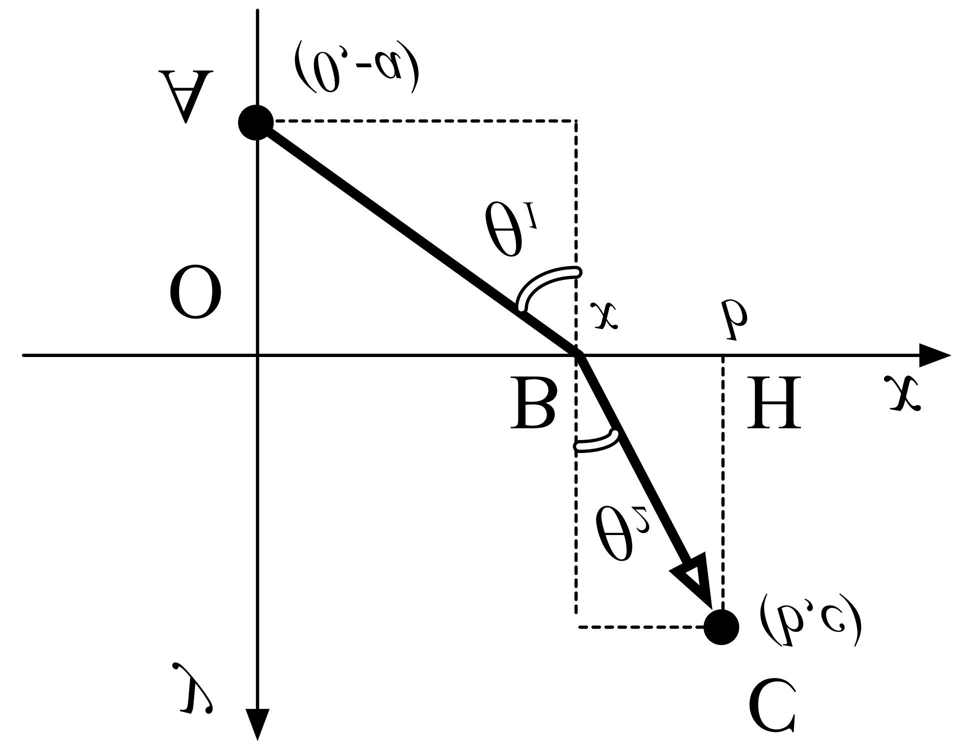
=SQRT((B2-F2)*(B2-F2)+C2*C2)/E2

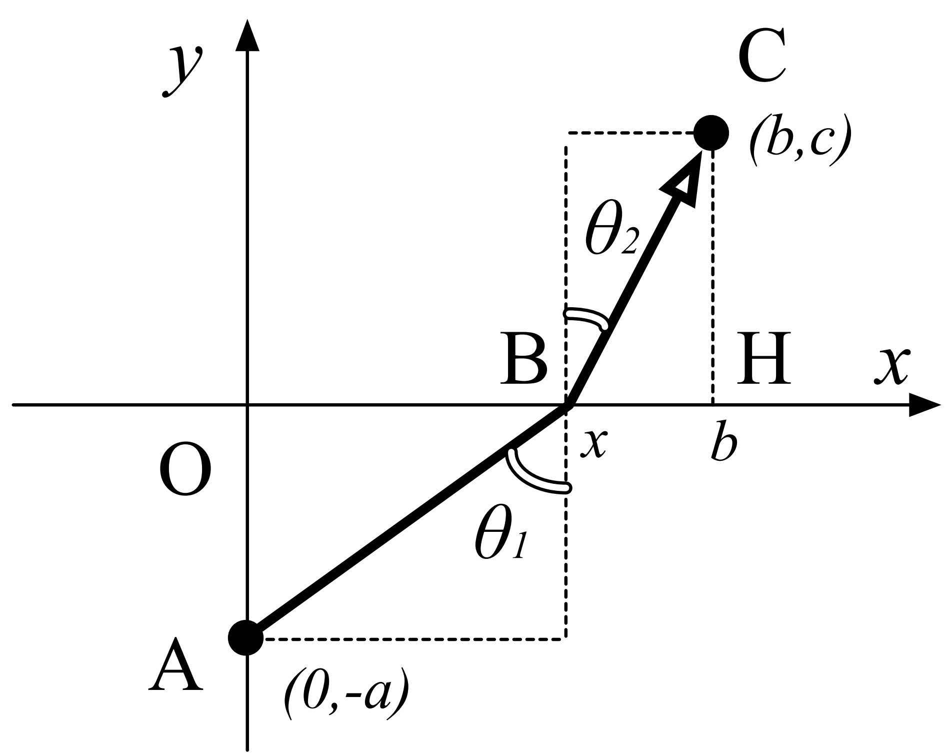
=SQRT((B3-F3)*(B3-F3)+C3*C3)/E3

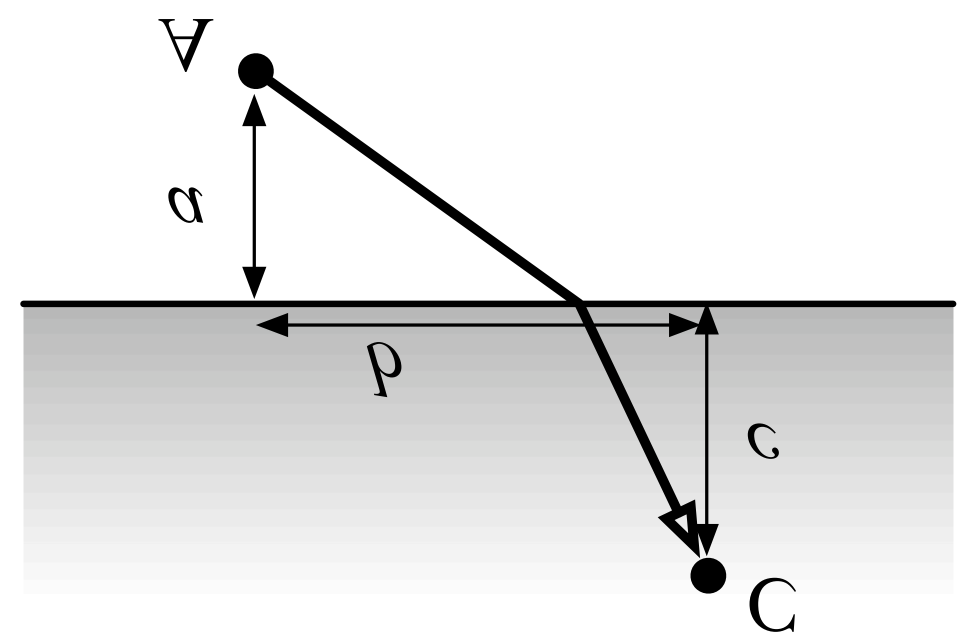
(3)

	A	B	C	D	E	F	G	H	I
1	distance a [m]	distance b [m]	distance c [m]	v1 [m/s]	v2 [m/s]	distance x [m]	t1 [s]	t2 [s]	t1+t2 [s]
2	50	100	50	3	1	50	23.570	70.711	94.281
3	50	100	50	3	1	51	23.807	70.007	93.814
4	50	100	50	3	1	52	24.046	69.311	93.357
5	50	100	50	3	1	53	24.288	68.622	92.910
6	50	100	50	3	1	54	24.531	67.941	92.472
7	50	100	50	3	1	55	24.777	67.268	92.045
8	50	100	50	3	1	56	25.024	66.603	91.628
9	50	100	50	3	1	57	25.274	65.947	91.221
10	50	100	50	3	1	58	25.526	65.299	90.825
11	50	100	50	3	1	59	25.779	64.661	90.440
24	50	100	50	3	1	72	29.219	57.306	86.526
25	50	100	50	3	1	73	29.494	56.824	86.318
26	50	100	50	3	1	74	29.769	56.356	86.125
27	50	100	50	3	1	75	30.046	55.902	85.948
28	50	100	50	3	1	76	30.324	55.462	85.786
29	50	100	50	3	1	77	30.603	55.036	85.640
30	50	100	50	3	1	78	30.883	54.626	85.509
31	50	100	50	3	1	79	31.164	54.231	85.395
32	50	100	50	3	1	80	31.447	53.852	85.298
33	50	100	50	3	1	81	31.730	53.488	85.218
34	50	100	50	3	1	82	32.014	53.141	85.155
35	50	100	50	3	1	83	32.299	52.811	85.110
36	50	100	50	3	1	84	32.585	52.498	85.083
37	50	100	50	3	1	85	32.872	52.202	85.073
38	50	100	50	3	1	86	33.160	51.923	85.083
39	50	100	50	3	1	87	33.448	51.662	85.111
40	50	100	50	3	1	88	33.738	51.420	85.157
41	50	100	50	3	1	89	34.028	51.196	85.223


