Adjusted ADM systems and their expected stability properties

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OUTLINE and KEYWORDs

- systematical understanding how to construct an evolution system which is robust against violation of errors
- idea of "adjusted system", adding constraints in RHS, why they work?
- (adjuted) constraint propagation equations and their eigenvalue analysis

Refs:

HS and G Yoneda, gr-qc/0110008

G Yoneda and HS, Phys Rev D 63 (2001) 120419

1 Background of the problem

Numerical Relativity – Necessary for unveiling the nature of strong gravity

- Gravitational Wave from colliding Black Holes, Neutron Stars, Supernovae, ...
- Relativistic Phenomena like Cosmology, Active Galactic Nuclei, ...
- Mathematical feedbacks to Singularity, Exact Solutions, Chaotic behavior, ...
- Labratory of Gravitational theories, Higher dimensional models, ...

Best Einstein formulation for long-term stable and accurate simulation?

Many (too many) trials and errors, not yet a systematical understanding

strategy 1 Shibata-Nakamura's (Baumgarte-Shapiro's) modifications to the standard ADM

strategy 2 Apply a formulation which reveals a hyperbolicity explicitly

strategy 3 Formulate a system which is "asymptotically constrained" against a violation of constraints

The direct use of the standard ADM equations is not recommended.
By adding constraints in RHS, we can kill error growing modes
$\Rightarrow$ Why?

strategy 1 Shibata-Nakamura's (Baumgarte-Shapiro's) modifications to the standard ADM

define new variables (ϕ, $\tilde{\gamma}_{ij}$, K, $\tilde{A}_{ij}$, $\tilde{\Gamma}^i$), instead of the ADM's (γ_{ij}, K_{ij}) where

$$\tilde{\gamma}_{ij} \equiv e^{-4\phi}\gamma_{ij}, \qquad \tilde{A}_{ij} \equiv e^{-4\phi}(K_{ij} - (1/3)\gamma_{ij}K), \qquad \tilde{\Gamma}^i \equiv \tilde{\Gamma}^i_{jk}\tilde{\gamma}^{jk},$$

use momentum constraint in Γ^i-eq., and impose $\det\tilde{\gamma}_{ij} = 1$ during the evolutions.

No explicit explanations why this formulation works better.

Potsdam group (2000): the replacement by momentum constraint is essential.

strategy 2 Apply a formulation which reveals a hyperbolicity explicitly.

For a first order partial differential equations on a vector u,

$$\partial_t \begin{bmatrix} u_1 \\ u_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \partial_x \begin{bmatrix} u_1 \\ u_2 \\ \vdots \end{bmatrix} + B \begin{bmatrix} u_1 \\ u_2 \\ \vdots \end{bmatrix}, \qquad (1)$$

	if the eigenvalues of A are
weakly hyperbolic	all real.
strongly hyperbolic	all real and $\exists$ a complete set of eigenvalues.
symmetric hyperbolic	if A is real and symmetric (Hermitian).

strategy 2 Apply a formulation which reveals a hyperbolicity explicitly. (cont.)

weakly hyperbolic $\ni$ strongly hyperbolic $\ni$ symmetric hyperbolic systems,

Are they actually helpful? Which level of hyperbolicity is necessary?

Using Ashtekar's variables between we found that [HS-Yoneda, CQG17(2000)4799]

(1) the three levels of hyperbolicity can be obtained by adding constraint terms and/or imposing gauge conditions

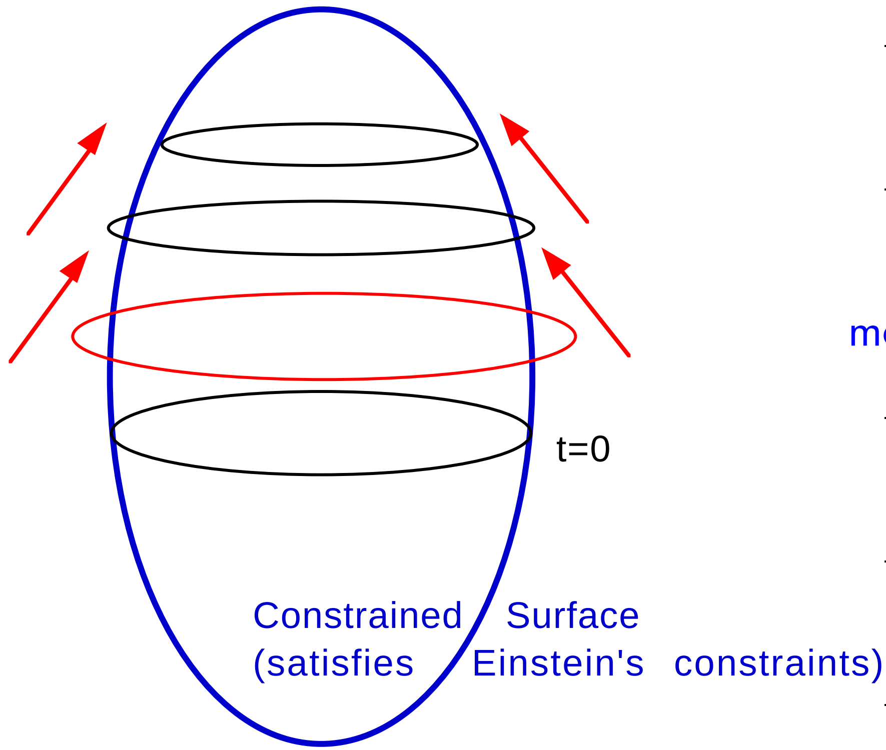
(2) there is no drastic difference in the accuracy of numerical evolutions in these three levels (comparison of nonlinear wave propagation in a plane symmetric spacetime)

