Black Objects and Hoop Conjecture in Five-dimensional Space-time

Hisa-aki Shinkai
(Osaka Institute of Technology, Japan)

work with Yuta Yamada (OIT)

Initial Data

Yamada & HS, CQG 27 (2010) 045012

Evolution

Yamada & HS, in preparation.

http://www.is.oit.ac.jp/~shinkai/

GR19 @ Mexico City, July 2010

1. Motivation and Goal

Higher-Dim Black Holes have Rich Structures

LHC experiments will (or will not) reveal Higher-Dim BHs in near future

Brane-World models give new viewpoints to gravity and cosmology

4-dim BH : horizon is S^2,
stable solutions
Schwarzschild --- Birkoff theorem (M)
Kerr --- uniqueness theorem (M, J)

1. Motivation and Goal

Higher-Dim Black Holes have Rich Structures

4-dim BHs		Higher-dim BHs :
Schwarzschild	$\longrightarrow$	Tangherlini
		--- unique & stable
Kerr	$\longrightarrow$	Myers-Perry
		--- maybe unstable in higher J
"Black Objects"		black ring (Emparan-Reall) black Saturn di-rings, orthogonal di-rings, ...

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Higher-Dim Black Holes have Rich Structures

"Black Objects"

black hole
black string
black ring
black Saturn
di-rings, orthogonal di-rings ...

Uniqueness (only in spherical sym.)
Stability?
Formation Process?
Dynamical Features? ...

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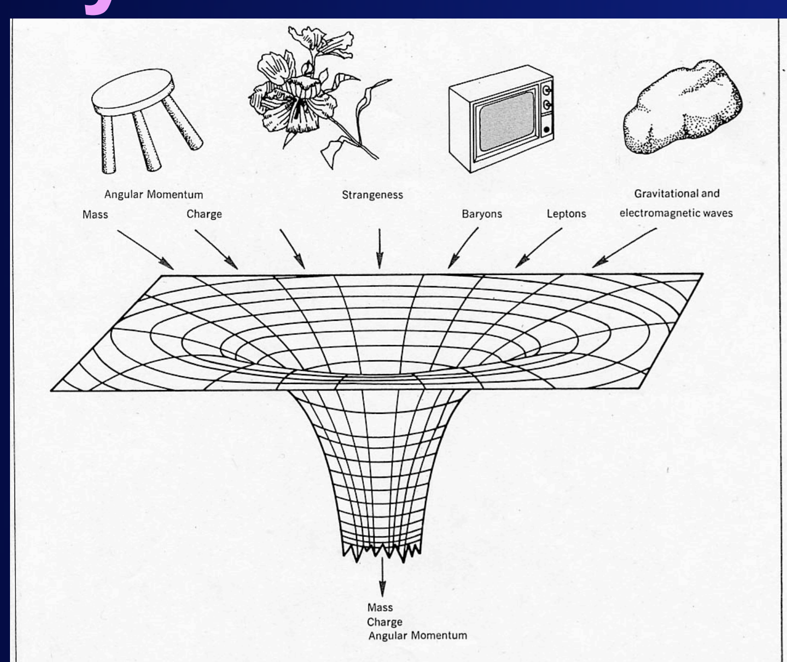
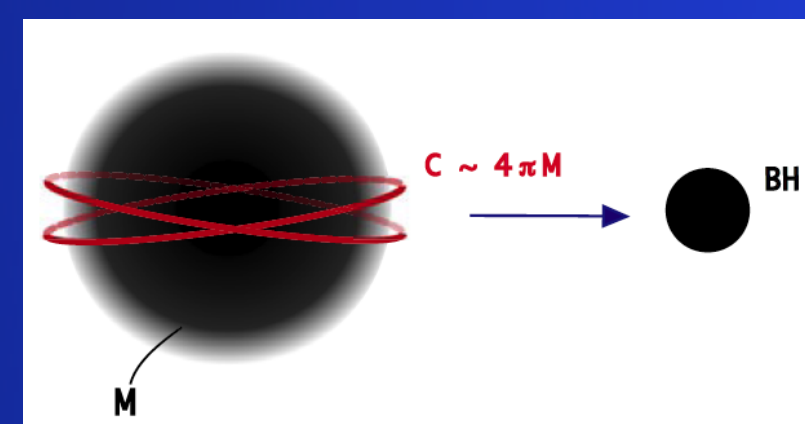
Stability?
Formation Process?
Dynamical Features? ...

No Hair Conjecture?
Cosmic Censorship?
Hoop Conjecture?

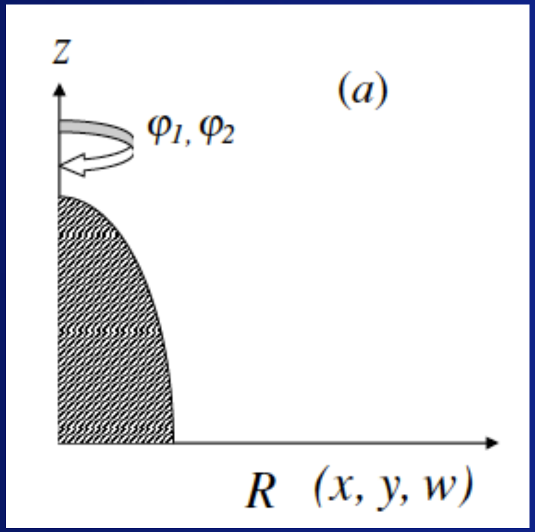


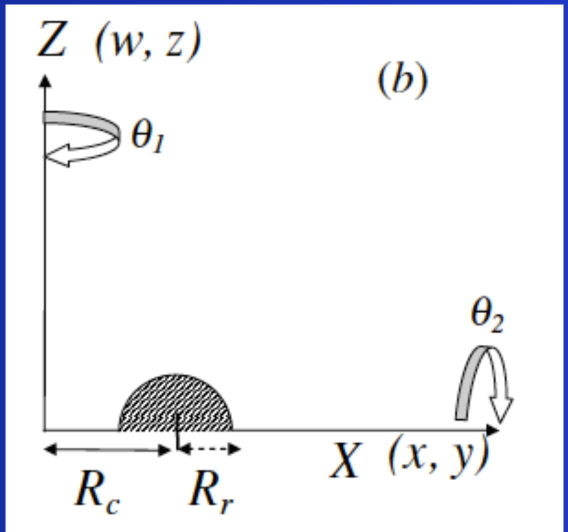
Figurative representation of a black hole in action. All details of the infalling matter are washed out. The final configuration is believed to be uniquely determined by mass, electric charge, and angular momentum. Figure 1



2. Initial Data Construction

- time symmetric, asymptotically flat
- conformal flat
- non-rotating homogeneous dust
- in spheroidal shape S^3 or ring shape $S^2 \times S^1$



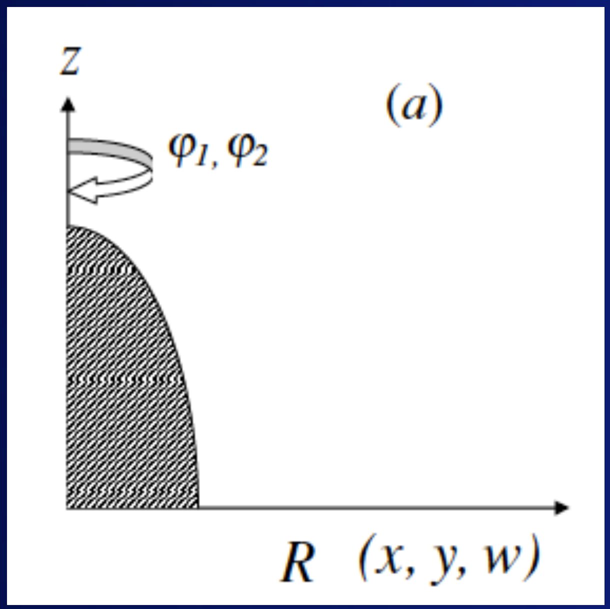


- solve the Hamiltonian constraint eq. 512^2 grids
- Apparent Horizon Search
  both for Ring Horizon and Common Horizon
- Define Hoop and check the Hoop Conjecture

2.A: Initial Data Construction

metric & Hamiltonian constraint

Spheroidal Cases

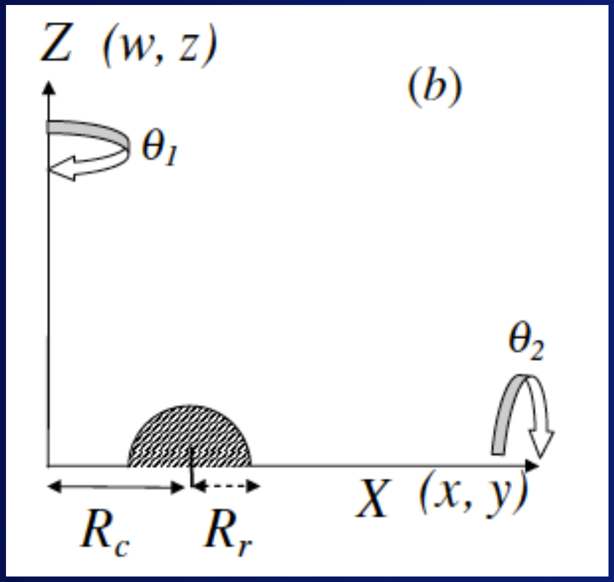


$$ds^2 = \psi(R,z)^2 \left[dR^2 + R^2(d\varphi_1^2 + \sin^2\varphi_1 d\varphi_2^2) + dz^2\right]$$

$$R = \sqrt{x^2+y^2+z^2}, \quad \varphi_1 = \tan^{-1}\left(\frac{w}{\sqrt{x^2+y^2}}\right), \; \varphi_2 = \tan^{-1}\left(\frac{y}{x}\right).$$

$$\frac{\partial^2\psi}{\partial R^2} + \frac{2}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial z^2} = -4\pi^2 G_5 \rho.$$

Toroidal Cases



$$ds^2 = \psi(X,Z)^2(dX^2 + dZ^2 + X^2 d\vartheta_1 + Z^2 d\vartheta_2)$$

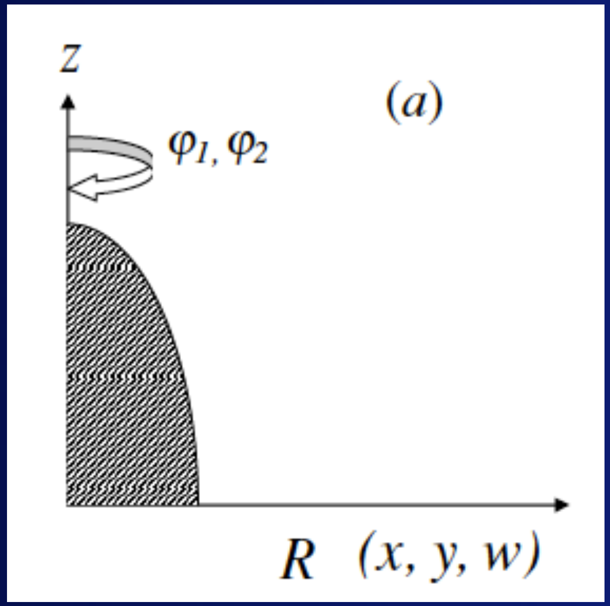
$$X = \sqrt{x^2+y^2}, \; Z = \sqrt{z^2+w^2}, \; \vartheta_1 = \tan^{-1}\left(\frac{y}{x}\right), \; \vartheta_2 = \tan^{-1}\left(\frac{z}{w}\right)$$

$$\frac{1}{X}\frac{\partial}{\partial X}\left(X\frac{\partial\psi}{\partial X}\right) + \frac{1}{Z}\frac{\partial}{\partial Z}\left(Z\frac{\partial\psi}{\partial Z}\right) = -4\pi^2 G_5 \rho.$$

2.A: Initial Data Construction

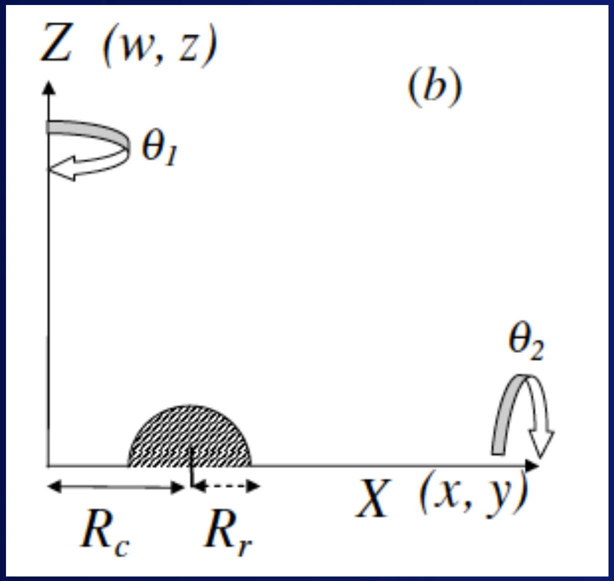
Apparent Horizons Search

Spheroidal Cases



$$\ddot{r}_M - \frac{4\dot{r}_M^2}{r_M} - 3r_M + \frac{r_M^2 + \dot{r}_M^2}{r_M}\left[\frac{2\dot{r}_M}{r_M}\cot\theta - \frac{3}{\psi}(\dot{r}_M\sin\theta + r_M\cos\theta)\frac{\partial\psi}{\partial z} + \frac{3}{\psi}(\dot{r}_M\cos\theta - r_M\sin\theta)\frac{\partial\psi}{\partial R}\right] = 0$$