C
c
b
a
A

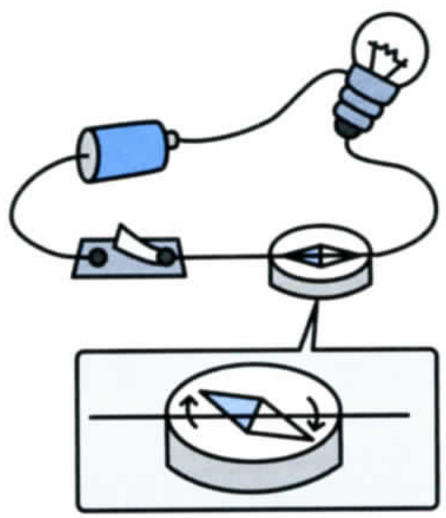

O
B
H
C


y
C
(b,c)
θ2
B
H
x
O
x
b
θ1
A
(0,-a)

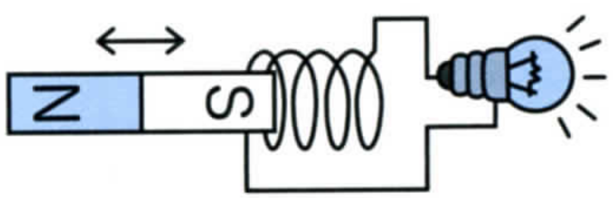

C

1 Special Relativity: Theory of time

1.1 Confusions from Maxwell's equation

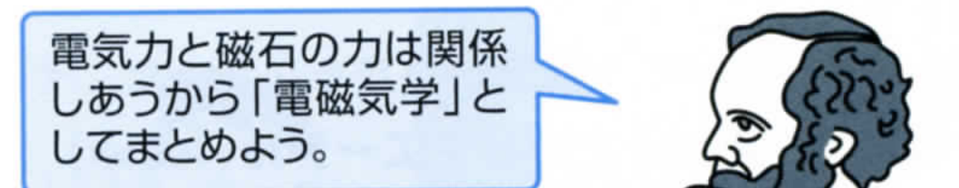


電流が流れると方位磁針の針が振れる。



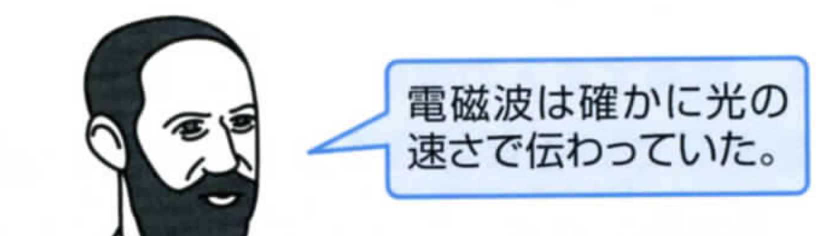
電磁誘導現象の発見（1831年）

ファラデー
Faraday



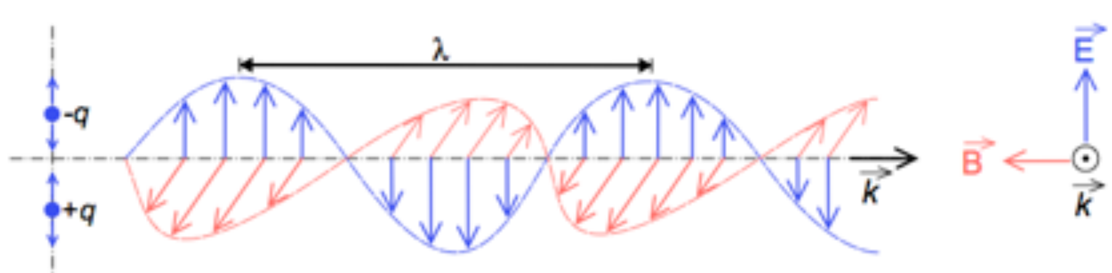
電磁気現象を説明する「マクスウェルの方程式」を完成させ（1864年）、電場と磁場が互いに作用して電磁波として伝わることを示す。

マクスウェル
Maxwell



電磁波（電波）の受発信に成功し、光の電磁波説を実証する（1888年）

ヘルツ
Hertz



Maxwell's equation（1864）

Figure 12: James C. Maxwell (1831-1879)

電磁気学の基本方程式

次の 4 本の方程式から成り立つ．$\boldsymbol{E}$ は電場ベクトル，$\boldsymbol{B}$ は磁場ベクトル，ρ は電荷密度，$\boldsymbol{j}$ は電流ベクトル，c は光速とする．また，∇ は微分演算子とする．

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho, \tag{2.20}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{2.21}$$

$$\nabla \times \boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{4\pi}{c}\boldsymbol{j}, \tag{2.22}$$

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0, \tag{2.23}$$

具体的には，各ベクトルは

$$\boldsymbol{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}, \quad \nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \tag{2.24}$$

などと書け，$\cdot$ と $\times$ はベクトルの内積と外積を表す．したがって，

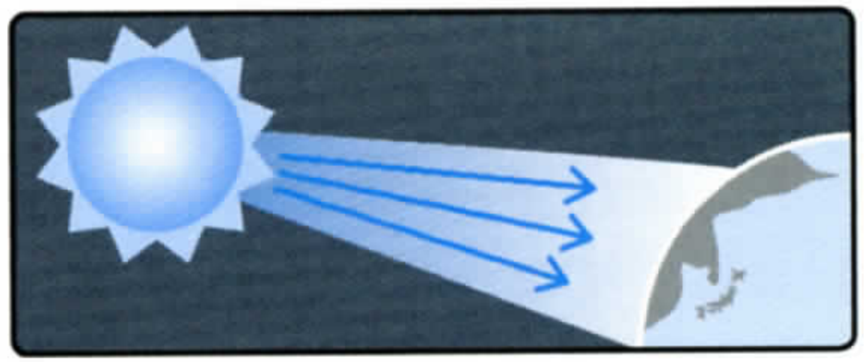
$$\nabla \cdot \boldsymbol{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}, \quad \nabla \times \boldsymbol{E} = \begin{pmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{pmatrix} \tag{2.25}$$

などとなる．

Ether? Why c in eqs?