(3) the symmetric hyperbolic system is not always the best for reducing numerical errors

Note that IBVP (Initial Boundary Value problem) requires "symmetric hyperbolicity" to be treated with.

strategy 3 Formulate a system which is "asymptotically constrained" against a violation of constraints

"Asymptotically Constrained System"– Constraint Surface as an Attractor



method 1: λ-system (Brodbeck et al, 2000)

– Add aritificial force to reduce the violation of constraints

– To be guaranteed if we apply the idea to a symmetric hyperbolic system.

method 2: Adjusted system (Yoneda HS, 2000, 2001)

– We can control the violation of constraints by adjusting constraints to EoM.

– Eigenvalue analysis of constraint propagation equations may prodict the violation of error.

– This idea is applicable even if the system is not symmetric hyperbolic. $\Rightarrow$ for the ADM formulation, too!!

2 Idea of "Adjusted system" and Our Conjecture

General Procedure

1. prepare a set of evolution eqs. $\partial_t u^a = f(u^a, \partial_b u^a, \cdots)$
2. add constraints in RHS $\partial_t u^a = f(u^a, \partial_b u^a, \cdots) + F(C^a, \partial_b C^a, \cdots)$
3. choose appropriate $F(C^a, \partial_b C^a, \cdots)$ to make the system stable evolution

How to specify $F(C^a, \partial_b C^a, \cdots)$?

4. prepare constraint propagation eqs. $\partial_t C^a = g(C^a, \partial_b C^a, \cdots)$
5. and its adjusted version $\partial_t C^a = g(C^a, \partial_b C^a, \cdots) + G(C^a, \partial_b C^a, \cdots)$
6. Fourier transform and evaluate eigenvalues $\partial_t \hat{C}^k = A(\hat{C}^a)\,\hat{C}^k$

Conjecture: Evaluate eigenvalues of (Fourier-transformed) constraint propagation eqs. If their (1) real part is non-positive, or (2) imaginary part is non-zero, then the system is more stable.

3 Adjusted ADM systems

We adjust the standard ADM system using constraints as:

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i, \tag{1}$$

$$+P_{ij}\mathcal{H} + Q^k{}_{ij}\mathcal{M}_k + p^k{}_{ij}(\nabla_k \mathcal{H}) + q^{kl}{}_{ij}(\nabla_k \mathcal{M}_l), \tag{2}$$

$$\partial_t K_{ij} = \alpha R^{(3)}_{ij} + \alpha K K_{ij} - 2\alpha K_{ik} K^k{}_j - \nabla_i \nabla_j \alpha + (\nabla_i \beta^k) K_{kj} + (\nabla_j \beta^k) K_{ki} + \beta^k \nabla_k K_{ij}, \tag{3}$$

$$+R_{ij}\mathcal{H} + S^k{}_{ij}\mathcal{M}_k + r^k{}_{ij}(\nabla_k \mathcal{H}) + s^{kl}{}_{ij}(\nabla_k \mathcal{M}_l), \tag{4}$$

with constraint equations

$$\mathcal{H} := R^{(3)} + K^2 - K_{ij}K^{ij}, \tag{5}$$

$$\mathcal{M}_i := \nabla_j K^j{}_i - \nabla_i K. \tag{6}$$

We can write the adjusted constraint propagation equations as

$$\partial_t \mathcal{H} = \text{(original terms)} + H_1^{mn}[(2)] + H_2^{imn}\partial_i[(2)] + H_3^{ijmn}\partial_i\partial_j[(2)] + H_4^{mn}[(4)], \tag{7}$$

$$\partial_t \mathcal{M}_i = \text{(original terms)} + M_{1i}{}^{mn}[(2)] + M_{2i}{}^{jmn}\partial_j[(2)] + M_{3i}{}^{mn}[(4)] + M_{4i}{}^{jmn}\partial_j[(4)]. \tag{8}$$

The constraint propagation equations of the original ADM equation:

- Expression using $\mathcal{H}$ and $\mathcal{M}_i$ (1)

$$\begin{aligned}\partial_t\mathcal{H} &= \beta^j(\partial_j\mathcal{H}) + 2\alpha K\mathcal{H} - 2\alpha\gamma^{ij}(\partial_i\mathcal{M}_j) + \alpha(\partial_l\gamma_{mk})(2\gamma^{ml}\gamma^{kj} - \gamma^{mk}\gamma^{lj})\mathcal{M}_j - 4\gamma^{ij}(\partial_j\alpha)\mathcal{M}_i,\\ \partial_t\mathcal{M}_i &= -(1/2)\alpha(\partial_i\mathcal{H}) - (\partial_i\alpha)\mathcal{H} + \beta^j(\partial_j\mathcal{M}_i) + \alpha K\mathcal{M}_i - \beta^k\gamma^{jl}(\partial_i\gamma_{lk})\mathcal{M}_j + (\partial_i\beta_k)\gamma^{kj}\mathcal{M}_j.\end{aligned}$$