Toroidal Cases



Common Horizon

$$\ddot{r}_m - 4\frac{\dot{r}_m^{\,2}}{r_m} - 3r_m - \frac{r_m^2 + \dot{r}_m^{\,2}}{r_m}\left[2\frac{\dot{r}_m}{r_m}\cot(2\phi) - \frac{3}{\psi}(\dot{r}_m\sin\phi + r\cos\phi)\frac{\partial\psi}{\partial X} + \frac{3}{\psi}(\dot{r}_m\cos\phi - r_m\sin\phi)\frac{\partial\psi}{\partial Z}\right] = 0$$

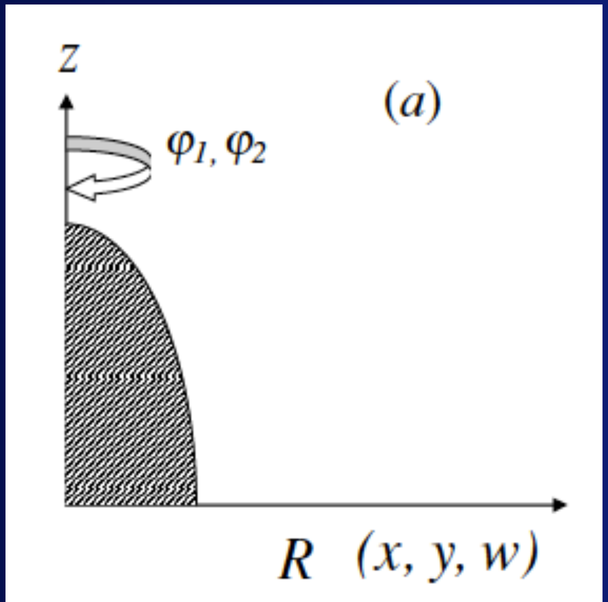
Ring Horizon

$$\ddot{r}_m - \frac{3\dot{r}_m^{\,2}}{r_m} - 2r_m - \frac{r_m^2 + \dot{r}_m^{\,2}}{r_m} \times \left[\frac{\dot{r}_m\sin\xi + r_m\cos\xi}{r_m\cos\xi + R_c} - \frac{\dot{r}_m}{r_m}\cot\xi + \frac{3}{\psi}(\dot{r}_m\sin\xi + r\cos\xi)\frac{\partial\psi}{\partial x} - \frac{3}{\psi}(\dot{r}_m\cos\xi - r\sin\xi)\frac{\partial\psi}{\partial z}\right] = 0$$

2.A: Initial Data Construction

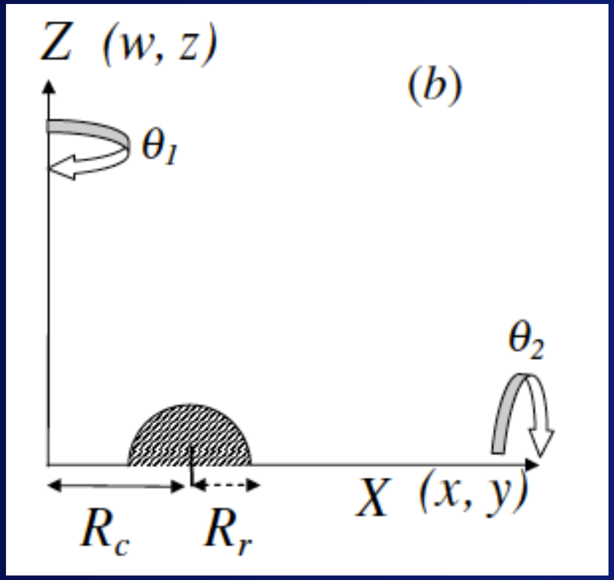
Area of Horizons

Spheroidal Cases



$$A_3^{(S)} = 8\pi \int_0^{\pi/2} \psi^3 r_M^2 \sin^2\theta \sqrt{\dot{r_M}^2 + r_M^2}\, d\theta$$

Toroidal Cases



Common Horizon

$$A_3^{(T1)} = 4\pi^2 \int_0^{\pi/2} \psi^3 r_m^2 \cos\phi \sin\phi \sqrt{\dot{r_m}^2 + r_m^2}\, d\phi$$

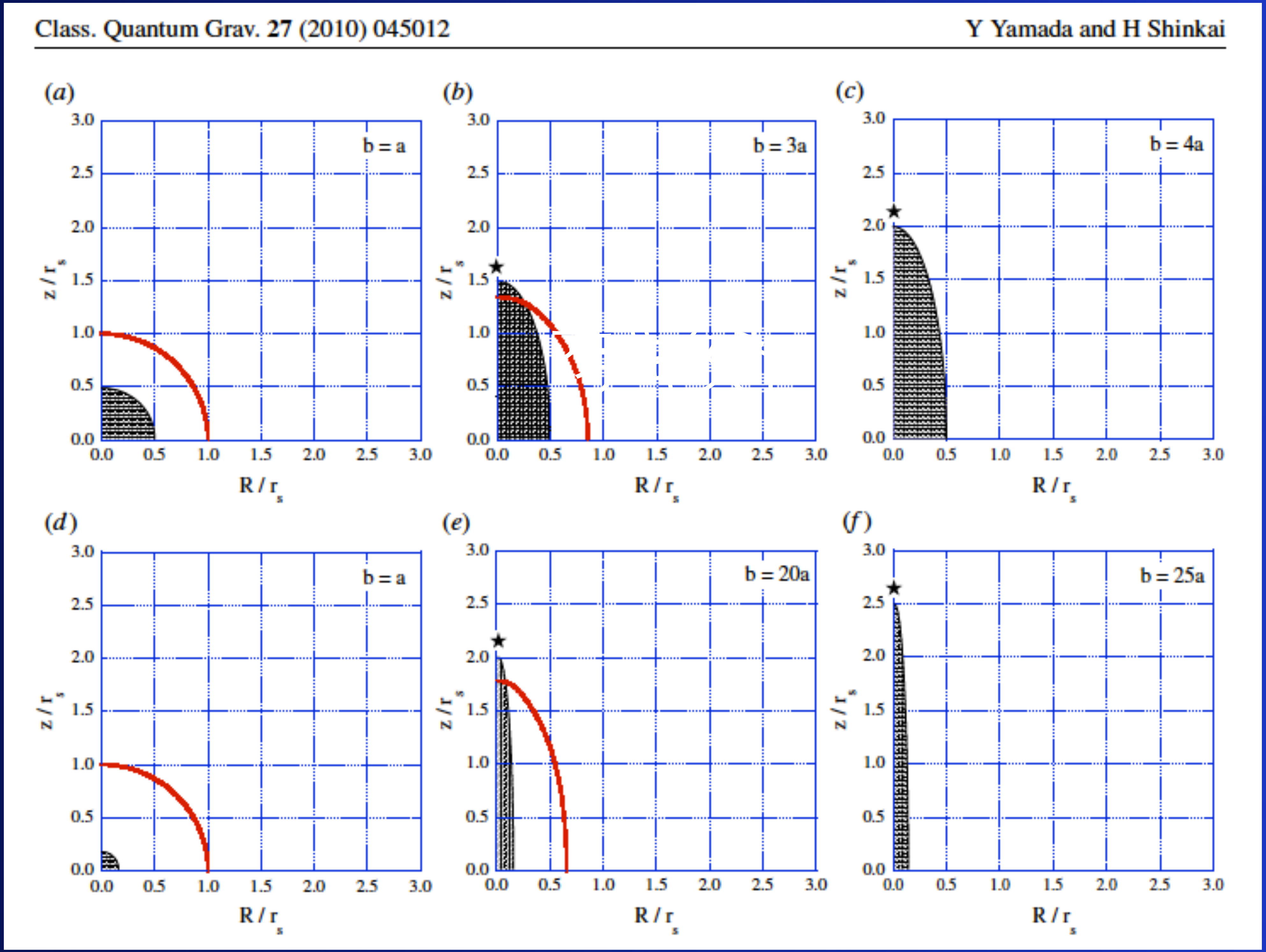
Ring Horizon

$$A_3^{(T2)} = 4\pi^2 \int_0^{\pi} \psi^3 (R_c + r_m \cos\xi) r_m \sin\xi \sqrt{\dot{r_m}^2 + r_m^2}\, d\xi$$

2.B: Initial Data Results

Spheroidal Cases

cf. (3-dim.) Nakamura-Shapiro-Teukolsky (1988)



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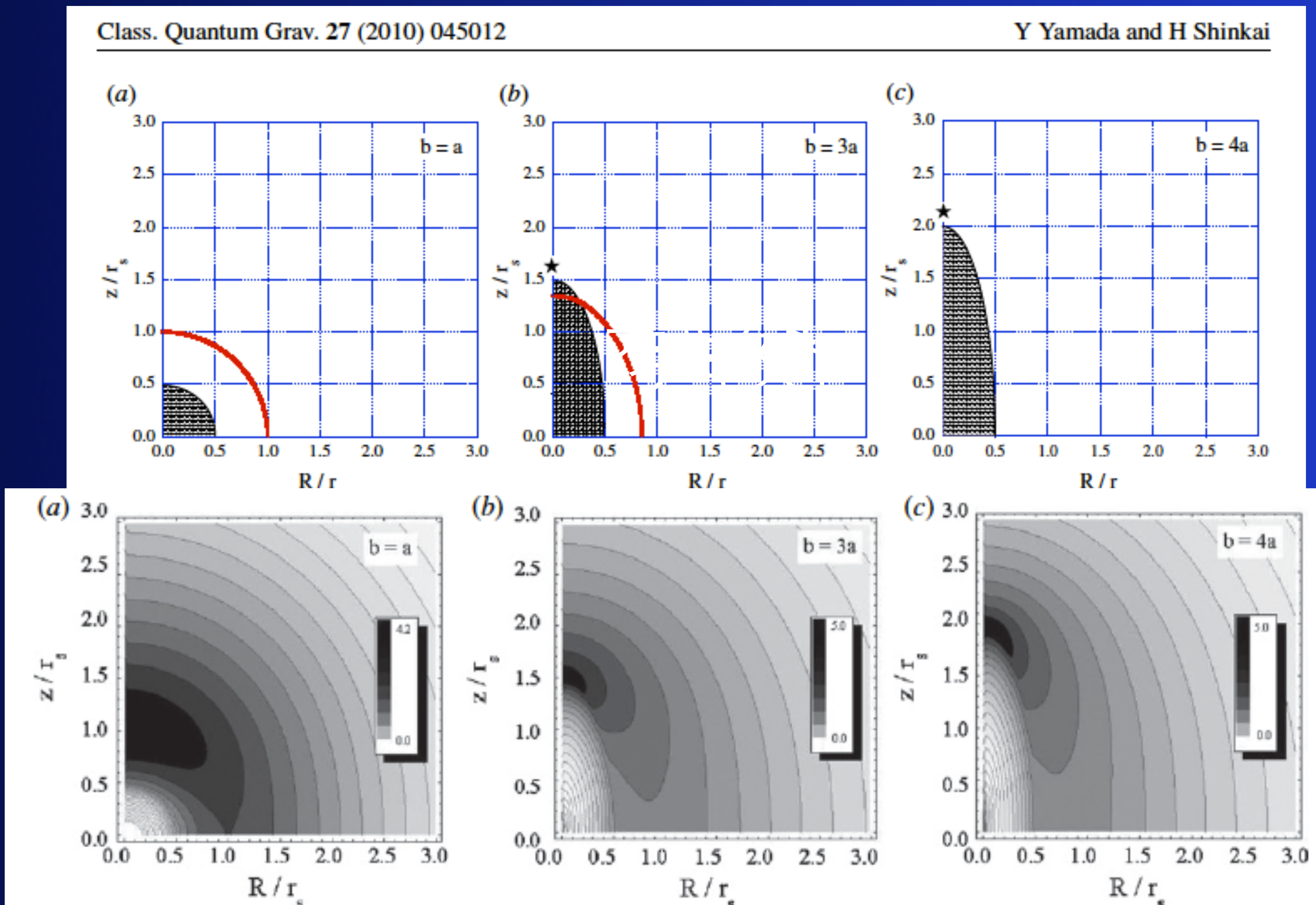
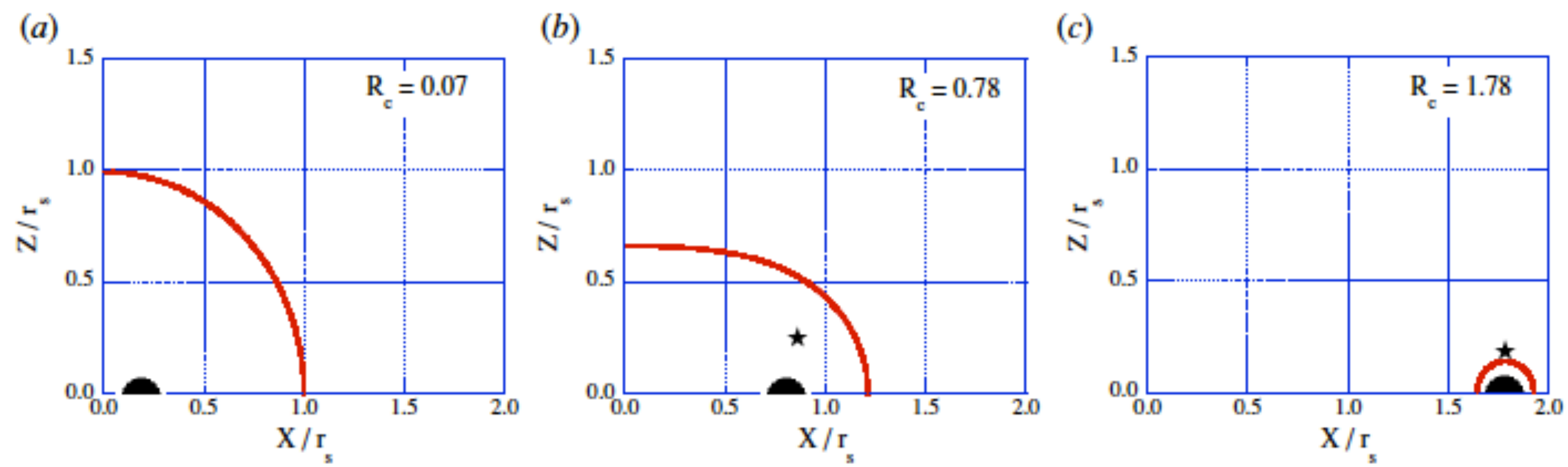