疑問1 電磁波を伝えるもの（媒質）は何か？

音波は空気中の分子の
振動が伝える。

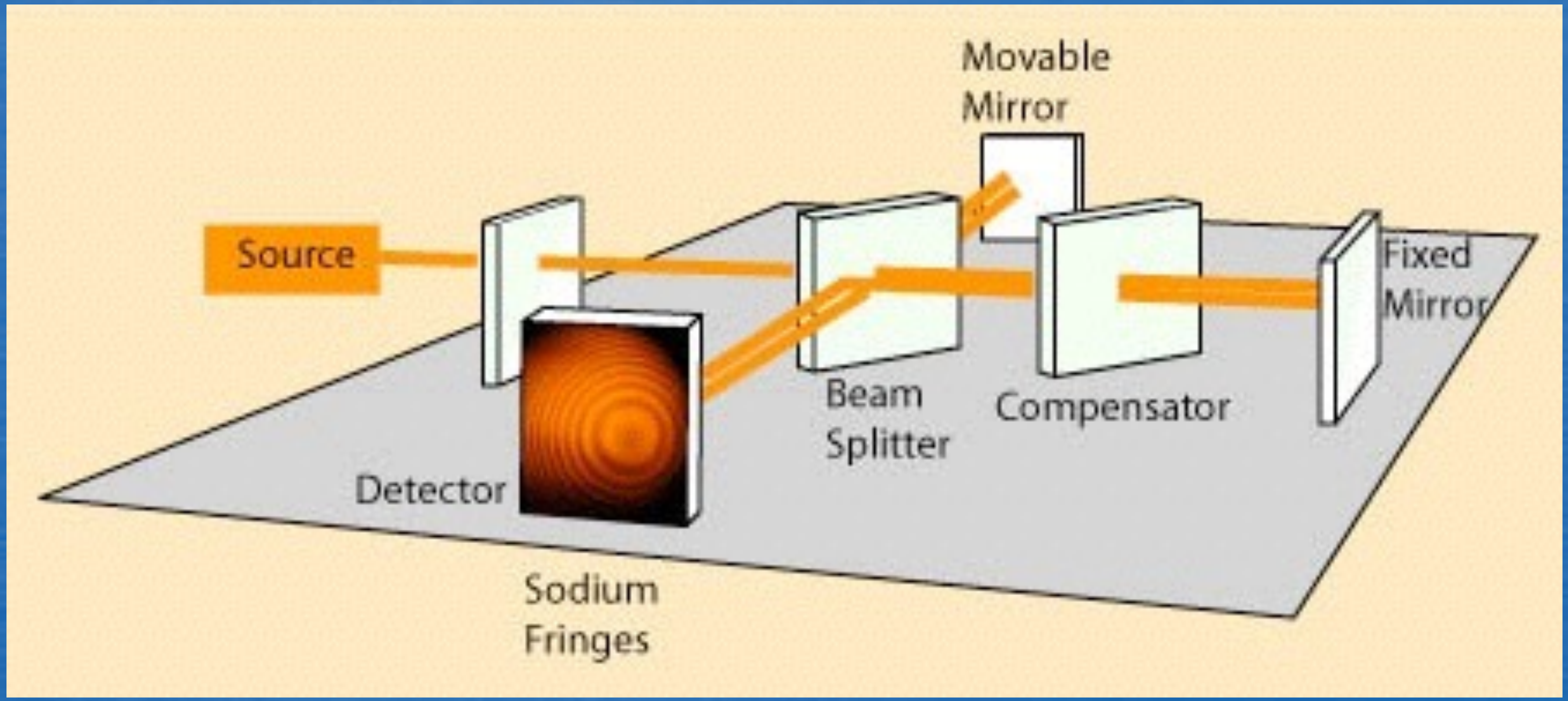


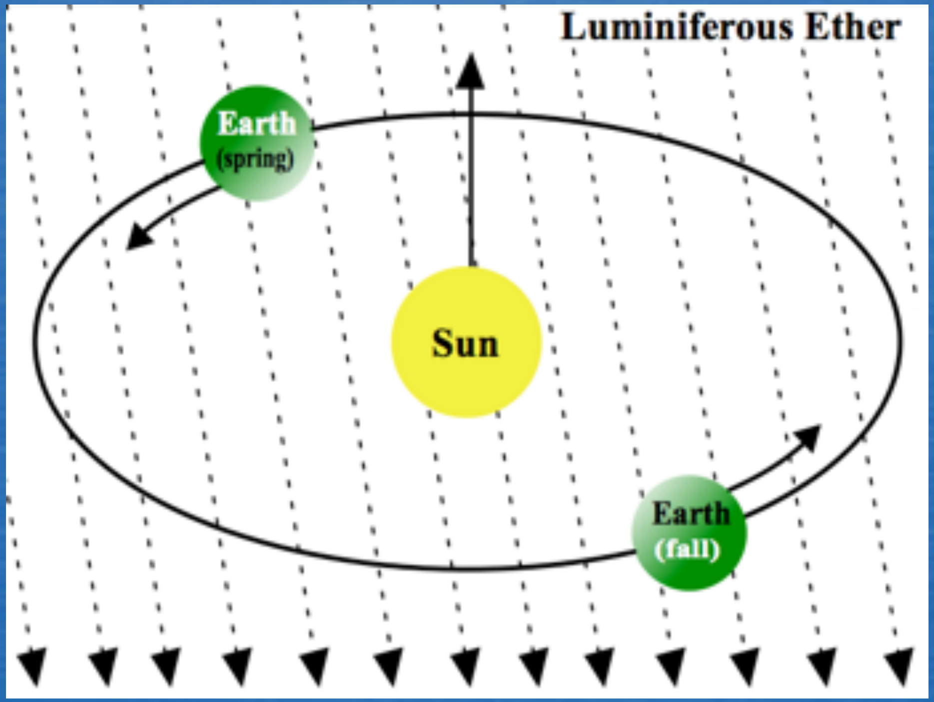
電磁波（光）は真空中を満たす
未知の媒質エーテルが伝える？

- First one is why the wave propagates in the vacuum. We need some medium, like molecules of air for sound wave. Physicists, therefore, named the medium, *Ether* [1], and began experiments to detect it.
- Second question is the appearance of the speed of light in the equations. The measure of speed certainly depends on the observers, so physicists had to define some special observer.

疑問2 電磁波が伝わる速度が「光速」であるとは、誰から見た時の光速なのか？

Michelson-Morley experiment 1887





If Ether exists, its effect will change by seasons. Such a tiny differences can be measured by interferometer.

double slit experiment of light

Experiment **Interference of waves**

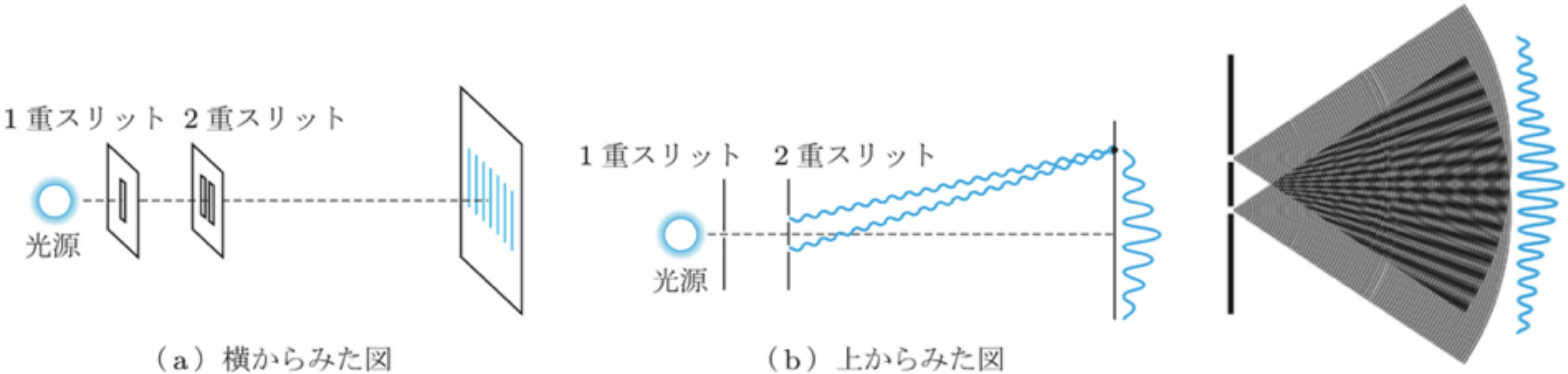
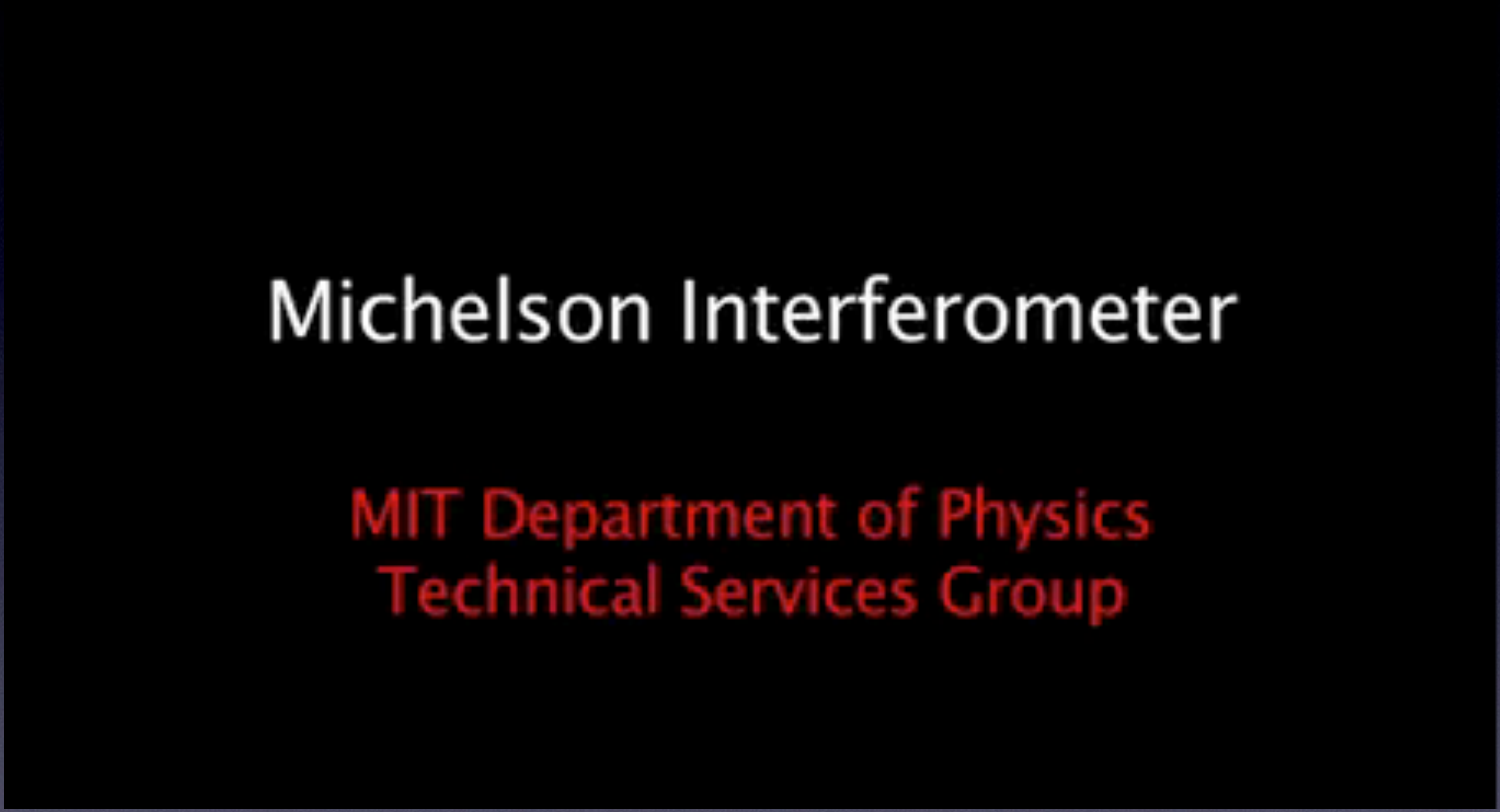