- Expression using $\mathcal{H}$ and $\mathcal{M}_i$ (2)

$$\begin{aligned}\partial_t\mathcal{H} &= \beta^l\partial_l\mathcal{H} + 2\alpha K\mathcal{H} - 2\alpha\gamma^{-1/2}\partial_l(\sqrt{\gamma}\mathcal{M}^l) - 4(\partial_l\alpha)\mathcal{M}^l\\ &= \beta^l\nabla_l\mathcal{H} + 2\alpha K\mathcal{H} - 2\alpha(\nabla_l\mathcal{M}^l) - 4(\nabla_l\alpha)\mathcal{M}^l,\\ \partial_t\mathcal{M}_i &= -(1/2)\alpha(\partial_i\mathcal{H}) - (\partial_i\alpha)\mathcal{H} + \beta^l\nabla_l\mathcal{M}_i + \alpha K\mathcal{M}_i + (\nabla_i\beta_l)\mathcal{M}^l\\ &= -(1/2)\alpha(\nabla_i\mathcal{H}) - (\nabla_i\alpha)\mathcal{H} + \beta^l\nabla_l\mathcal{M}_i + \alpha K\mathcal{M}_i + (\nabla_i\beta_l)\mathcal{M}^l,\end{aligned}$$

- Expression using $\mathcal{H}$ and $\mathcal{M}_i$ (3): by using Lie derivatives along αn^μ,

$$\begin{aligned}\pounds_{\alpha n^\mu}\mathcal{H} &= +2\alpha K\mathcal{H} - 2\alpha\gamma^{-1/2}\partial_l(\sqrt{\gamma}\mathcal{M}^l) - 4(\partial_l\alpha)\mathcal{M}^l,\\ \pounds_{\alpha n^\mu}\mathcal{M}_i &= -(1/2)\alpha(\partial_i\mathcal{H}) - (\partial_i\alpha)\mathcal{H} + \alpha K\mathcal{M}_i.\end{aligned}$$

- Expression using γ_{ij} and K_{ij}

$$\begin{aligned}\partial_t\mathcal{H} &= H_1^{mn}(\partial_t\gamma_{mn}) + H_2^{imn}\partial_i(\partial_t\gamma_{mn}) + H_3^{ijmn}\partial_i\partial_j(\partial_t\gamma_{mn}) + H_4^{mn}(\partial_t K_{mn}),\\ \partial_t\mathcal{M}_i &= M_{1i}{}^{mn}(\partial_t\gamma_{mn}) + M_{2i}{}^{jmn}\partial_j(\partial_t\gamma_{mn}) + M_{3i}{}^{mn}(\partial_t K_{mn}) + M_{4i}{}^{jmn}\partial_j(\partial_t K_{mn}),\end{aligned}$$

where

$$
\begin{aligned}
H_1^{mn} &:= -2R^{(3)mn} - \Gamma^p_{kj}\Gamma^k_{pi}\gamma^{mi}\gamma^{nj} + \Gamma^m\Gamma^n \\
&\quad +\gamma^{ij}\gamma^{np}(\partial_i\gamma^{mk})(\partial_j\gamma_{kp}) - \gamma^{mp}\gamma^{ni}(\partial_i\gamma^{kj})(\partial_j\gamma_{kp}) - 2KK^{mn} + 2K^n{}_jK^{mj}, \\
H_2^{imn} &:= -2\gamma^{mi}\Gamma^n - (3/2)\gamma^{ij}(\partial_j\gamma^{mn}) + \gamma^{mj}(\partial_j\gamma^{in}) + \gamma^{mn}\Gamma^i, \\
H_3^{ijmn} &:= -\gamma^{ij}\gamma^{mn} + \gamma^{in}\gamma^{mj}, \\
H_4^{mn} &:= 2(K\gamma^{mn} - K^{mn}), \\
M_{1i}{}^{mn} &:= \gamma^{nj}(\partial_i K^m{}_j) - \gamma^{mj}(\partial_j K^n{}_i) + (1/2)(\partial_j\gamma^{mn})K^j{}_i + \Gamma^n K^m{}_i, \\
M_{2i}{}^{jmn} &:= -\gamma^{mj}K^n{}_i + (1/2)\gamma^{mn}K^j{}_i + (1/2)K^{mn}\delta^j_i, \\
M_{3i}{}^{mn} &:= -\delta^n_i\Gamma^m - (1/2)(\partial_i\gamma^{mn}), \\
M_{4i}{}^{jmn} &:= \gamma^{mj}\delta^n_i - \gamma^{mn}\delta^j_i,
\end{aligned}
$$

where we expressed $\Gamma^m = \Gamma^m_{ij}\gamma^{ij}$.

4 Constraint propagations in spherically symmetric spacetime

4.1 The procedure

The discussion becomes clear if we expand the constraint $C_\mu := (\mathcal{H}, \mathcal{M}_i)^T$ using vector harmonics.

$$C_\mu = \sum_{l,m} \left(A^{lm}(t,r) a_{lm}(\theta,\varphi) + B^{lm} b_{lm} + C^{lm} c_{lm} + D^{lm} d_{lm} \right), \tag{1}$$

where we choose the basis of the vector harmonics as

$$a_{lm} = \begin{pmatrix} Y_{lm} \\ 0 \\ 0 \\ 0 \end{pmatrix}, b_{lm} = \begin{pmatrix} 0 \\ Y_{lm} \\ 0 \\ 0 \end{pmatrix}, c_{lm} = \frac{r}{\sqrt{l(l+1)}} \begin{pmatrix} 0 \\ 0 \\ \partial_\theta Y_{lm} \\ \partial_\varphi Y_{lm} \end{pmatrix}, d_{lm} = \frac{r}{\sqrt{l(l+1)}} \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{\sin\theta}\partial_\varphi Y_{lm} \\ \sin\theta\, \partial_\theta Y_{lm} \end{pmatrix}.$$