Contour Plot of the Kretchmann invariant, $R_{abcd}R^{abcd}$

2.B: Initial Data Results

Toroidal Cases

Class. Quantum Grav. **27** (2010) 045012 Y Yamada and H Shinkai



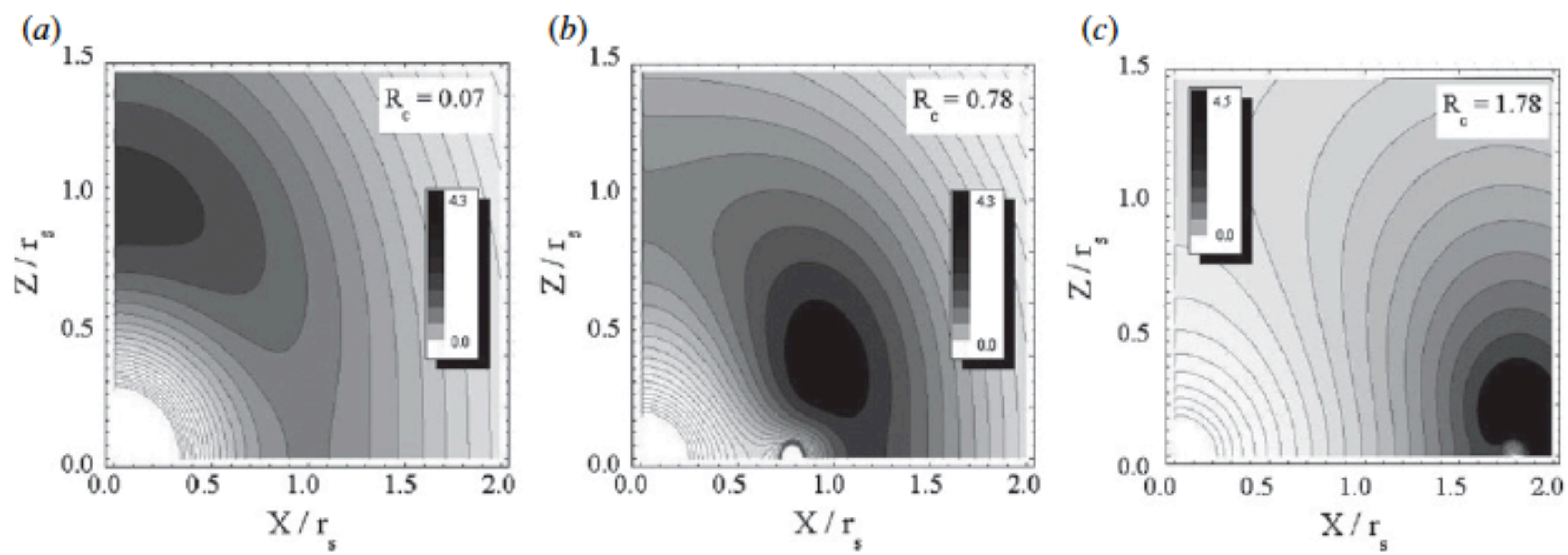
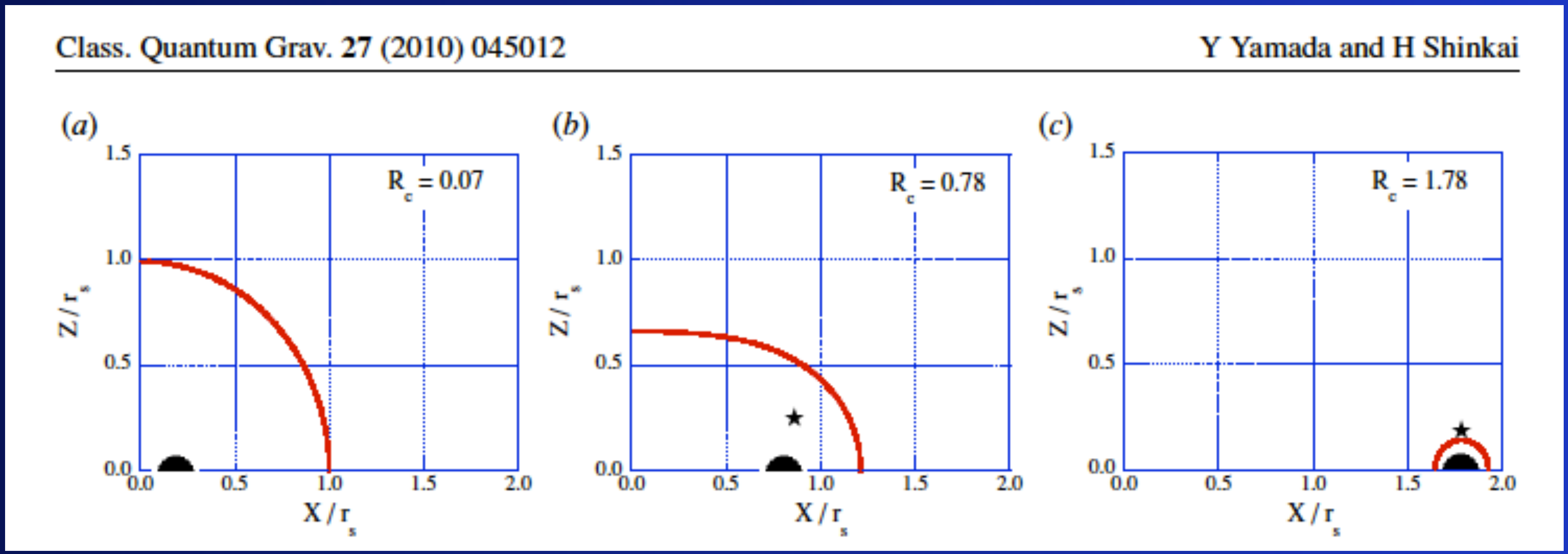
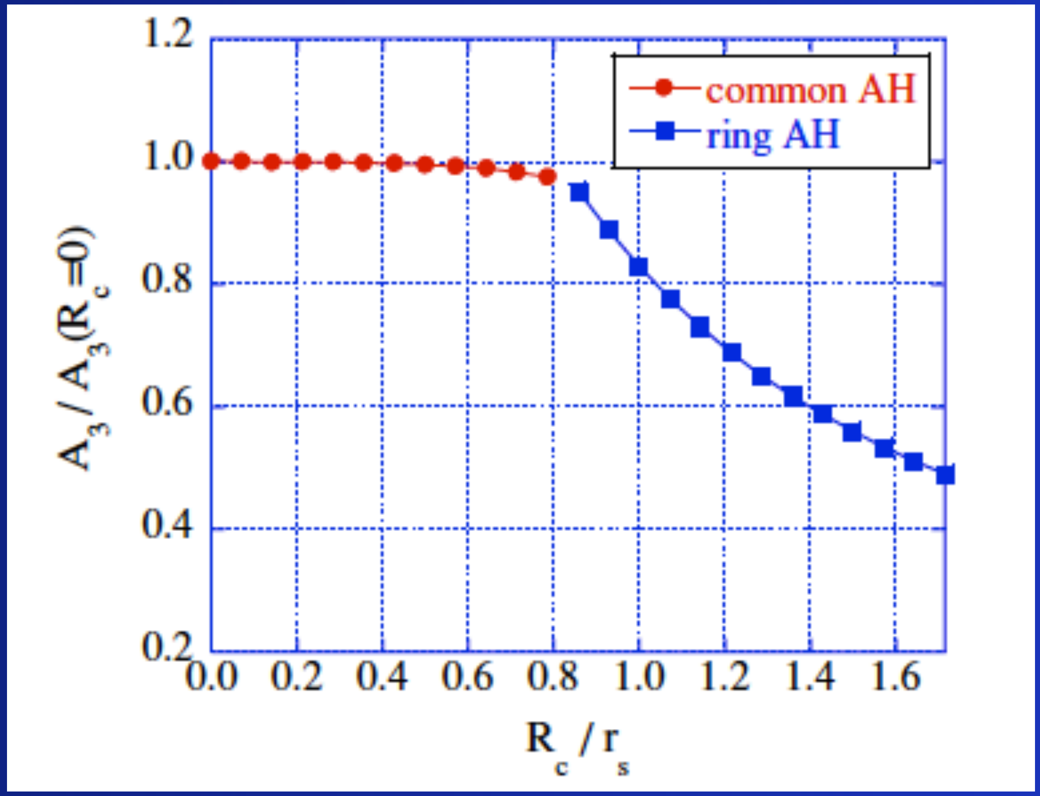


Figure 8. Contours of Kretchmann invariant, $\log_{10} \mathcal{I}^{(4)}$, corresponding to figure 7.

2.B: Initial Data Results

Toroidal Cases

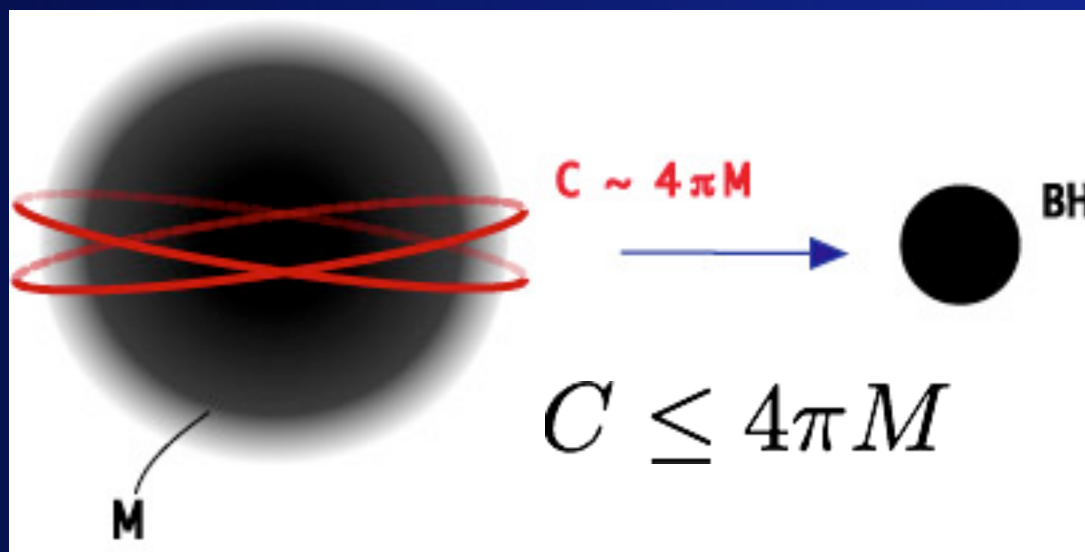




2.C. Initial Data Analysis

Hyper-Hoop conjecture ?