Figure 15: Double-slit experiment of light.

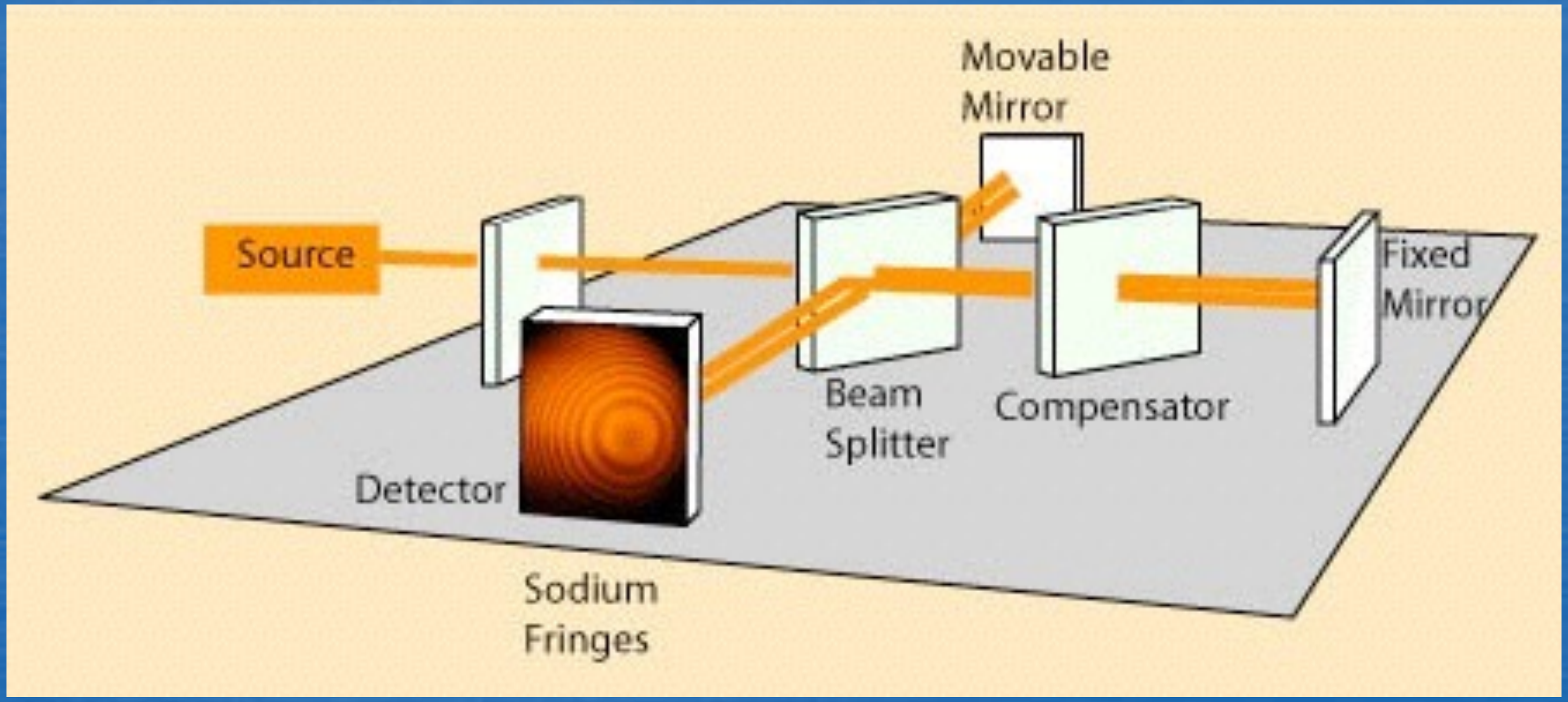
Michelson Interferometer

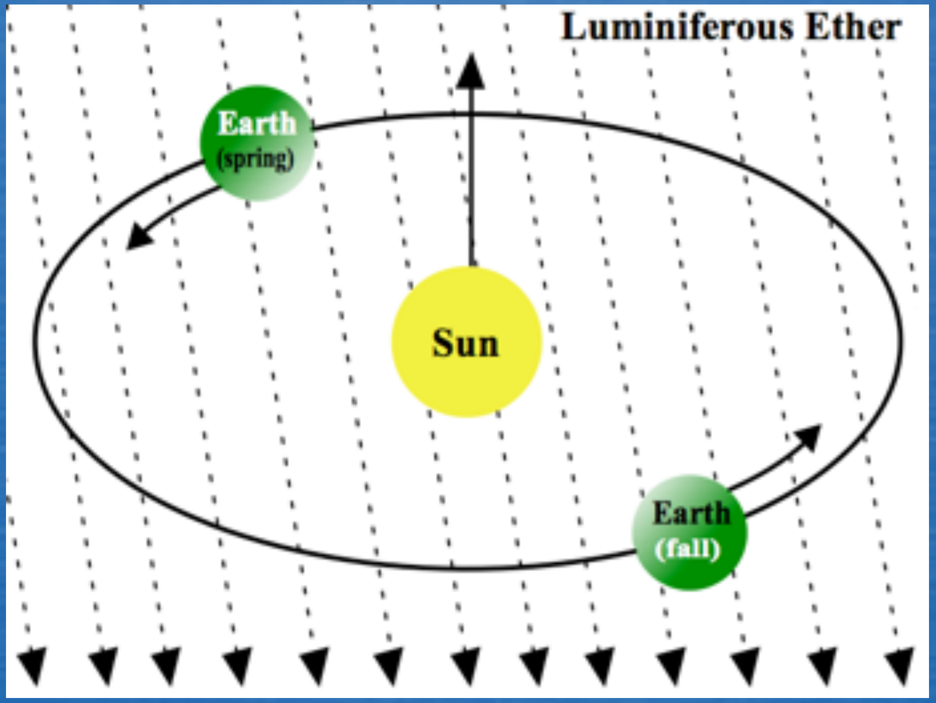


start on click, last half, 1'45"

http://techtv.mit.edu/videos/9823-michelson-interferometer

Michelson-Morley experiment　1887





If Ether exists, its effect will change by seasons. Such a tiny differences can be measured by interferometer.

→「Failed」 for detection

Lorentz's idea "contraction"

The result of Michelson and Morley made physicists in confusion. The result indicated that the wind of *Ether* is not observable. Many physicists proposed explanations for this fact which were 'consistent' with the existence of *Ether*. For example, if *Ether* will circulate together with the Earth, the experiment had no contradiction. This explanation predicts that *Ether* moves due to the gravity of the Earth. Michelson tried his experiment again with the heavy gravity source at one arm of interferometer, but there was no differences in the interference patterns.