The basis are normalized so that they satisfy

$$\langle C_\mu, C_\nu \rangle = \int_0^{2\pi} d\varphi \int_0^\pi C_\mu^* C_\rho \eta^{\nu\rho} \sin\theta d\theta,$$

where $\eta^{\nu\rho}$ is Minkowskii metric and the asterisk denotes the complex conjugate. Therefore

$$A^{lm} = \langle a^{lm}_{(\nu)}, C_\nu \rangle, \quad \partial_t A^{lm} = \langle a^{lm}_{(\nu)}, \partial_t C_\nu \rangle, \quad \text{etc.}$$

We also express these evolution equations using the Fourier expansion on the radial coordinate,

$$A^{lm} = \sum_k \hat{A}^{lm}_{(k)}(t)\, e^{ikr} \quad \text{etc.} \tag{2}$$

So that we will be able to obtain the RHS of the evolution equations for $(\hat{A}^{lm}_{(k)}(t), \cdots, \hat{D}^{lm}_{(k)}(t))^T$ in a homogeneous form.

4.2 Constraint propagations in Schwarzschild spacetime

1. the standard Schwarzschild coordinate

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{1 - 2M/r} + r^2 d\Omega^2, \qquad \text{(the standard expression)}$$

2. the isotropic coordinate, which is given by, $r = (1 + M/2r_{iso})^2 r_{iso}$:

$$ds^2 = -(\frac{1 - M/2r_{iso}}{1 + M/2r_{iso}})^2 dt^2 + (1 + \frac{M}{2r_{iso}})^4 [dr_{iso}^2 + r_{iso}^2 d\Omega^2], \qquad \text{(the isotropic expression)}$$

3. the ingoing Eddington-Finkelstein (iEF) coordinate, by $t_{iEF} = t + 2M \log(r - 2M)$:

$$ds^2 = -(1 - \frac{2M}{r})dt_{iEF}^2 + \frac{4M}{r} dt_{iEF} dr + (1 + \frac{2M}{r})dr^2 + r^2 d\Omega^2 \qquad \text{(the iEF expression)}$$

4. the Painlevé-Gullstrand (PG) coordinates,

$$ds^2 = -\left(1 - \frac{2\,M}{r}\right) dt_{PG}^2 + 2\sqrt{\frac{2\,M}{r}} dt_{PG}\, dr + dr^2 + r^2 d\Omega^2 \,, \qquad \text{(the PG expression)}$$

which is given by $t_{PG} = t + \sqrt{8Mr} - 2M \log\{(\sqrt{r/2M} + 1)/(\sqrt{r/2M} - 1)\}$

Example 1: standard ADM vs original ADM (in Schwarzschild coordinate)

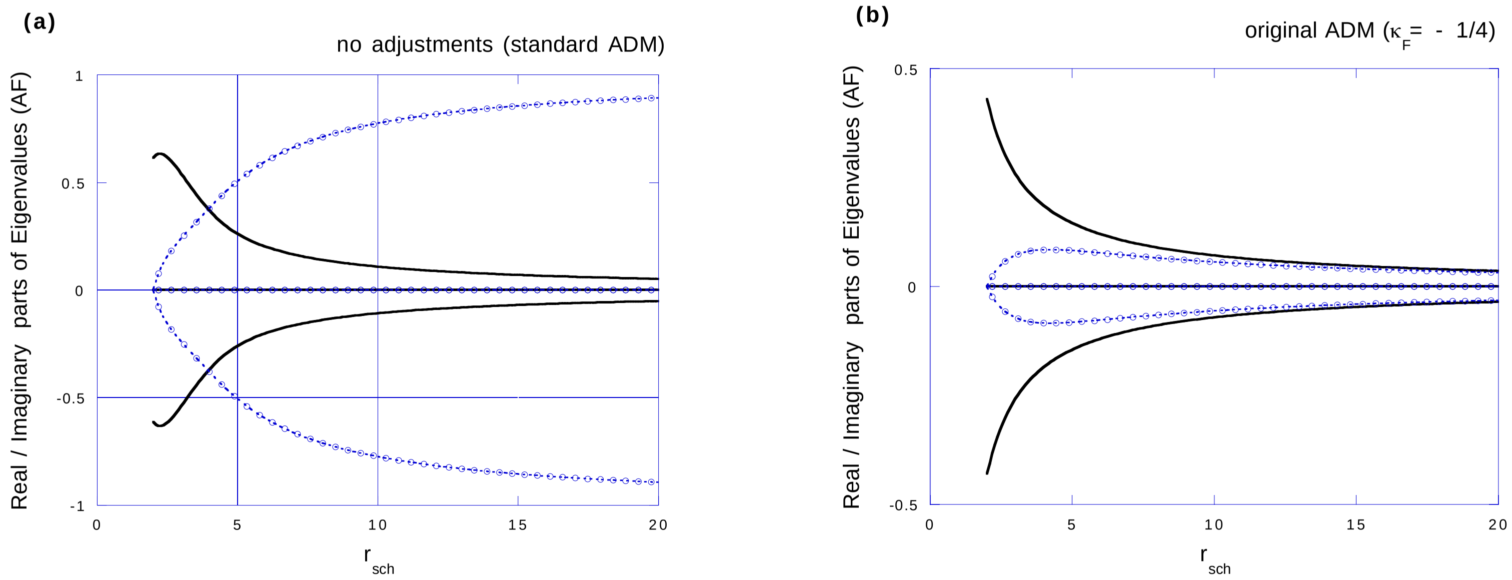


Figure 1: Amplification factors (AFs, eigenvalues of homogenized constraint propagation equations) are shown for the standard Schwarzschild coordinate, with (a) no adjustments, i.e., standard ADM, (b) original ADM ($\kappa_F = -1/4$). The solid lines and the dotted lines with circles are real parts and imaginary parts, respectively. They are four lines each, but actually the two eigenvalues are zero for all cases. Plotting range is $2 < r \leq 20$ using Schwarzschild radial coordinate. We set $k = 1, l = 2$, and $m = 2$ throughout the article.