Hoop Conjecture Thorne (1972)



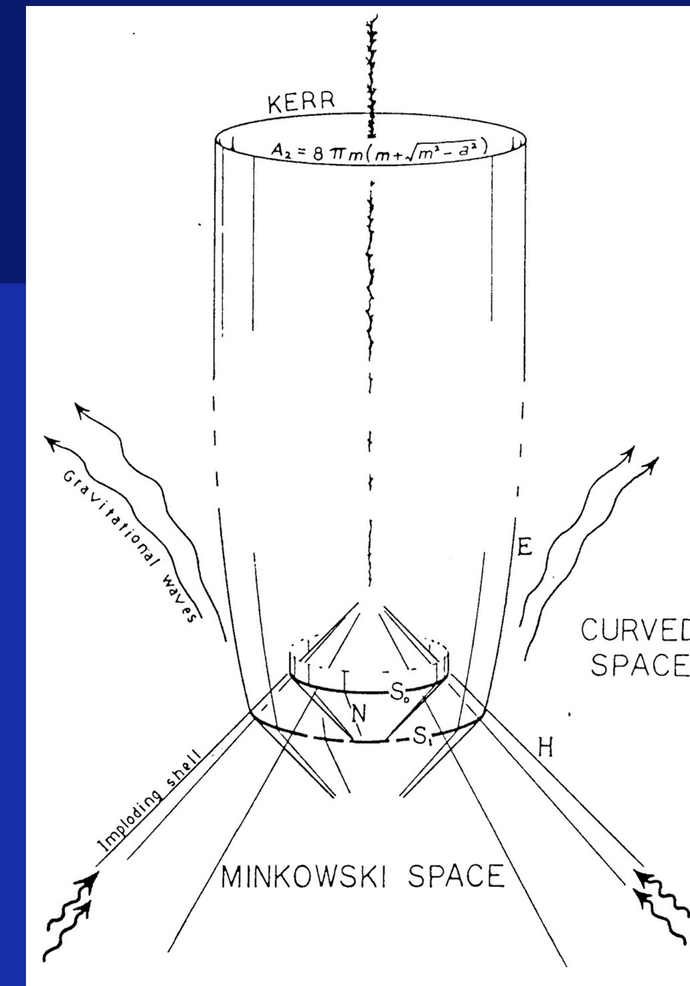


FIGURE 3. An attempt to derive a contradiction with the "establishment" viewpoint. A shell of photons collapses into a flat space-time. The assumption that a K black hole results implies the existence of a certain inequality on the initial geor

Penrose (1969)

$$A \leq 16\pi M^2$$

Hyper-Hoop Conjecture

Ida-Nakao (2002)

$$V_{D-3} \leq G_D M$$

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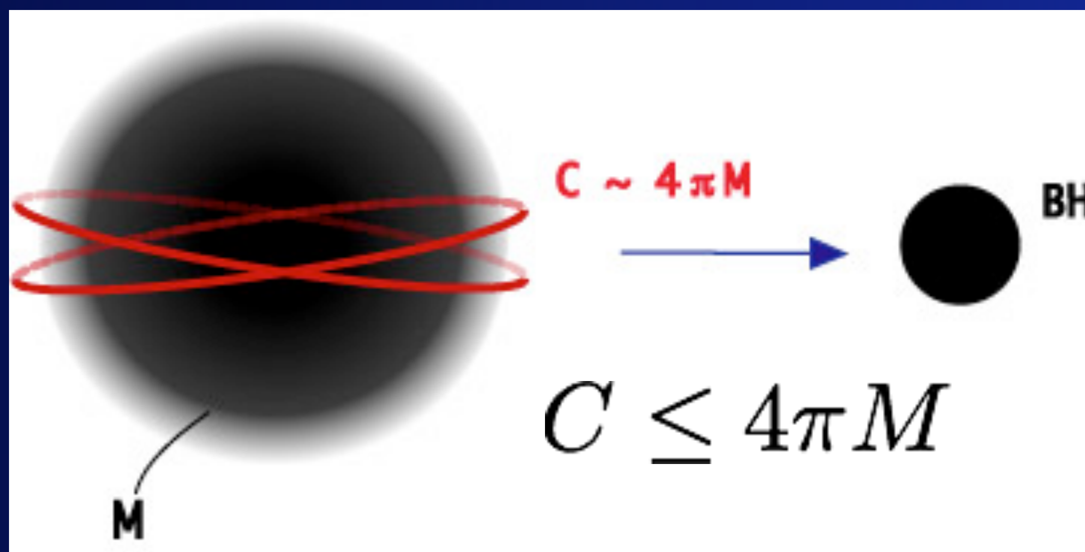

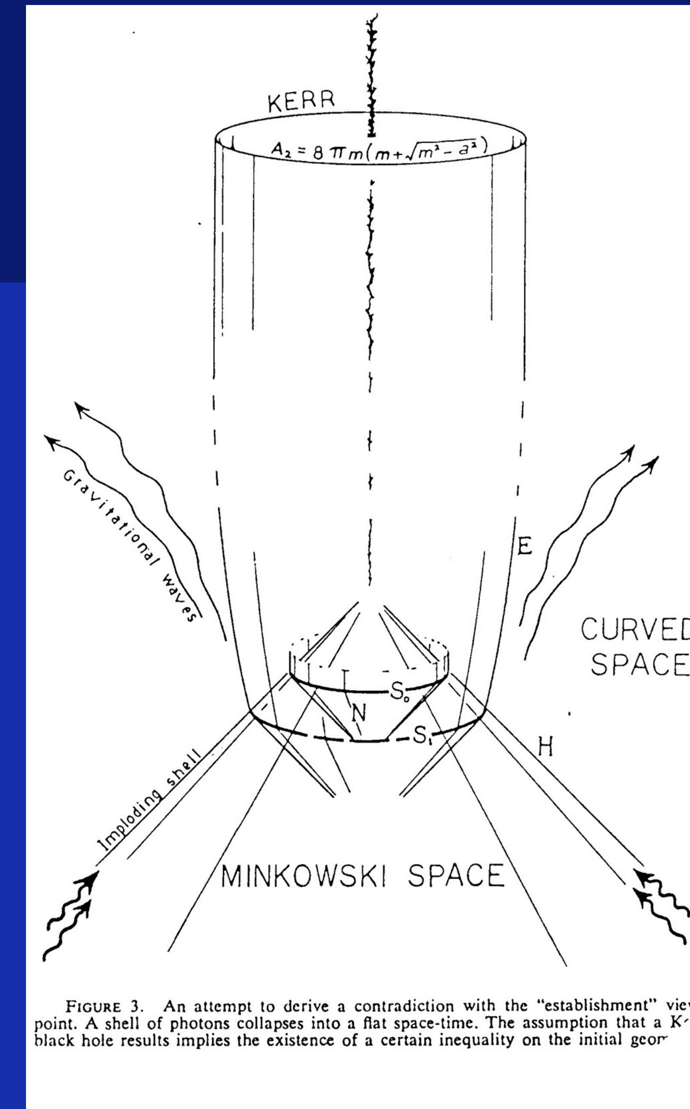

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Hyper-Hoop Conjecture

Ida-Nakao (2002)

$$V_{D-3} \leq G_D M$$

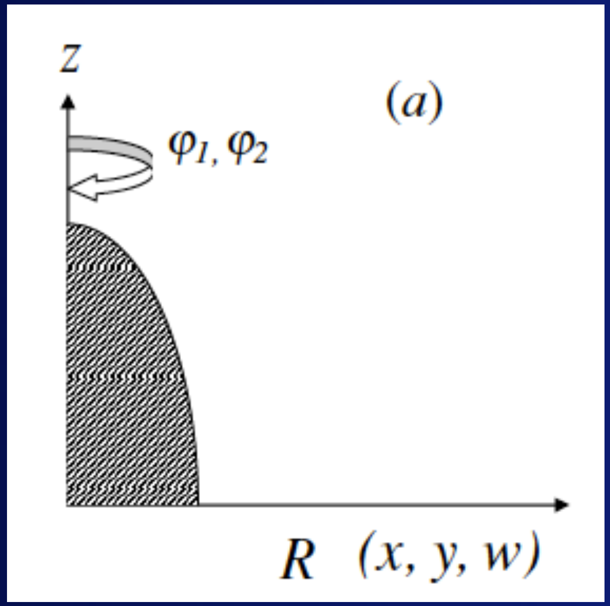
In 5-D, if mass gets compacted
in some area, ...

2.C. Initial Data Analysis

Hyper-Hoop conjecture ?

$$V_2 \leq \frac{\pi}{2} 16\pi G_5 M$$

Spheroidal Cases



Define Hyper-Hoop as the surface $\delta V_2 = 0$

$$V_2^{(A)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \sin\theta \, d\theta$$

$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h + \frac{r_h^2 + \dot{r_h}^2}{r_h} \left[\frac{\dot{r_h}}{r_h} \cot\theta - \frac{2}{\psi} (\dot{r_h} \sin\theta + r_h \cos\theta) \frac{\partial \psi}{\partial z} - \frac{2}{\psi} (r_h \sin\theta - \dot{r_h} \cos\theta) \frac{\partial \psi}{\partial R} \right] = 0$$

$$V_2^{(B)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \cos\theta \, d\theta$$

$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h - \frac{r_h^2 + \dot{r_h}^2}{r_h} \left[\frac{\dot{r_h}}{r_h} \tan\theta + \frac{2}{\psi} (r_h \sin\theta - \dot{r_h} \cos\theta) \frac{\partial \psi}{\partial R} + \frac{2}{\psi} (r_h \cos\theta + \dot{r_h} \sin\theta) \frac{\partial \psi}{\partial z} \right] = 0$$

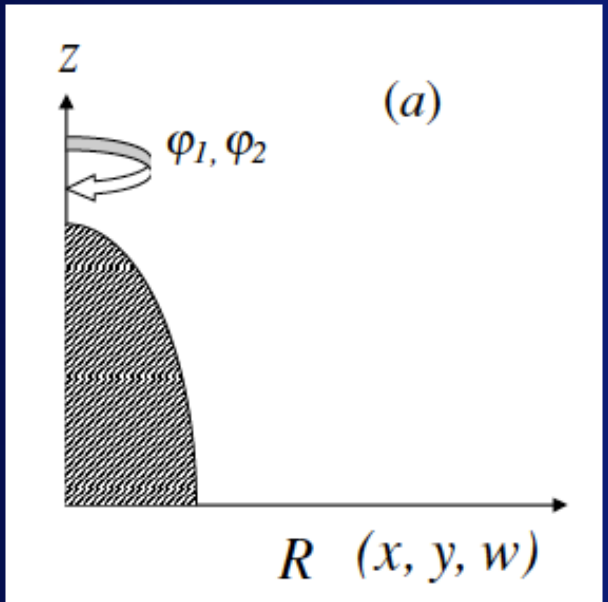
2.C. Initial Data Analysis

Hyper-Hoop conjecture ?

$$V_2 \le \frac{\pi}{2} 16\pi G_5 M$$

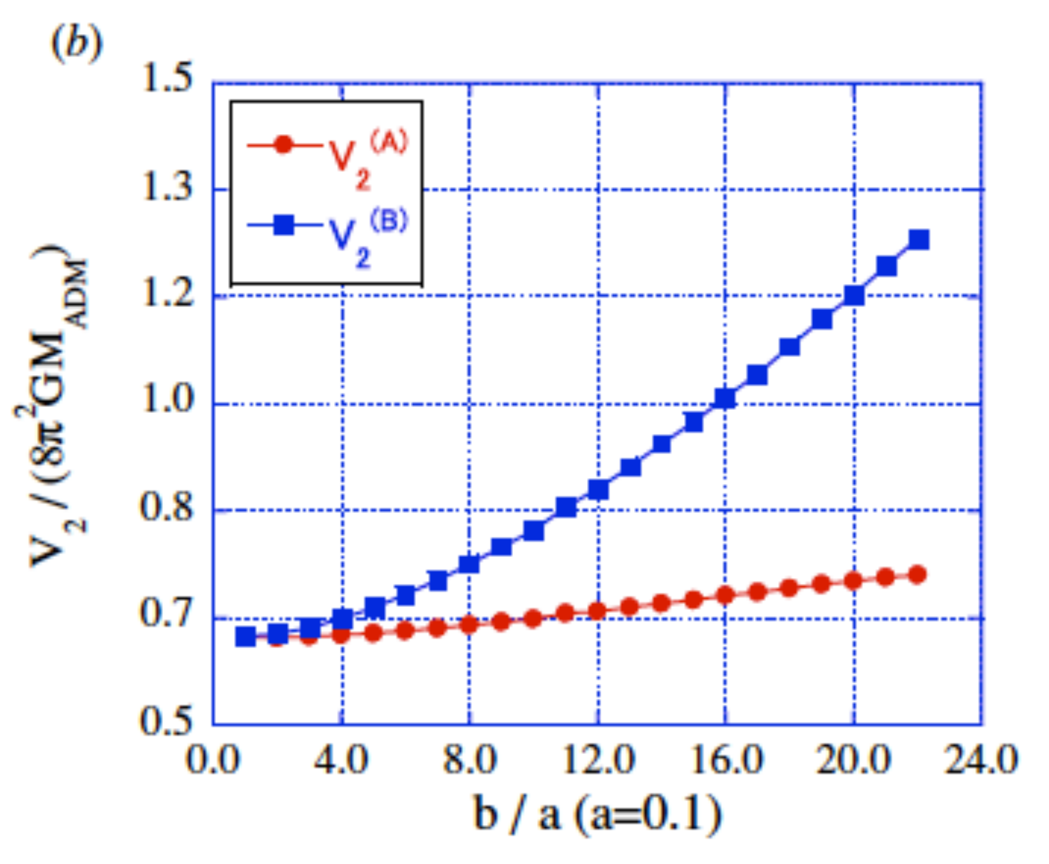
Spheroidal Cases

Define Hyper-Hoop as the surface $\delta V_2 = 0$



$$V_2^{(A)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \sin\theta \, d\theta$$

$$V_2^{(B)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \cos\theta \, d\theta$$