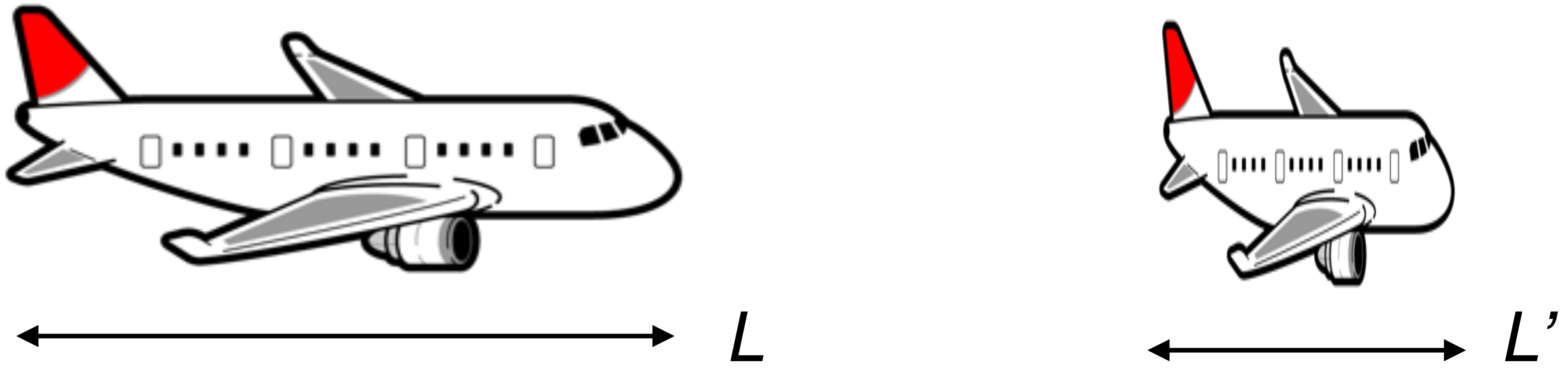
The theory of *Ether* became difficult. FitzGerald and Lorentz began changing the Newton's physics. They proposed that all the matter will shrink its length if they move at large speed, which is called the Lorentz-FitzGerald contraction. If we suppose that the matter of length L at the speed $v = 0$ will change its length if it moves with the speed v in its moving direction as

$$L' = \sqrt{1 - (v/c)^2} L \tag{1.5}$$

where c is the speed of light, then there is no contradiction with the experiments. This relation can be interpreted also as the clock timing of the observer who moves at the speed v will be longer with the factor $1/\sqrt{1-(v/c)^2}$.

Figure 16: Hendrik A. Lorentz (1853–1928)

George F. FitzGerald (1851–1901)



Lorentz's idea "contraction"

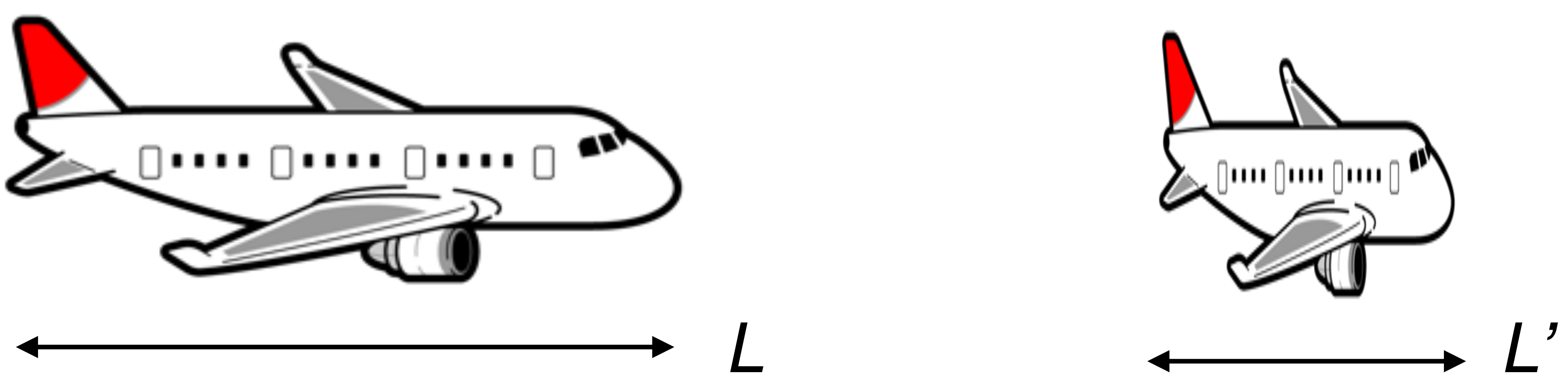
【Detail explanation】 (you can skip this part)
Lorentz explained this proposal with coordinate transformation (1904). That is, if we change our coordinate (t, x, y, z) into the new coordinate (t', x', y', z') which moves in x-direction with the speed v, then we obtain

$$\begin{aligned} t' &= \frac{t-(v/c^2)x}{\sqrt{1-(v/c)^2}} \\ x' &= \frac{x-vt}{\sqrt{1-(v/c)^2}}, \\ y' &= y, \\ z' &= z. \end{aligned} \tag{1.6}$$

These relation can be written in a matrix form as

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-(v/c)^2}} & \frac{-v/c^2}{\sqrt{1-(v/c)^2}} & 0 & 0 \\ \frac{-v}{\sqrt{1-(v/c)^2}} & \frac{1}{\sqrt{1-(v/c)^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}. \tag{1.7}$$

Figure 16: Hendrik A. Lorentz (1853–1928)



Einstein's idea "principle of relativity"

Two principles by Einstein

Albert Einstein proposed a new interpretation to the problem. He starts from proposing two principles, which *derive* the Lorentz transformation (1.7), and also conclude that we do not need *Ether* for explaining the Maxwell equations. Two principles are the followings:

(a) **Principle of Relativity** : All the physics laws should be the same equation, independent from the observers' coordinate.

(b) **Principle of the constant speed of light** : The speed of light in vacuum is the constant at any coordinate in the Universe.

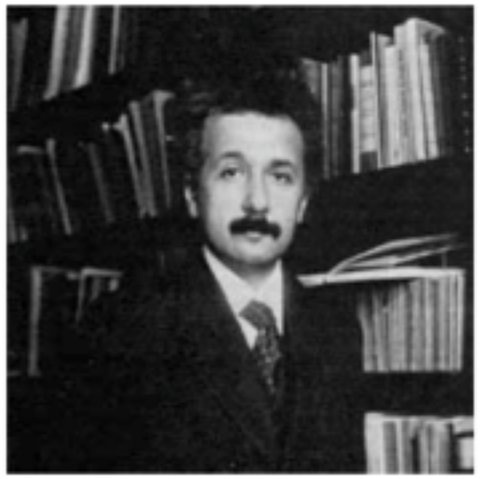


Figure 17: Albert Einstein (1879-1955)

By assuming these two simple principles, we do not have any problem.

疑問2 電磁波が伝わる速度が「光速」であるとは、誰から見た時の光速なのか?

c is constant.
Time is relative!