$$\begin{aligned}
\partial_t \gamma_{ij} &= -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i, \\
\partial_t K_{ij} &= \alpha R^{(3)}_{ij} + \alpha K K_{ij} - 2\alpha K_{ik} K^k{}_j - \nabla_i \nabla_j \alpha + (\nabla_i \beta^k) K_{kj} + (\nabla_j \beta^k) K_{ki} + \beta^k \nabla_k K_{ij} + \kappa_F \alpha \gamma_{ij} \mathcal{H},
\end{aligned}$$

Example 2: Detweiler-type adjusted (in Schwarzschild coord.)

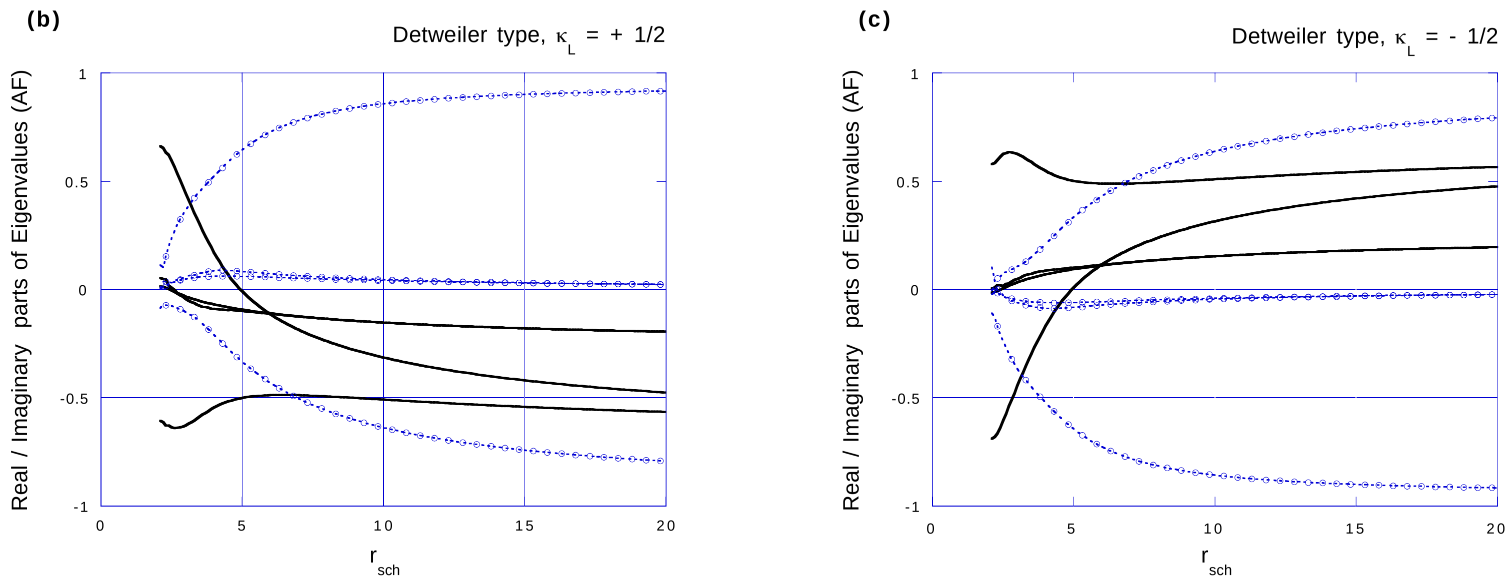


Figure 2: Amplification factors of the standard Schwarzschild coordinate, with Detweiler type adjustments. Multipliers used in the plot are (b) $\kappa_L = +1/2$, and (c) $\kappa_L = -1/2$.

$$\partial_t \gamma_{ij} = \text{(original terms)} + P_{ij}\mathcal{H},$$

$$\partial_t K_{ij} = \text{(original terms)} + R_{ij}\mathcal{H} + S^k{}_{ij}\mathcal{M}_k + s^{kl}{}_{ij}(\nabla_k \mathcal{M}_l),$$

where $P_{ij} = -\kappa_L \alpha^3 \gamma_{ij}, \quad R_{ij} = \kappa_L \alpha^3 (K_{ij} - (1/3) K \gamma_{ij}),$

$$S^k{}_{ij} = \kappa_L \alpha^2 [3(\partial_{(i}\alpha)\delta^k_{j)} - (\partial_l \alpha)\gamma_{ij}\gamma^{kl}], \quad s^{kl}{}_{ij} = \kappa_L \alpha^3 [\delta^k_{(i}\delta^l_{j)} - (1/3)\gamma_{ij}\gamma^{kl}],$$

Example 3: standard ADM (in isotropic/iEF coord.)