2.C. Initial Data Analysis

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$$V_2 \le \frac{\pi}{2} 16\pi G_5 M$$

Spheroidal Cases

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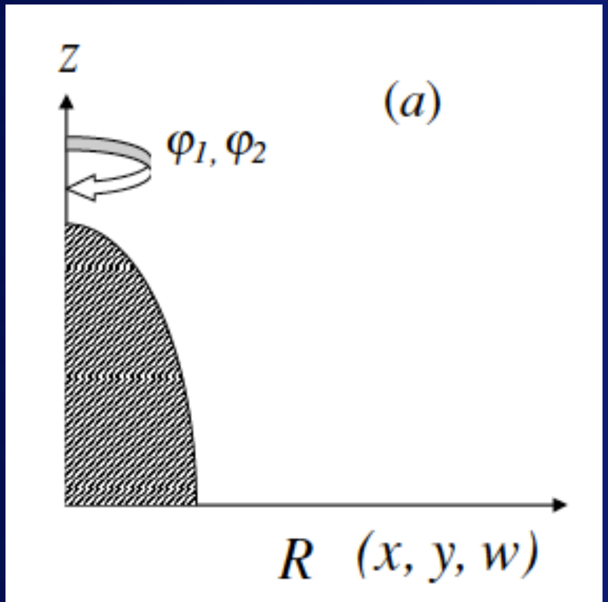

$$V_2^{(A)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \sin\theta \, d\theta$$

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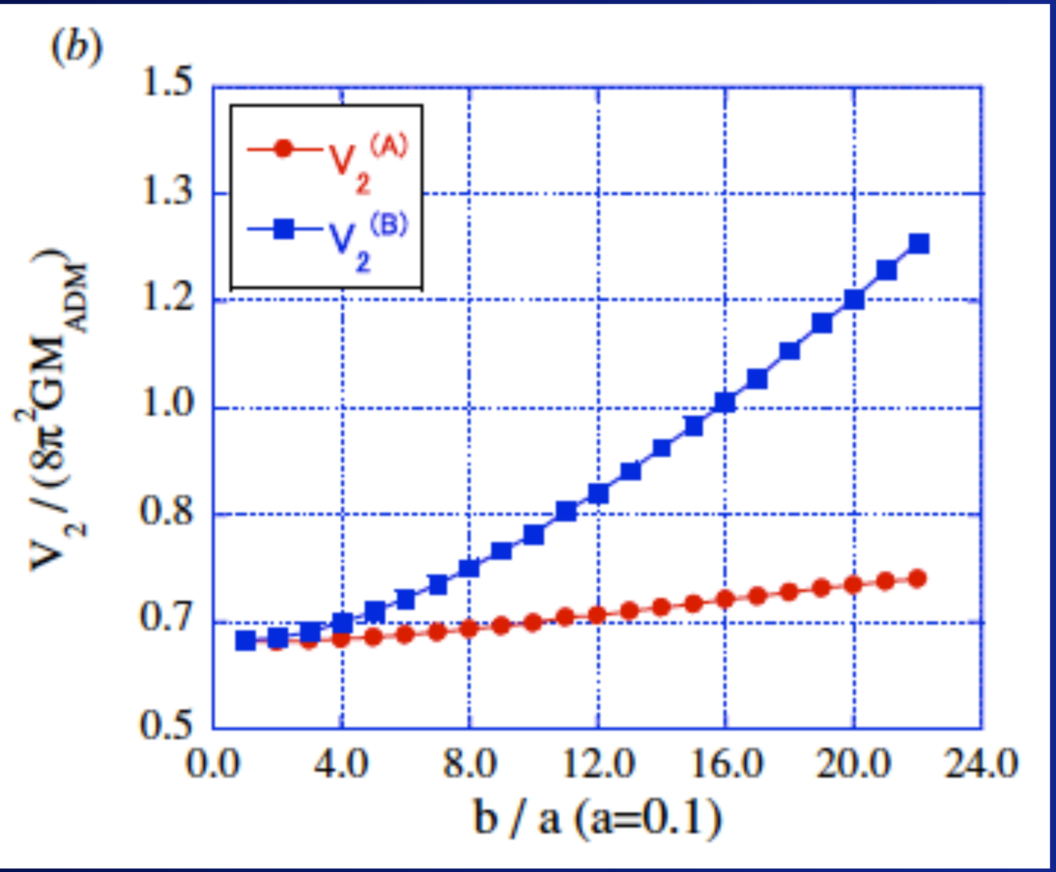
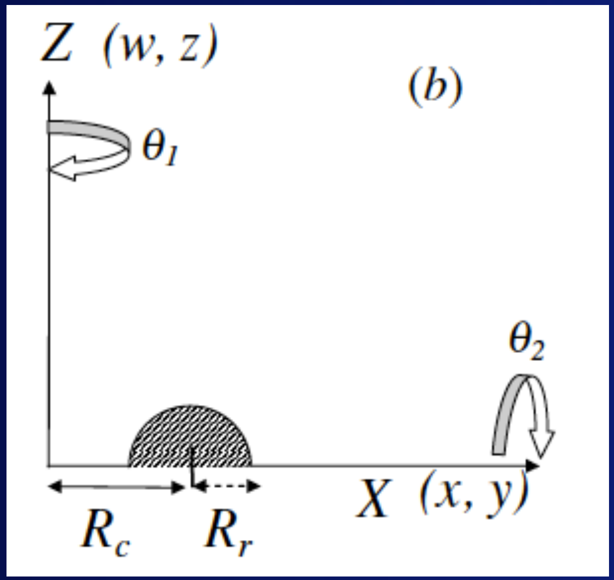


Hyper-Hoop $V_2^{(A)}$ does work for spheroidal horizons.

2.C. Initial Data Analysis

Hyper-Hoop conjecture ? $V_2 \leq \frac{\pi}{2} 16\pi G_5 M$

Toroidal Cases



$$V_2^{(C)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \cos\phi \, d\phi$$

$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h + \frac{r_h^2 + \dot{r_h}^2}{r_h} \left[\frac{\dot{r_h}}{r_h} \cot\phi - \frac{2}{\psi}(\dot{r_h} \sin\phi + r_h \cos\phi) \frac{\partial \psi}{\partial X} - \frac{2}{\psi}(r_h \sin\phi - \dot{r_h} \cos\phi) \frac{\partial \psi}{\partial Z} \right] = 0$$

$$V_2^{(D)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} r_h \sin\phi \, d\phi$$

$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h - \frac{r_h^2 + \dot{r_h}^2}{r_h} \left[\frac{\dot{r_h}}{r_h} \tan\phi + \frac{2}{\psi}(r_h \sin\phi - \dot{r_h} \cos\phi) \frac{\partial \psi}{\partial X} + \frac{2}{\psi}(r_h \cos\phi + \dot{r_h} \sin\phi) \frac{\partial \psi}{\partial Z} \right] = 0$$

$$V_2^{(E)} = 2\pi \int_0^{\pi} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2} (r_h \cos\xi + R_c) \, d\xi$$

$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h - \frac{r_h^2 + \dot{r_h}^2}{r_h} \left[\frac{-R_c + \dot{r_h} \sin\xi}{R_c + r_h \cos\xi} + \frac{2}{\psi}(\dot{r_h} \sin\xi + r_h \cos\xi) \frac{\partial \psi}{\partial X} + \frac{2}{\psi}(r_h \sin\xi - \dot{r_h} \cos\xi) \frac{\partial \psi}{\partial Z} \right] = 0$$

2.C. Initial Data Analysis

Hyper-Hoop conjecture ?

$$V_2 \le \frac{\pi}{2} 16\pi G_5 M$$

Toroidal Cases

$$V_2^{(C)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \cos\phi \, d\phi$$

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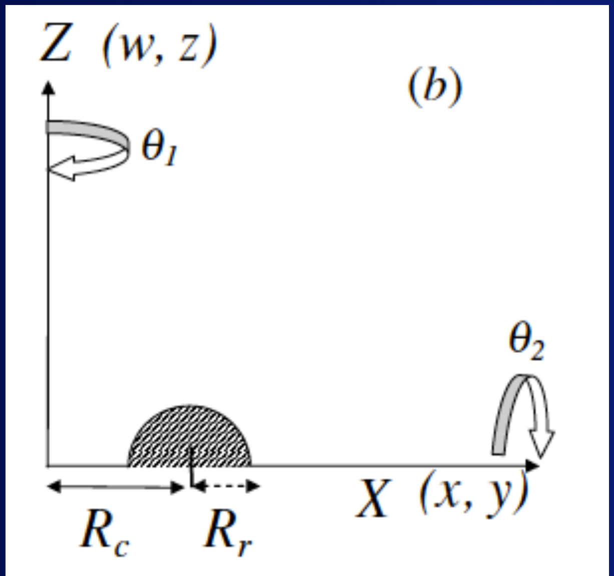

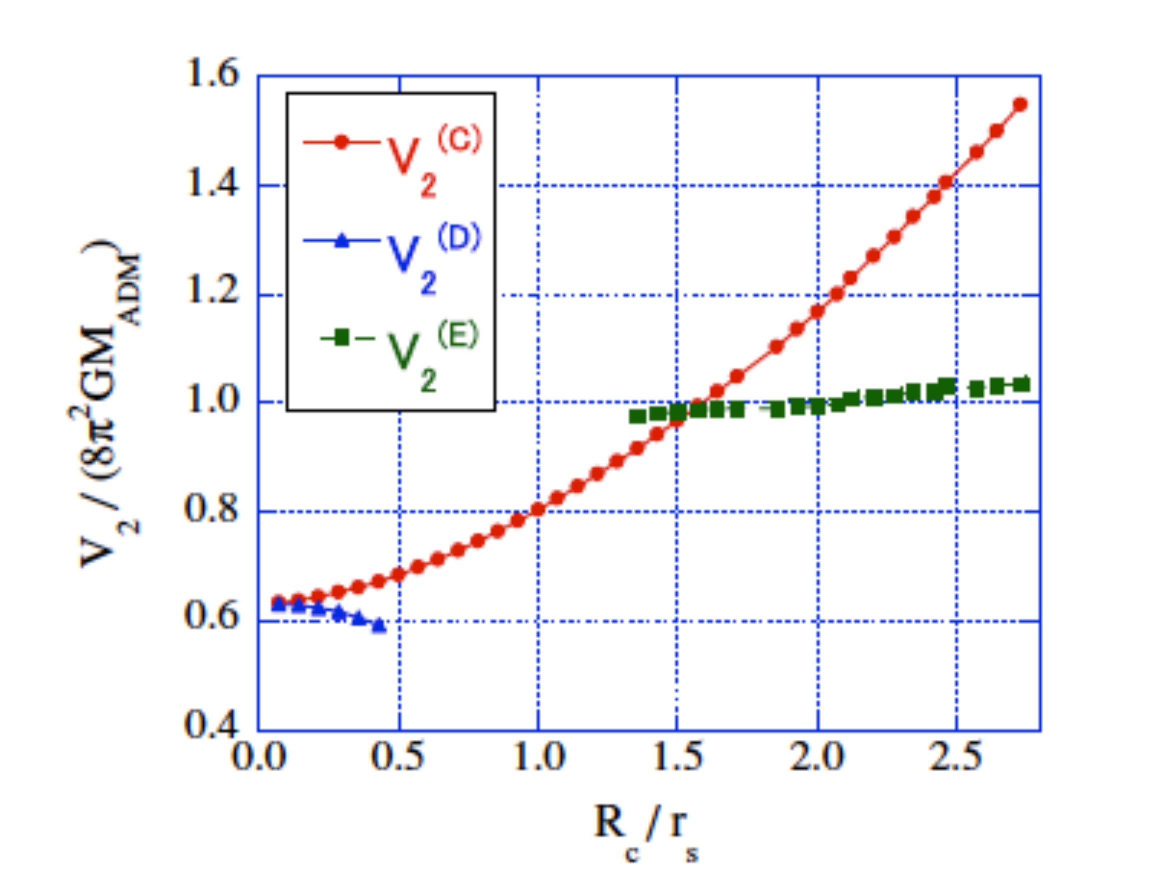

2.C. Initial Data Analysis

Hyper-Hoop conjecture ? $V_2 \leq \frac{\pi}{2} 16\pi G_5 M$

Toroidal Cases

$$V_2^{(C)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \cos\phi \, d\phi$$