$c + c = c$

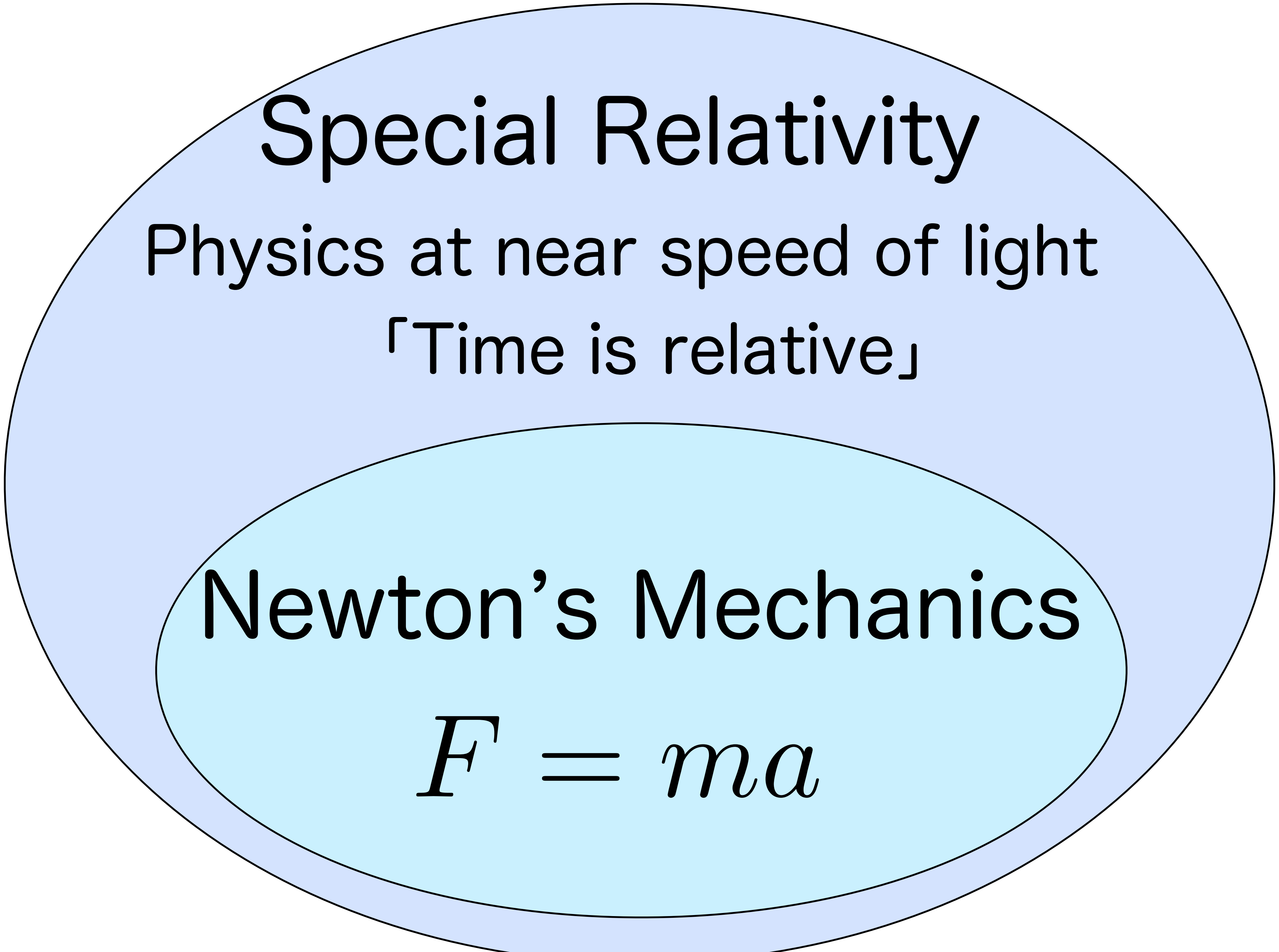
In our daily life, if we throw a ball with speed v_1 in the train with speed v_2, then the ball moves at the speed $v_1 + v_2$ (measured from the ground observer). Einstein's theory, however, says that this is not true. The true additive calculation should be

$$v_1 + v_2 \implies \frac{v_1 + v_2}{1 + (v_1 v_2 / c^2)} \tag{1.8}$$

which is derived from Lorentz transformation, (1.7). This rule says if the speed is small compared with c, then $v_1 + v_2$ is *approximately* true. Actually, this approximation is valid unless speed is close to c.

Exercise 1 Using the additive rule (1.8), fill in the blanks.

v_1	v_2	$v_1 + v_2$
0.1% of light	0.1% of light	0.1999998% of light
$0.10c$	$0.10c$	
$0.50c$	$0.50c$	
$0.90c$	$0.90c$	
$0.99c$	$0.99c$	
c	c	


Special Relativity
Physics at near speed of light
「Time is relative」
Newton's Mechanics
$F = ma$

1.3 Time is Relative

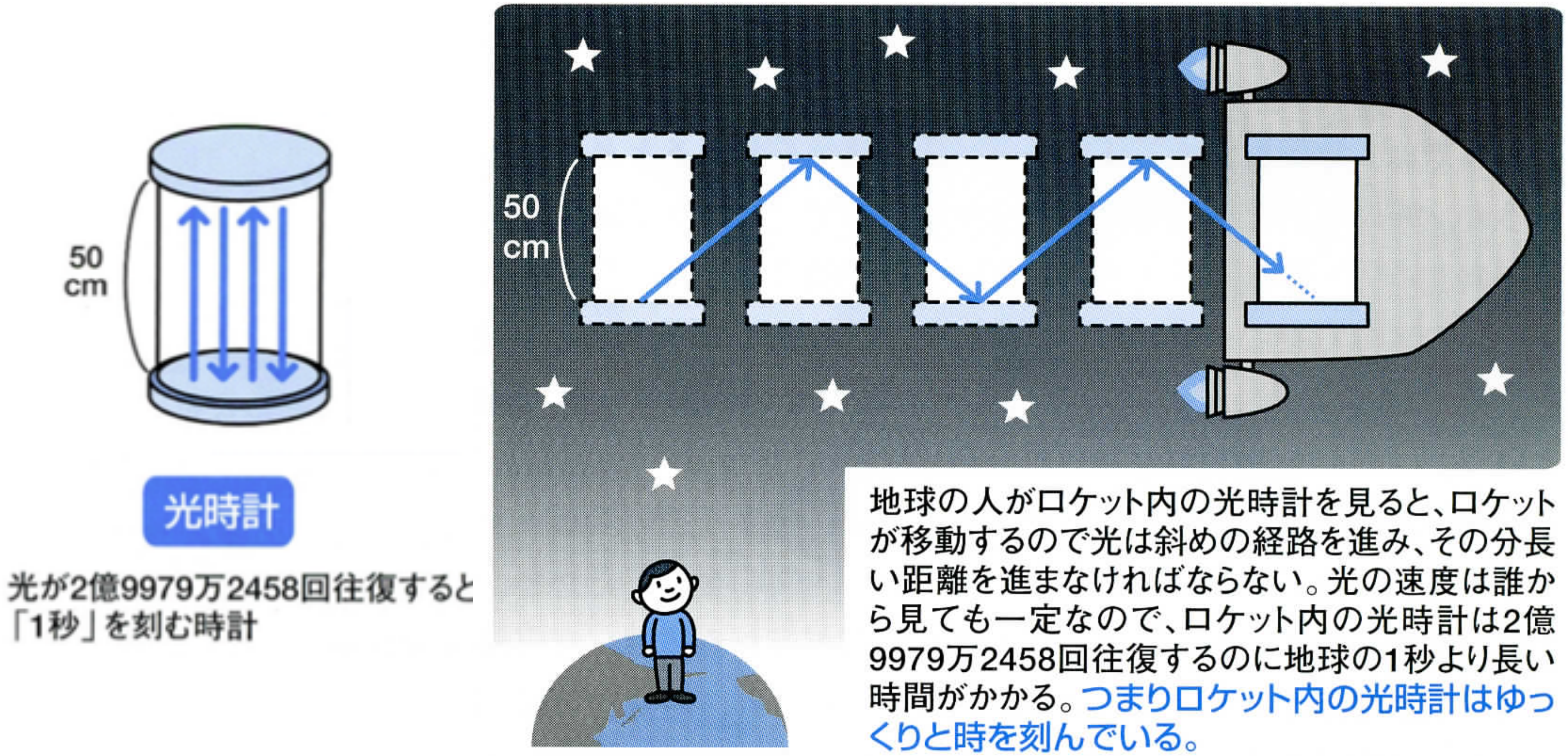


Figure 18: (Left) A light-clock. (Right) When the clock is in a rocket at large speed, a light-ray has to move longer distance for traveling between two mirrors, which means one second in a rocket is longer than that on the Earth.

Moving-clock delays than a clock on the Earth.

Time Travel to the Future

Time travel to the future

Actual difference of time can be expressed by the equation

$$T' = \sqrt{1-(v/c)^2}\,T \qquad (1.9)$$

where T is the clock time (say, one second) in a rocket moving at speed v and T is the time (say, one second) at rest observer.

The right table is examples of the value of $\sqrt{1-(v/c)^2}$. This table shows that the difference of time will crucially large when the speed v is close to c.

v	$\sqrt{1-(v/c)^2}$
0.1 c	0.99499
0.5 c	0.86603
0.9 c	0.43589
0.99c	0.14107

Exercise 2a The ISS (International Space Station) circulates around the Earth with a speed 7.8 [km/s], which is 0.000026c. How much is the difference of time for a staff who stayed a year in ISS comparing to a person on the Earth?

Ex.2 **Time travel to the future.**

Time Travel to the Future

Story of Mr. Taro Urashima

Time Travel to the Future

Exercise 2b According to a story of *Mr. Taro Urashima*, a famous fairy-tale in Japan, Mr. Urashima spended three-years travel to Ryugu castle and traveled back to his town, but he found that he suddenly got old and all the people he knew turned out to have died. If we interpreted this story as a time travel to the future (say 300 years) using a rocket, what is the speed of the rocket? Use the approximation,

$$\sqrt{1-(v/c)^2} \approx 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{1.10}$$

if your calculator is not smart.