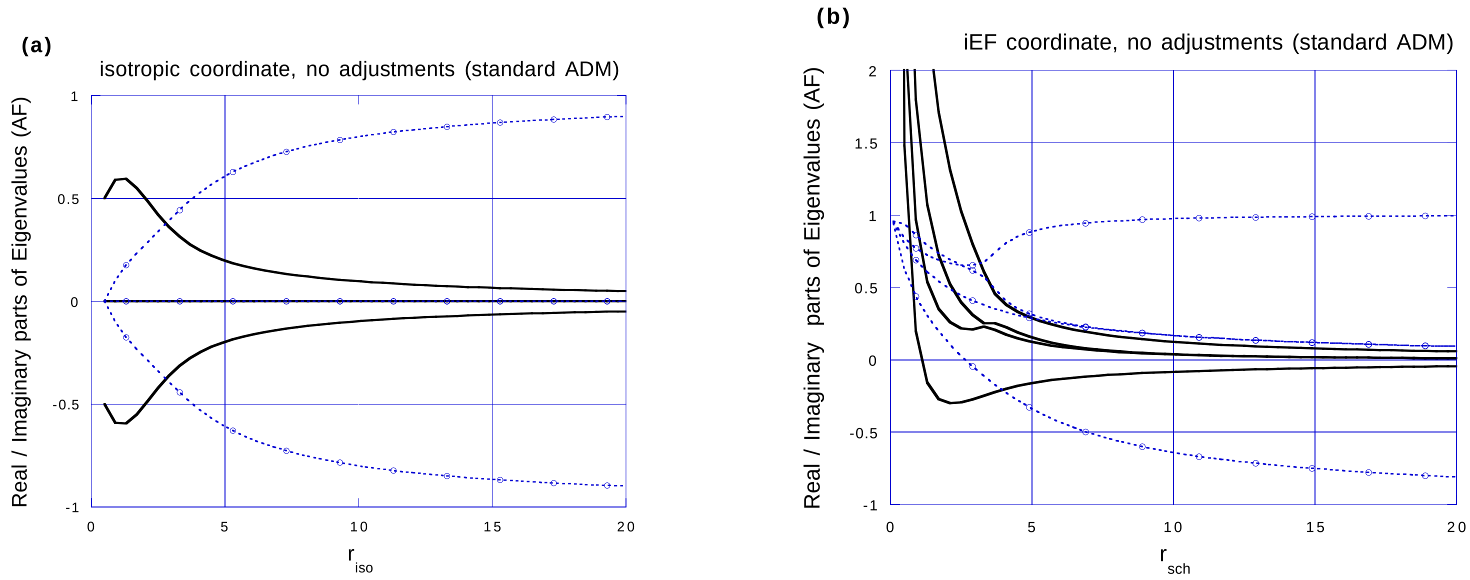


Figure 3: Comparison of amplification factors between different coordinate expressions for the standard ADM formulation (i.e. no adjustments). Fig. (a) is for the isotropic coordinate (1), and the plotting range is $1/2 \leq r_{iso}$. Fig. (b) is for the iEF coordinate (1) and we plot lines on the $t = 0$ slice for each expression. The solid four lines and the dotted four lines with circles are real parts and imaginary parts, respectively.

Example 4: Detweiler-type adjusted (in iEF/PG coord.)

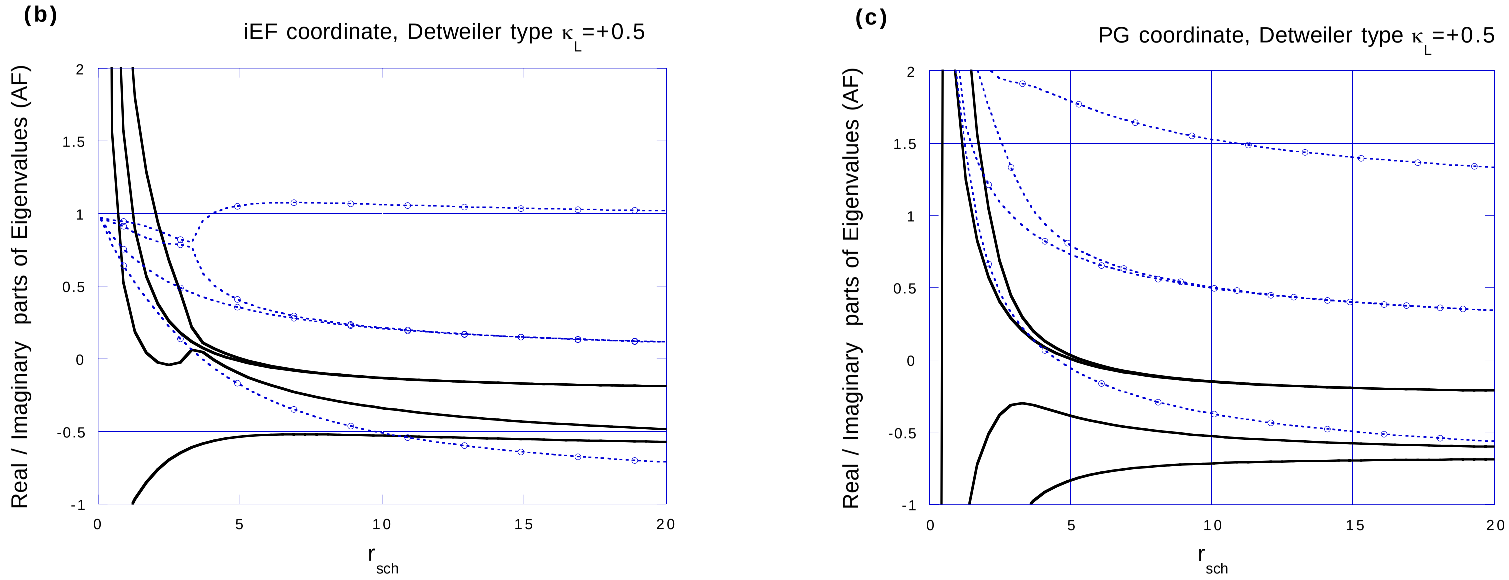


Figure 4: Similar comparison for Detweiler adjustments. $\kappa_L = +1/2$ for all plots.