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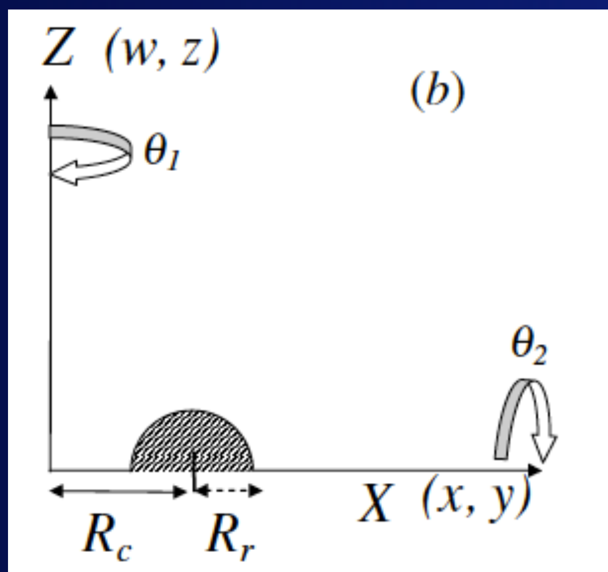
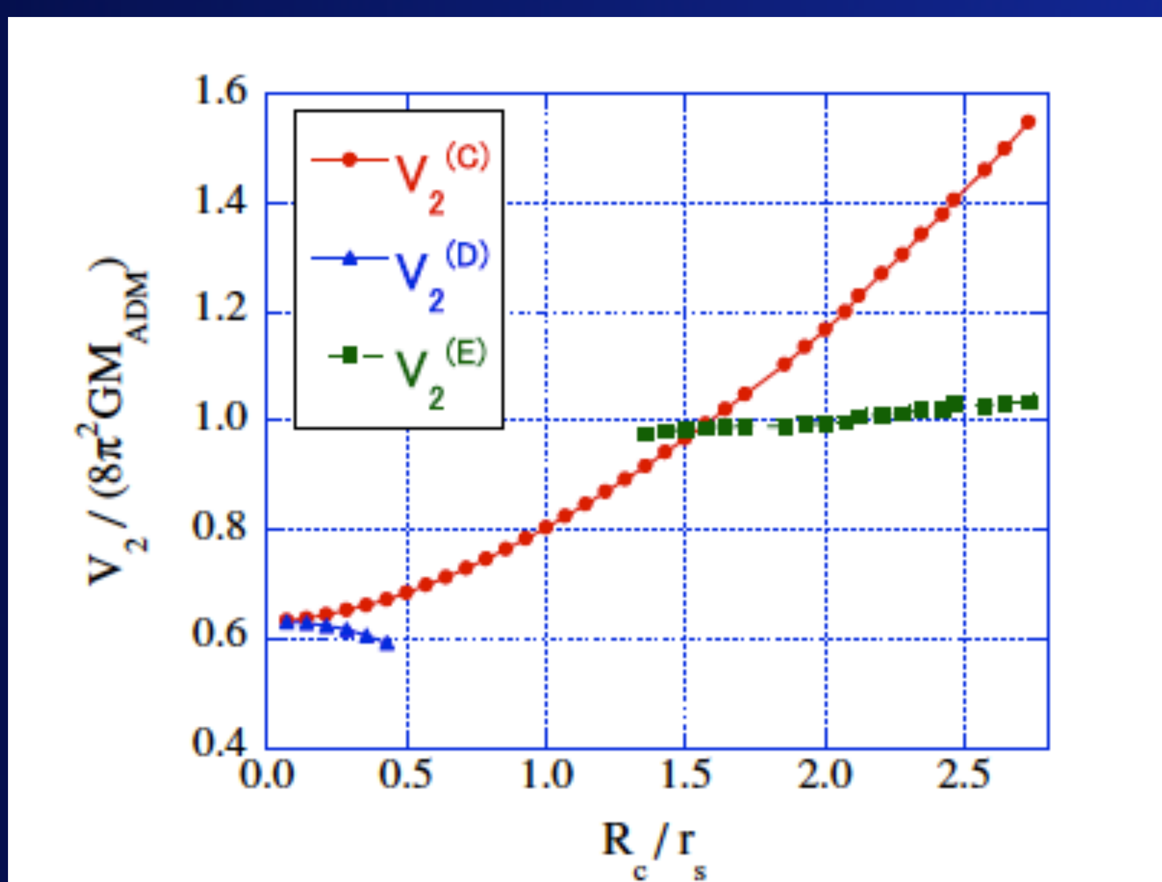




Hyper-Hoop does not work for ring horizons.

3. Evolution Code

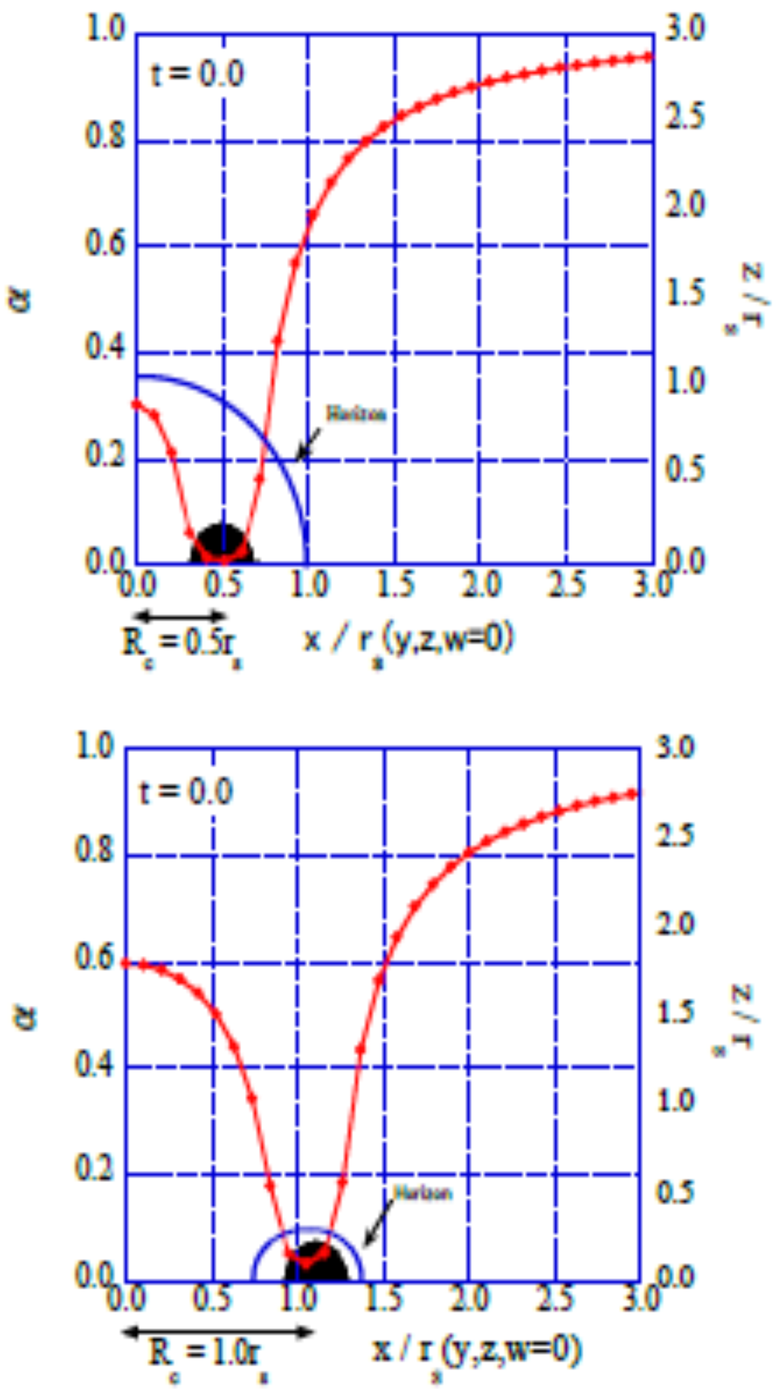
- ADM full 4+1,   ADM 2+1 Double Axisym Cartoon
- 33^4 grids, 65^2 x 2^2 grids

- Maximal slicing condition, zero shift vectors
- asymptically flat

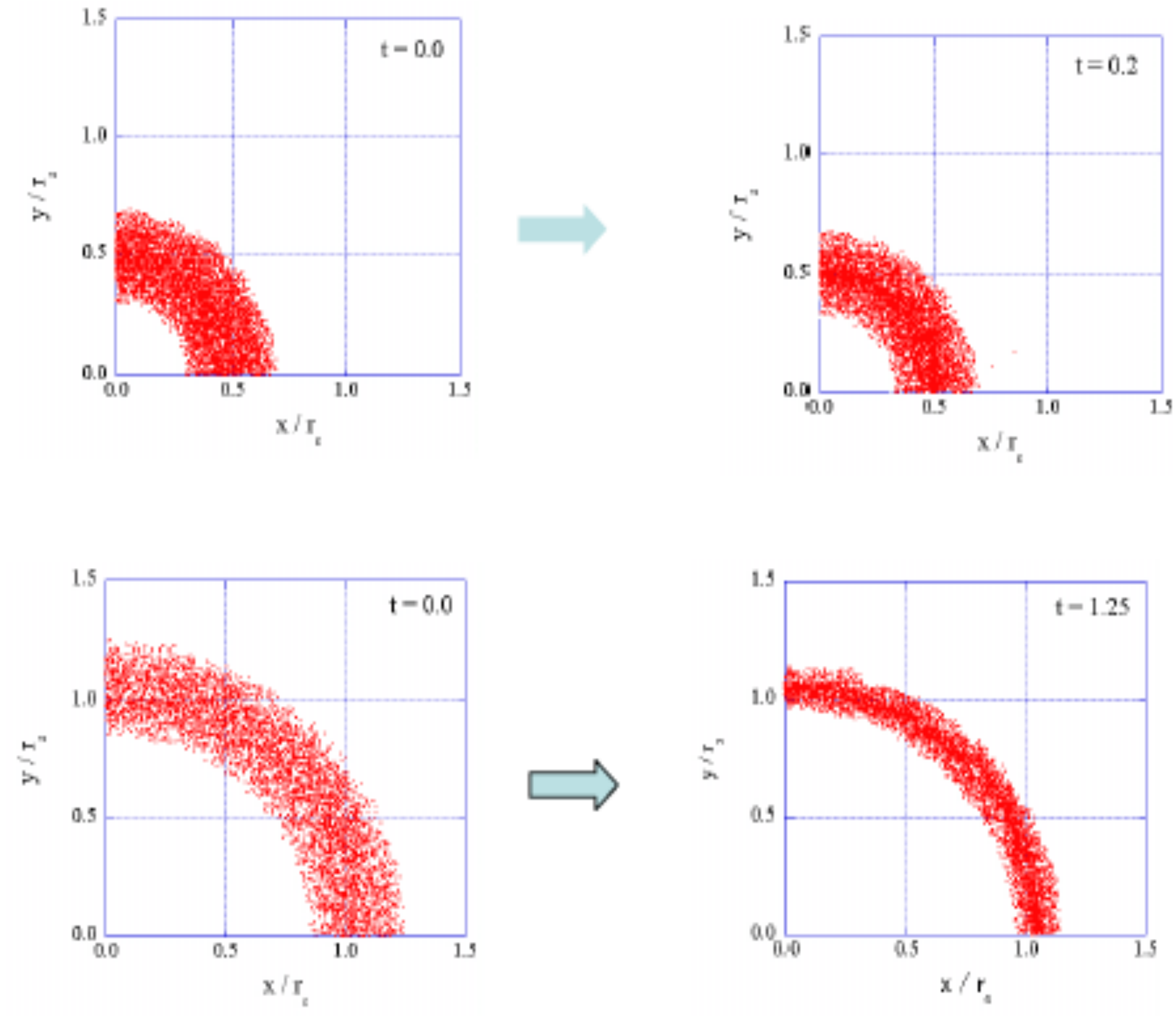
- Collisionless Particles (5000)
- the same total mass
- no rotation