Ex.2 Time travel to the future.

Exercise 2a The ISS (International Space Station) circulates around the Earth with a speed 7.8 [km/s], which is $0.000026c$. How much is the difference of time for a staff who stayed a year in ISS comparing to a person on the Earth?

```
In[5]:= 365 * 24 * 60 * 60 * (1 - Sqrt[1 - 0.000026 ^ 2])

Out[5]= 0.0106592
```

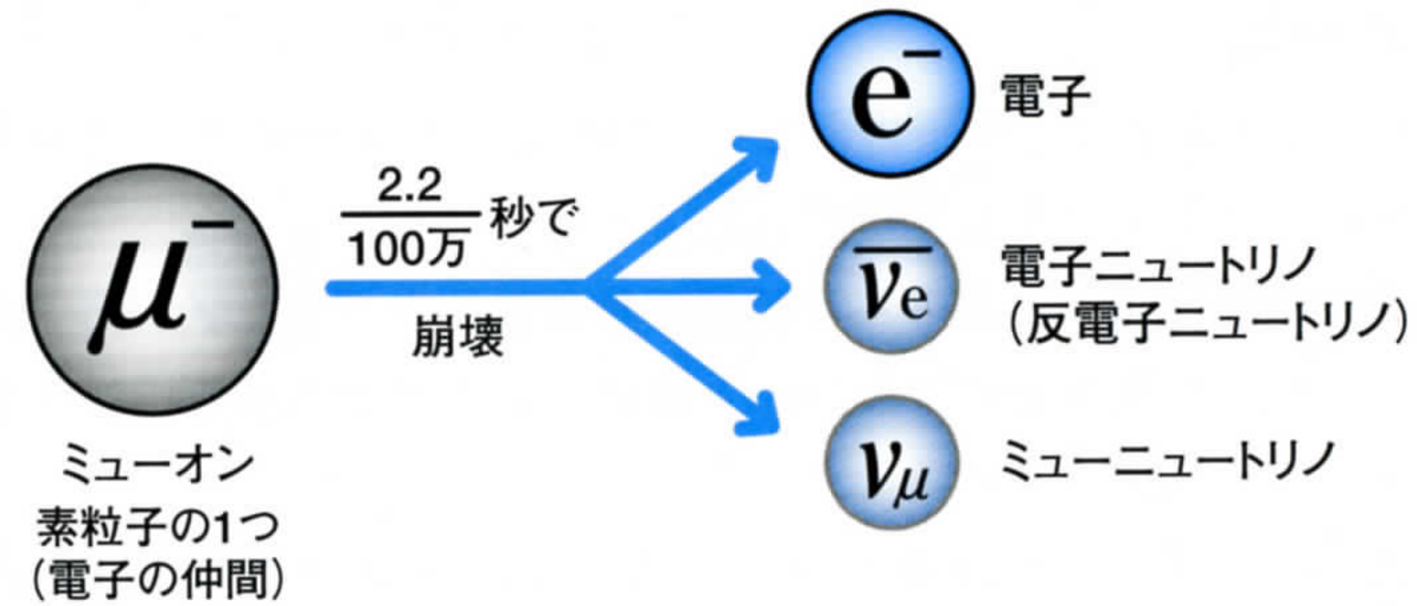
Exercise 2b According to a story of *Mr. Taro Urashima*, a famous fairy-tale in Japan, Mr. Urashima spended three-years travel to Ryugu castle and traveled back to his town, but he found that he suddenly got old and all the people he knew turned out to have died. If we interpreted this story as a time travel to the future (say 300 years) using a rocket, what is the speed of the rocket? Use the approximation,

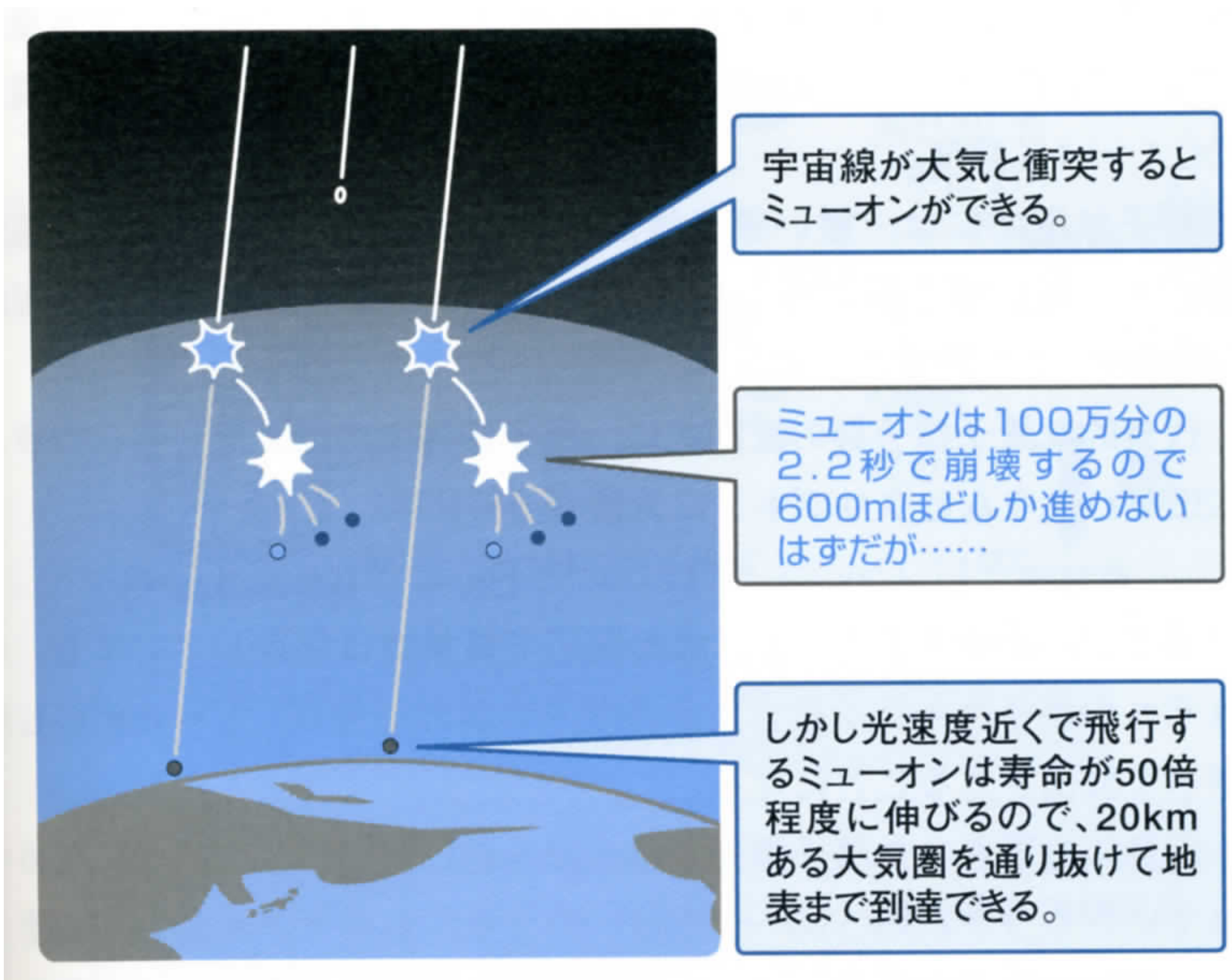
$$\sqrt{1-(v/c)^2} \approx 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2 \qquad (1.10)$$

if your calculator is not smart.

```
In[7]:= NSolve[300 * Sqrt[1 - (v / c) ^ 2] == 3, v]
Out[7]= {{v → -0.99995 c}, {v → 0.99995 c}}
```

Lifetime of high-speed particle is longer !