No.	No. in Table.??	adjustment		1st?	Sch/iso coords.			iEF/PG coords.	
					TRS	real.	imag.	real.	imag.
0	0	–	no adjustments	yes	–	–	–	–	–
P-1	2-P	P_{ij}	$-\kappa_L\alpha^3\gamma_{ij}$	no	no	makes 2 Neg.	not apparent	makes 2 Neg.	not apparent
P-2	3	P_{ij}	$-\kappa_L\alpha\gamma_{ij}$	no	no	makes 2 Neg.	not apparent	makes 2 Neg.	not apparent
P-3	-	P_{ij}	$P_{rr} = -\kappa$ or $P_{rr} = -\kappa\alpha$	no	no	slightly enl.Neg.	not apparent	slightly enl.Neg.	not apparent
P-4	-	P_{ij}	$-\kappa\gamma_{ij}$	no	no	makes 2 Neg.	not apparent	makes 2 Neg.	not apparent
P-5	-	P_{ij}	$-\kappa\gamma_{rr}$	no	no	red. Pos./enl.Neg.	not apparent	red.Pos./enl.Neg.	not apparent
Q-1	-	$Q^k{}_{ij}$	$\kappa\alpha\beta^k\gamma_{ij}$	no	no	N/A	N/A	$\kappa \sim 1.35$ min. vals.	not apparent
Q-2	-	$Q^k{}_{ij}$	$Q^r{}_{rr} = \kappa$	no	yes	red. abs vals.	not apparent	red. abs vals.	not apparent
Q-3	-	$Q^k{}_{ij}$	$Q^r{}_{ij} = \kappa\gamma_{ij}$ or $Q^r{}_{ij} = \kappa\alpha\gamma_{ij}$	no	yes	red. abs vals.	not apparent	enl.Neg.	enl. vals.
Q-4	-	$Q^k{}_{ij}$	$Q^r{}_{rr} = \kappa\gamma_{rr}$	no	yes	red. abs vals.	not apparent	red. abs vals.	not apparent
R-1	1	R_{ij}	$\kappa_F\alpha\gamma_{ij}$	yes	yes	$\kappa_F = -1/4$ min. abs vals.		$\kappa_F = -1/4$ min. vals.	
R-2	4	R_{ij}	$R_{rr} = -\kappa_\mu\alpha$ or $R_{rr} = -\kappa_\mu$	yes	no	not apparent	not apparent	red.Pos./enl.Neg.	enl. vals.
R-3	-	R_{ij}	$R_{rr} = -\kappa\gamma_{rr}$	yes	no	enl. vals.	not apparent	red.Pos./enl.Neg.	enl. vals.
S-1	2-S	$S^k{}_{ij}$	$\kappa_L\alpha^2[3(\partial_{(i}\alpha)\delta^k_{j)} - (\partial_l\alpha)\gamma_{ij}\gamma^{kl}]$	yes	no	not apparent	not apparent	not apparent	not apparent
S-2	-	$S^k{}_{ij}$	$\kappa\alpha\gamma^{lk}(\partial_l\gamma_{ij})$	yes	no	makes 2 Neg.	not apparent	makes 2 Neg.	not apparent
p-1	-	$p^k{}_{ij}$	$p^r{}_{ij} = -\kappa\alpha\gamma_{ij}$	no	no	red. Pos.	red. vals.	red. Pos.	enl. vals.
p-2	-	$p^k{}_{ij}$	$p^r{}_{rr} = \kappa\alpha$	no	no	red. Pos.	red. vals.	red.Pos/enl.Neg.	enl. vals.
p-3	-	$p^k{}_{ij}$	$p^r{}_{rr} = \kappa\alpha\gamma_{rr}$	no	no	makes 2 Neg.	enl. vals.	red. Pos. vals.	red. vals.
q-1	-	$q^{kl}{}_{ij}$	$q^{rr}{}_{ij} = \kappa\alpha\gamma_{ij}$	no	no	$\kappa = 1/2$ min. vals.	red. vals.	not apparent	enl. vals.
q-2	-	$q^{kl}{}_{ij}$	$q^{rr}{}_{rr} = -\kappa\alpha\gamma_{rr}$	no	yes	red. abs vals.	not apparent	not apparent	not apparent
r-1	-	$r^k{}_{ij}$	$r^r{}_{ij} = \kappa\alpha\gamma_{ij}$	no	yes	not apparent	not apparent	not apparent	enl. vals.
r-2	-	$r^k{}_{ij}$	$r^r{}_{rr} = -\kappa\alpha$	no	yes	red. abs vals.	enl. vals.	red. abs vals.	enl. vals.
r-3	-	$r^k{}_{ij}$	$r^r{}_{rr} = -\kappa\alpha\gamma_{rr}$	no	yes	red. abs vals.	enl. vals.	red. abs vals.	enl. vals.
s-1	2-s	$s^{kl}{}_{ij}$	$\kappa_L\alpha^3[\delta^k_{(i}\delta^l_{j)} - (1/3)\gamma_{ij}\gamma^{kl}]$	no	no	makes 4 Neg.	not apparent	makes 4 Neg.	not apparent
s-2	-	$s^{kl}{}_{ij}$	$s^{rr}{}_{ij} = -\kappa\alpha\gamma_{ij}$	no	no	makes 2 Neg.	red. vals.	makes 2 Neg.	red. vals.
s-3	-	$s^{kl}{}_{ij}$	$s^{rr}{}_{rr} = -\kappa\alpha\gamma_{rr}$	no	no	makes 2 Neg.	red. vals.	makes 2 Neg.	red. vals.