- Apparent Horizon Search
  both for Ring Horizon and Common Horizon

・lapse function at t=0.0

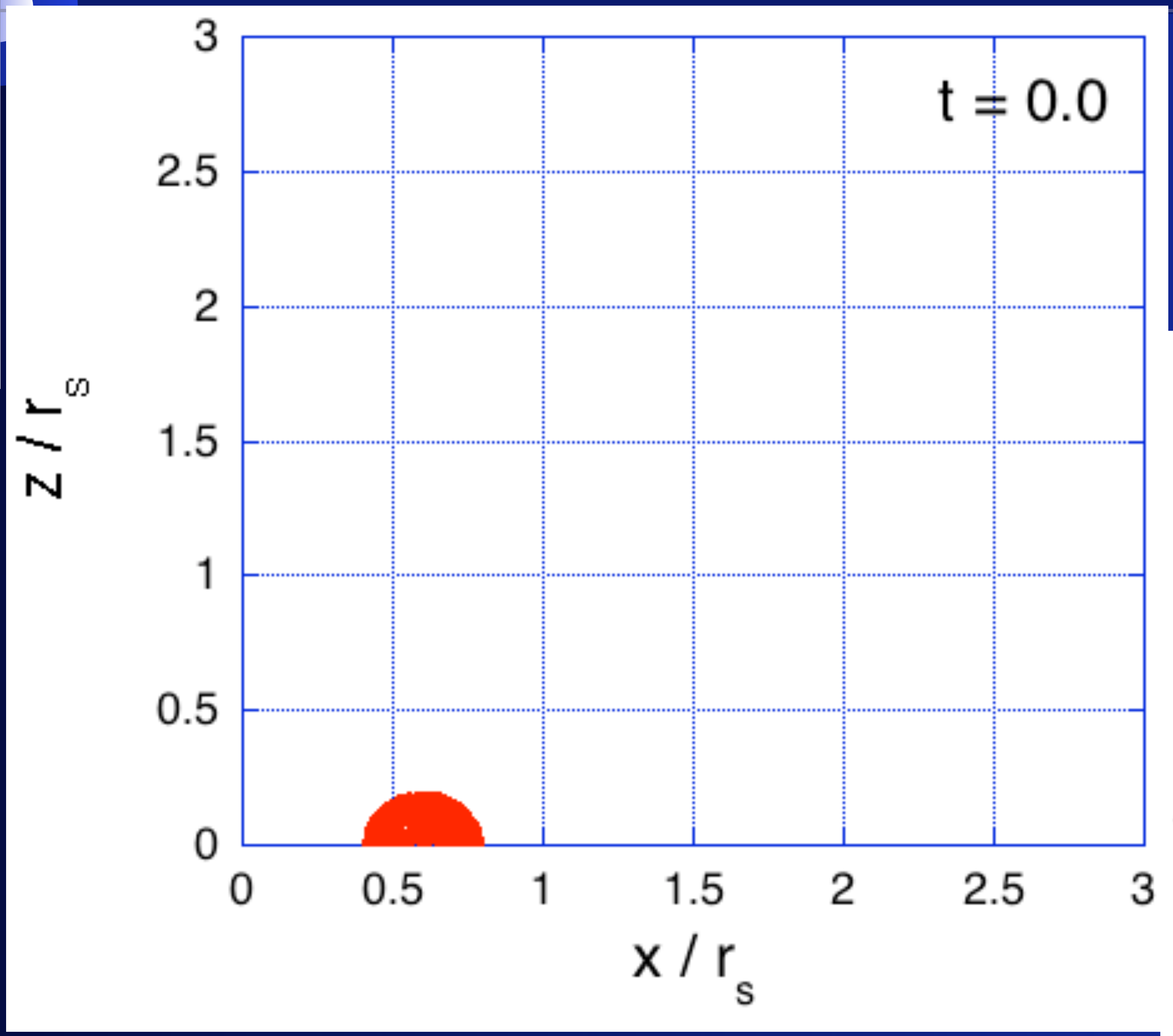


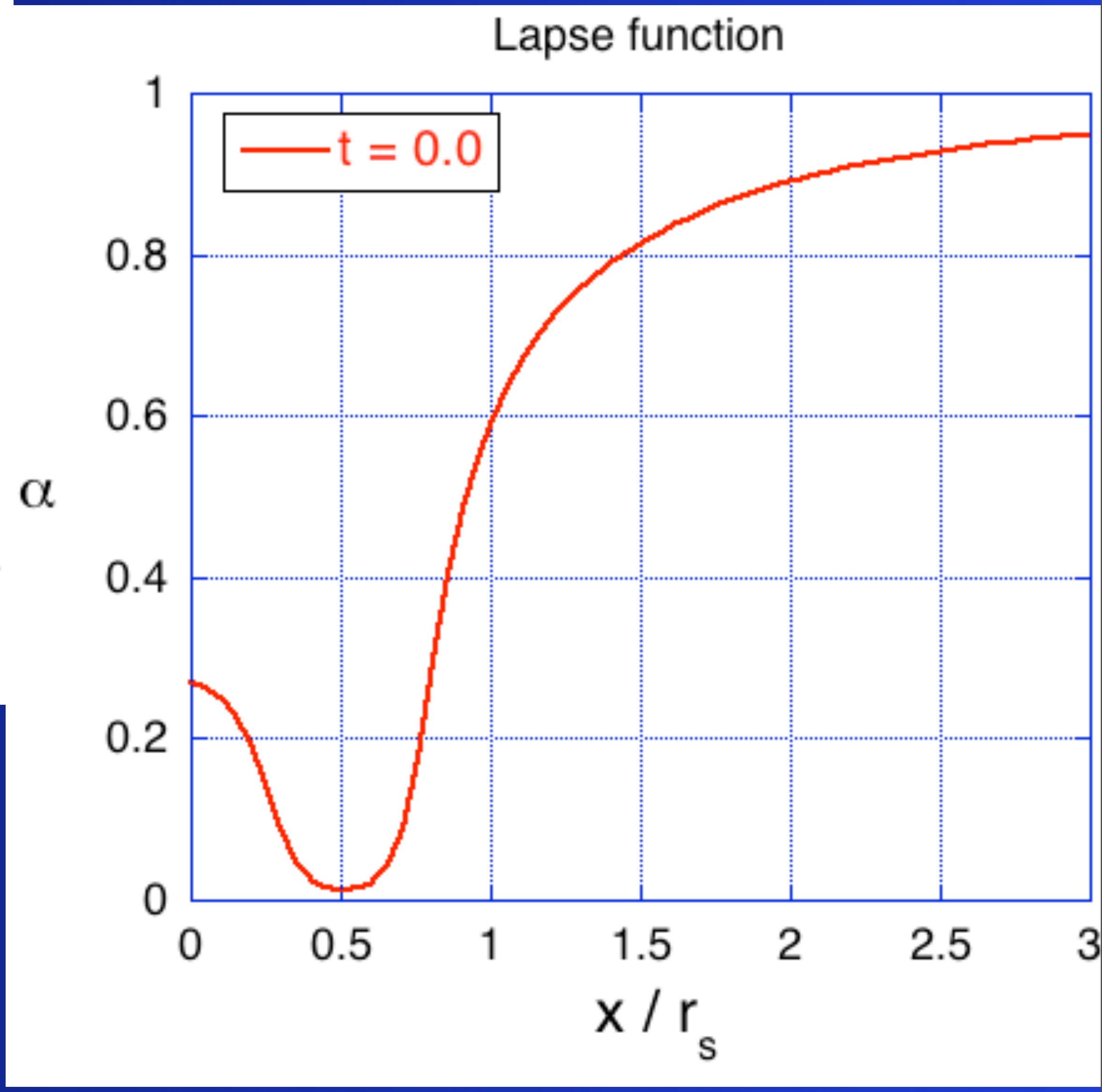
・time evolution of particle



3. Evolution (case I)

t=0 No Horizon

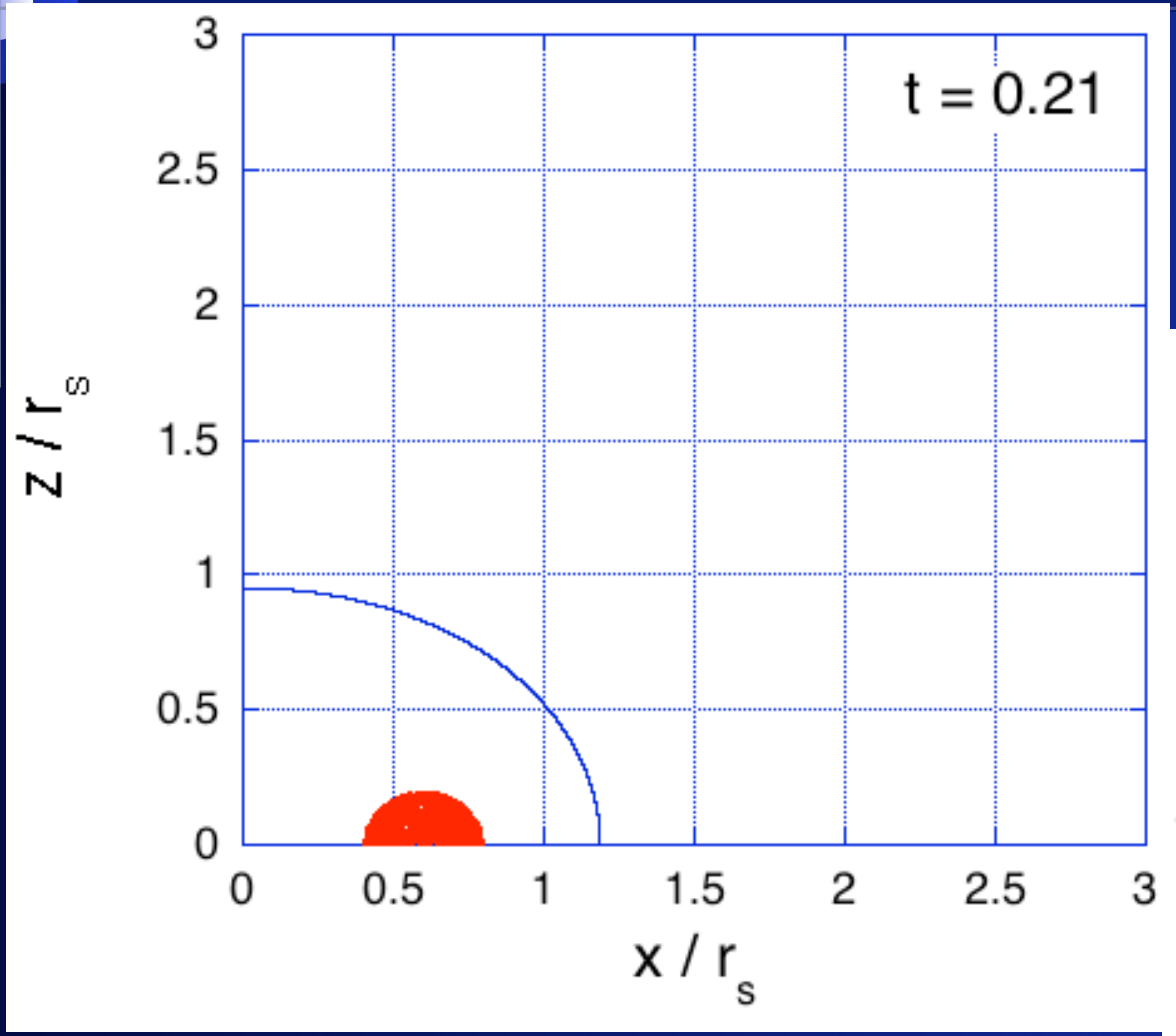