The most famous physics formula

$$E = mc^2$$

Energy **mass x light-speed ^2**

Energy = mass

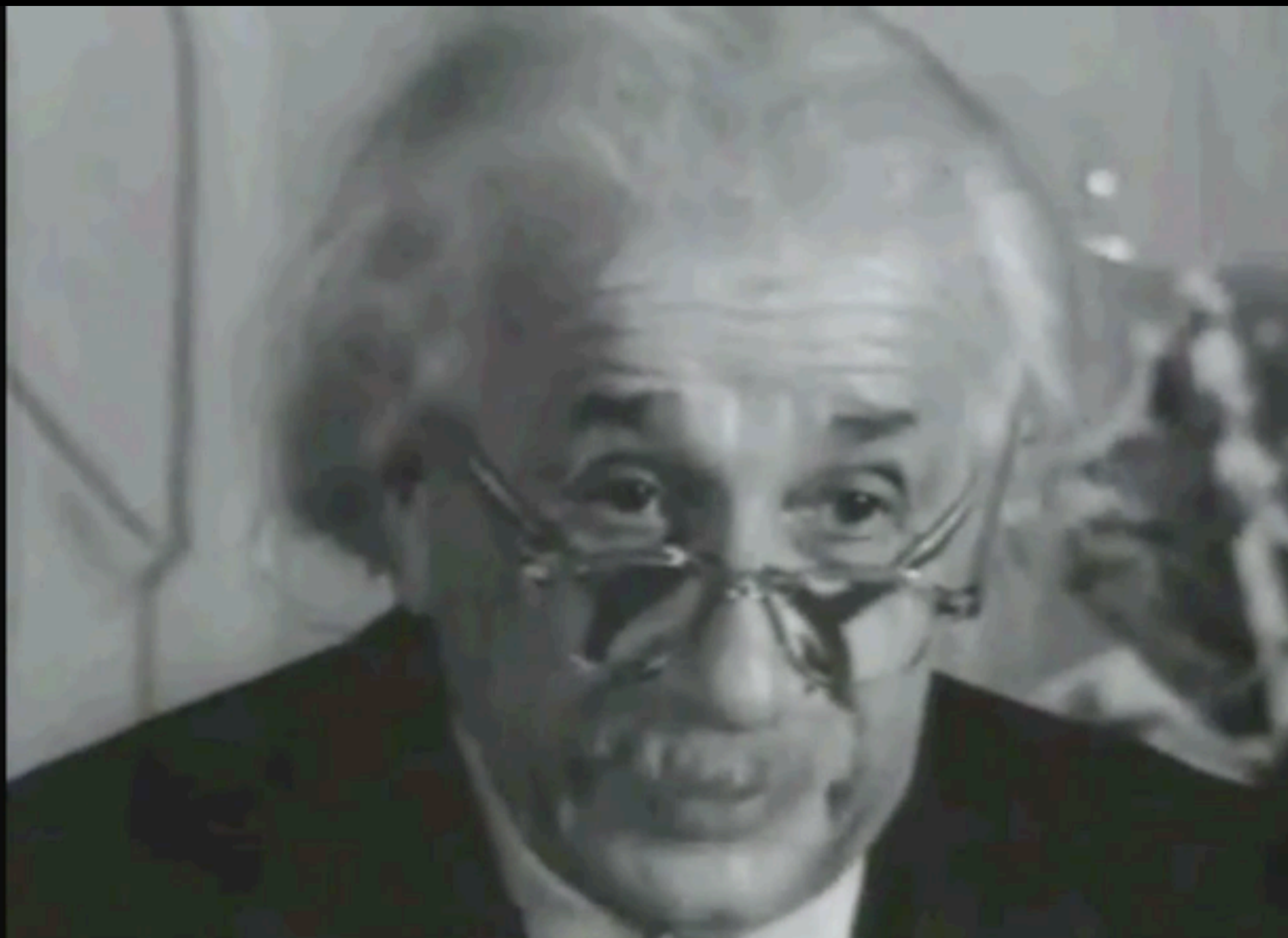
mass can be converted to energy!

$$\frac{dp^i}{dt} = F^i$$
$$(ma = F)$$

→

$$\frac{dp^i}{d\tau} = f^i$$

$$E \equiv mc^2 \frac{dt}{d\tau} = m \frac{c^2}{\sqrt{1-(v/c)^2}}$$
$$= mc^2 + \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \cdots$$



$E = mc^2$ by Albert Einstein himself

It followed from the special theory of relativity that mass and energy are both but different manifestations of the same thing — a somewhat unfamiliar conception for the average mind.

Furthermore, the equation $E = mc^2$, in which energy is put equal to mass, multiplied by the square of the velocity of light, showed that very small amounts of mass may be converted into a very large amount of energy and vice versa.

The mass and energy were in fact equivalent, according to the formula mentioned before.

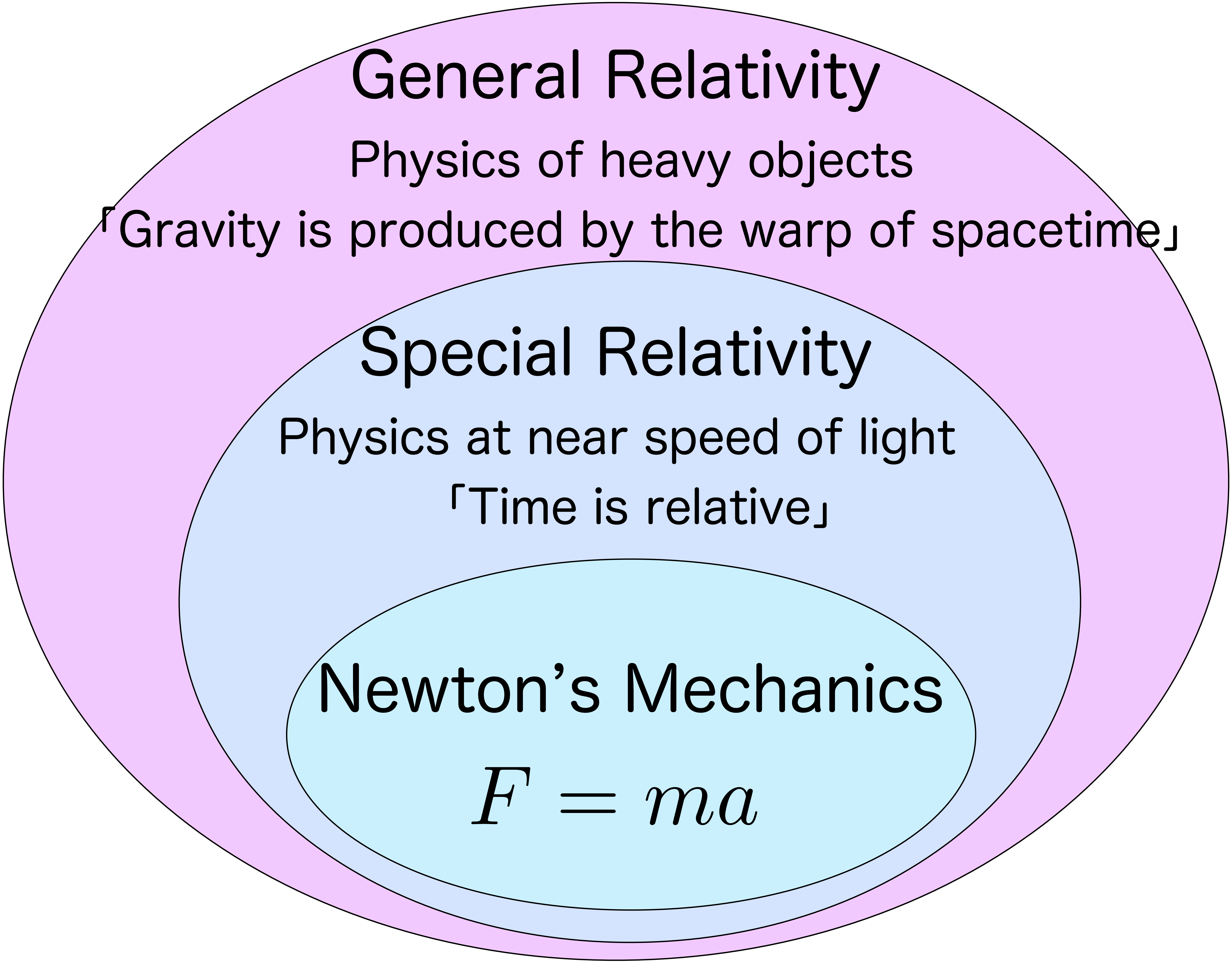
This was demonstrated by Cockcroft and Walton in 1932, experimentally.

Special Relativity (1905)

- **Motivation :**
  Why the light speed c appears in Maxwell equation?
  Why the light propagates in vacuum?

- **Major theories :**
  The light propagates in Ether.
  The speed c changes by observers.
  But Ether can not be observed ….

- **Einstein's idea :**
  The physics law should be simple.
  The light propagates in vacuum.
  The speed c is constant in any observers.


General Relativity
Physics of heavy objects
「Gravity is produced by the warp of spacetime」
Special Relativity
Physics at near speed of light
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