Table 1: List of adjustments we tested in the Schwarzschild spacetime. The column of adjustments are nonzero multipliers. The effects to amplification factors (when $\kappa > 0$) are commented for each coordinate system and for real/imaginary parts of AFs, respectively. The 'N/A' means that there is no effect due to the coordinate properties; 'not apparent' means the adjustment does not change the AFs effectively according to our conjecture; 'enl./red./min.' means enlarge/reduce/minimize, and 'Pos./Neg.' means positive/negative, respectively. These judgements are made at the $r \sim O(10M)$ region on their $t = 0$ slice.

Example 5:
On Maximally-sliced hypersurfaces (standard ADM in Sch. coord.)

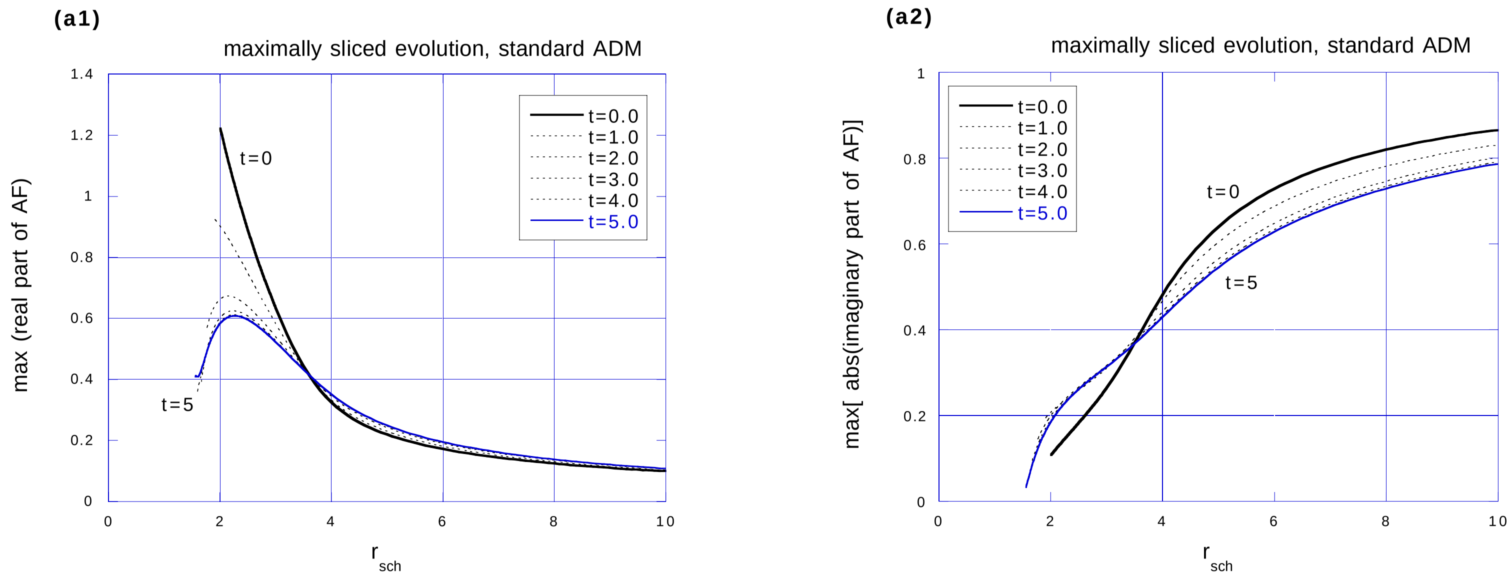


Figure 5: Amplification factors of snapshots of maximally-sliced evolving Schwarzschild spacetime. Fig (a1) and (a2) are of the standard ADM formulation (real and imaginary parts, respectively), Lines in (a1) are the largest (positive) AF on each time slice, while lines in (a2) are the maximum imaginary part of AF on each time slice. The lines start at $r_{min} = 2$ ($\bar{t} = 0$) and $r_{min} = 1.55$ ($\bar{t} = 5$).

Conclusion

Towards a stable and accurate formulation for numerical relativity

- Several reports say numerical stabilities depend on the formulations to apply, although they are mathematically equivalent.
- status = chaotic, many trials and errors
  We tried to understand the background systematically.
- Our proposal = "Evaluate eigenvalues of constraint propagation eqns"
  We give satisfactory conditions for stable evolutions.
  Fourier transformation allows to discuss lower-order terms.
- Our Observation = "Stability will change by adding constraints in RHS"
  We named "Adjusted System".
  Numerically confirmed in Maxwell system and Ashtekar system.
- Our Observation 2= The idea works even for the ADM formulation
  We explain the effective parameter range of Detweiler's system (1987).
  We proposed variety of adjustments, predicted their expected stability.

(A workshop on this subject will be held at Mexico, May 2002.)