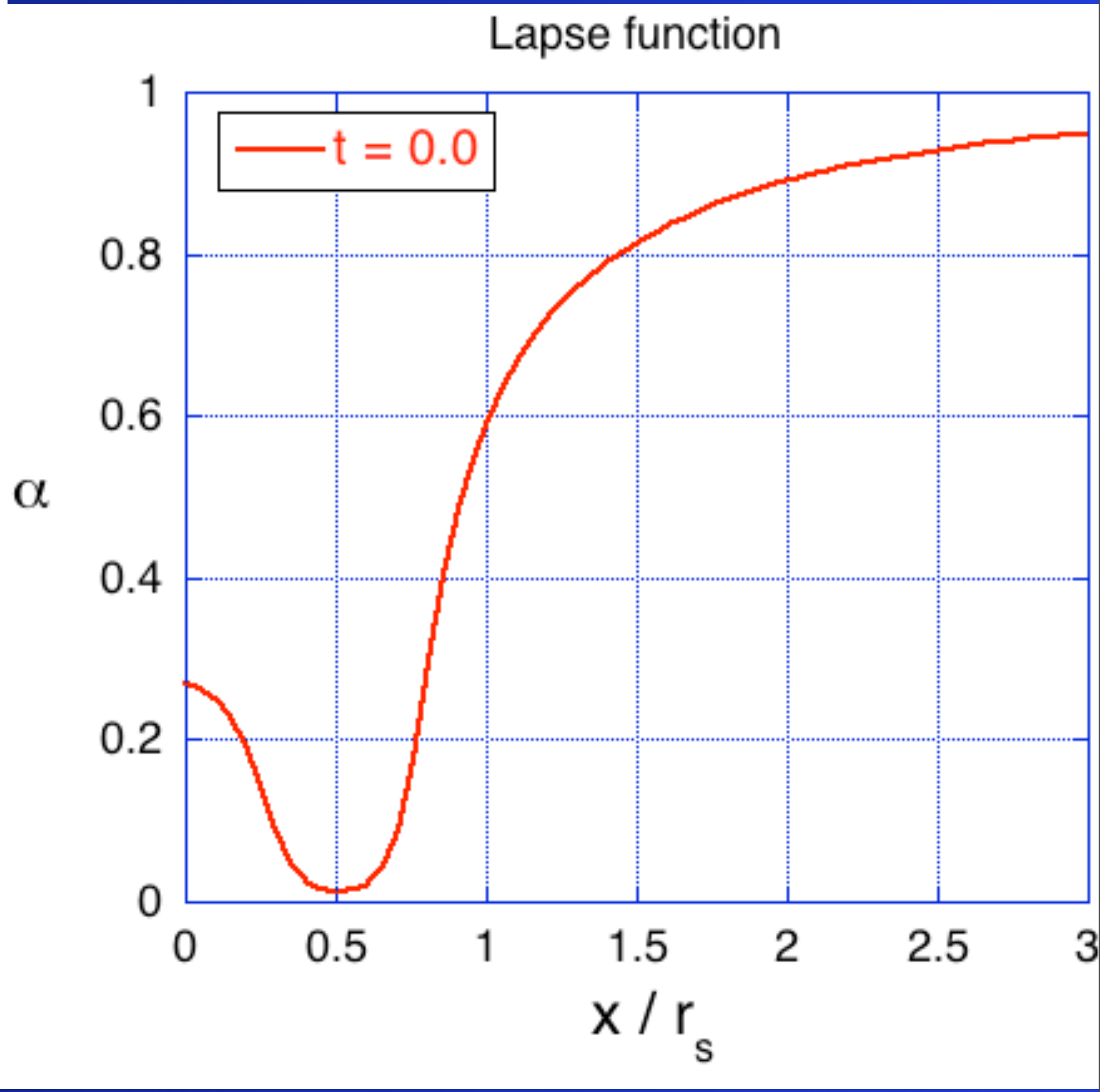


3. Evolution (case I)

t=0 No Horizon

t=0.2 Common Horizon

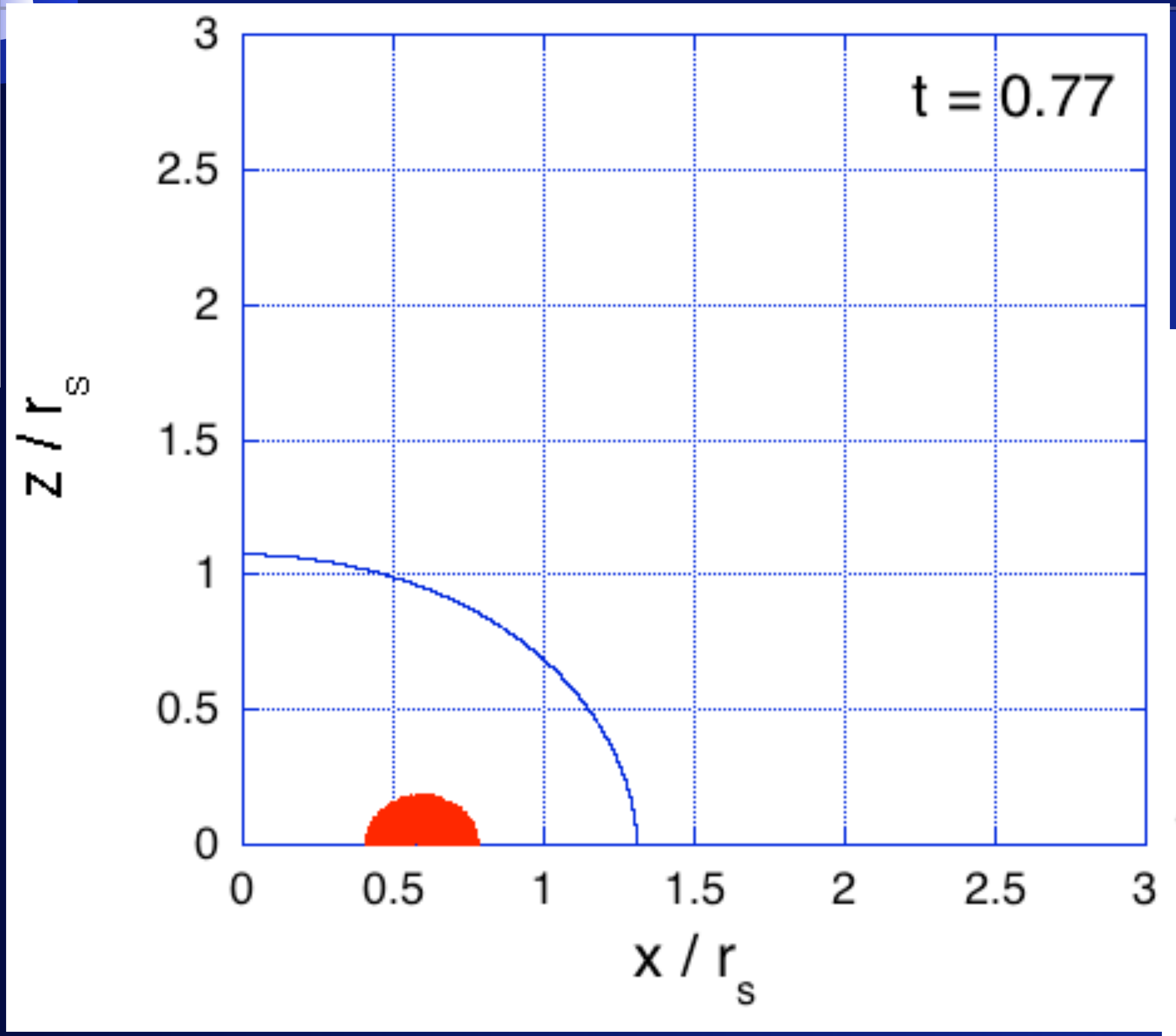


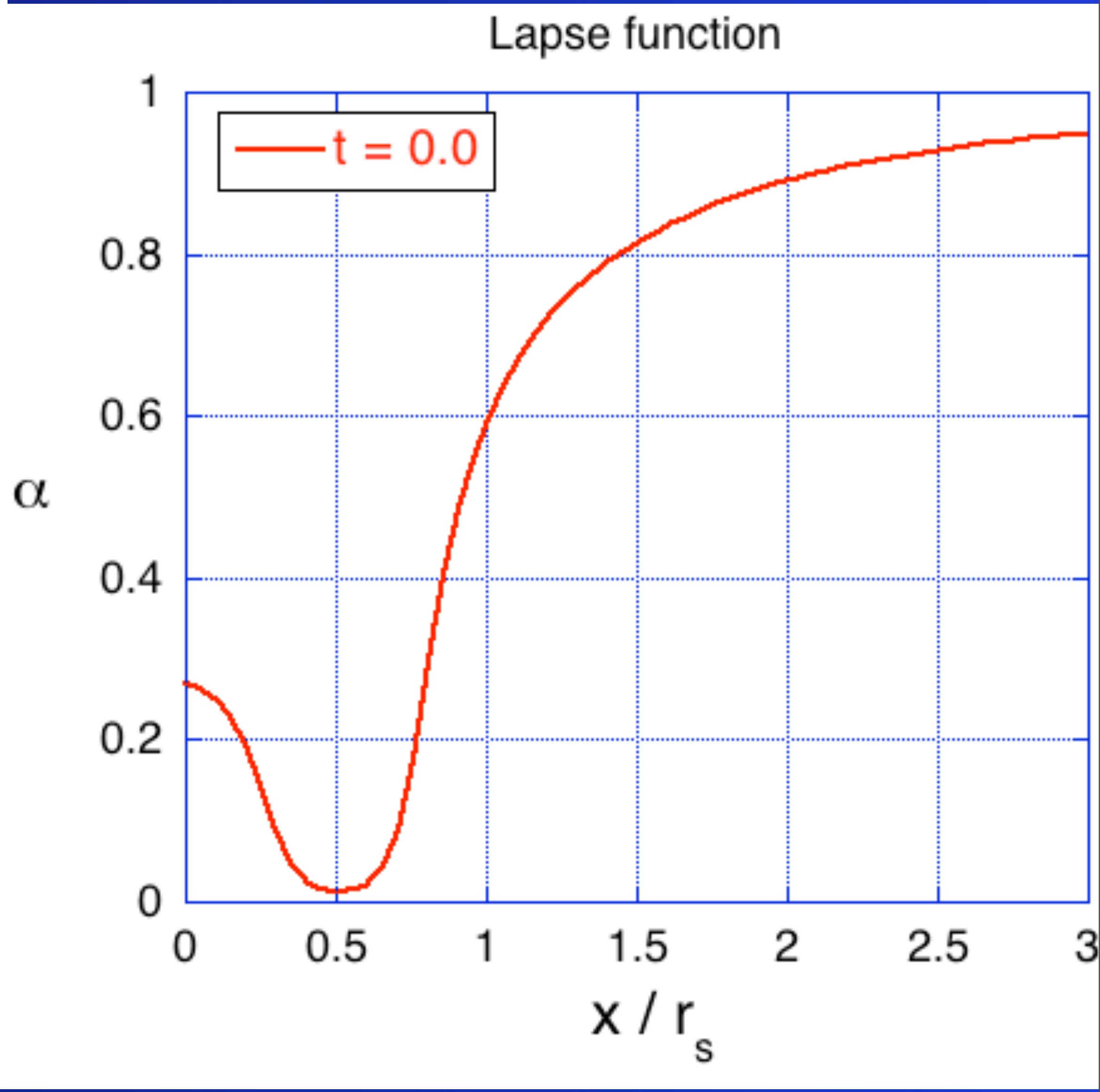


3. Evolution (case I)

t=0 No Horizon

t=0.2 Common Horizon

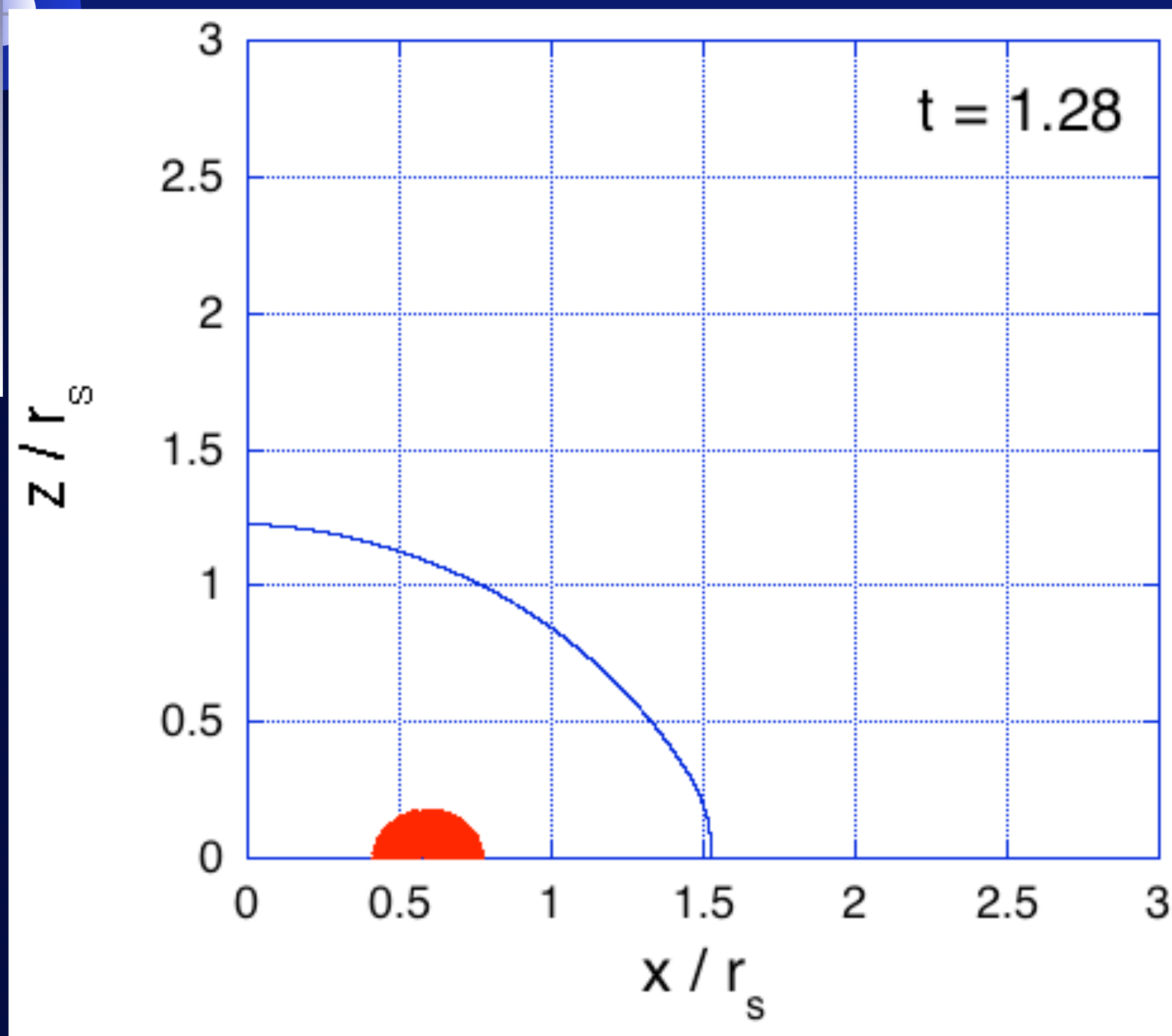


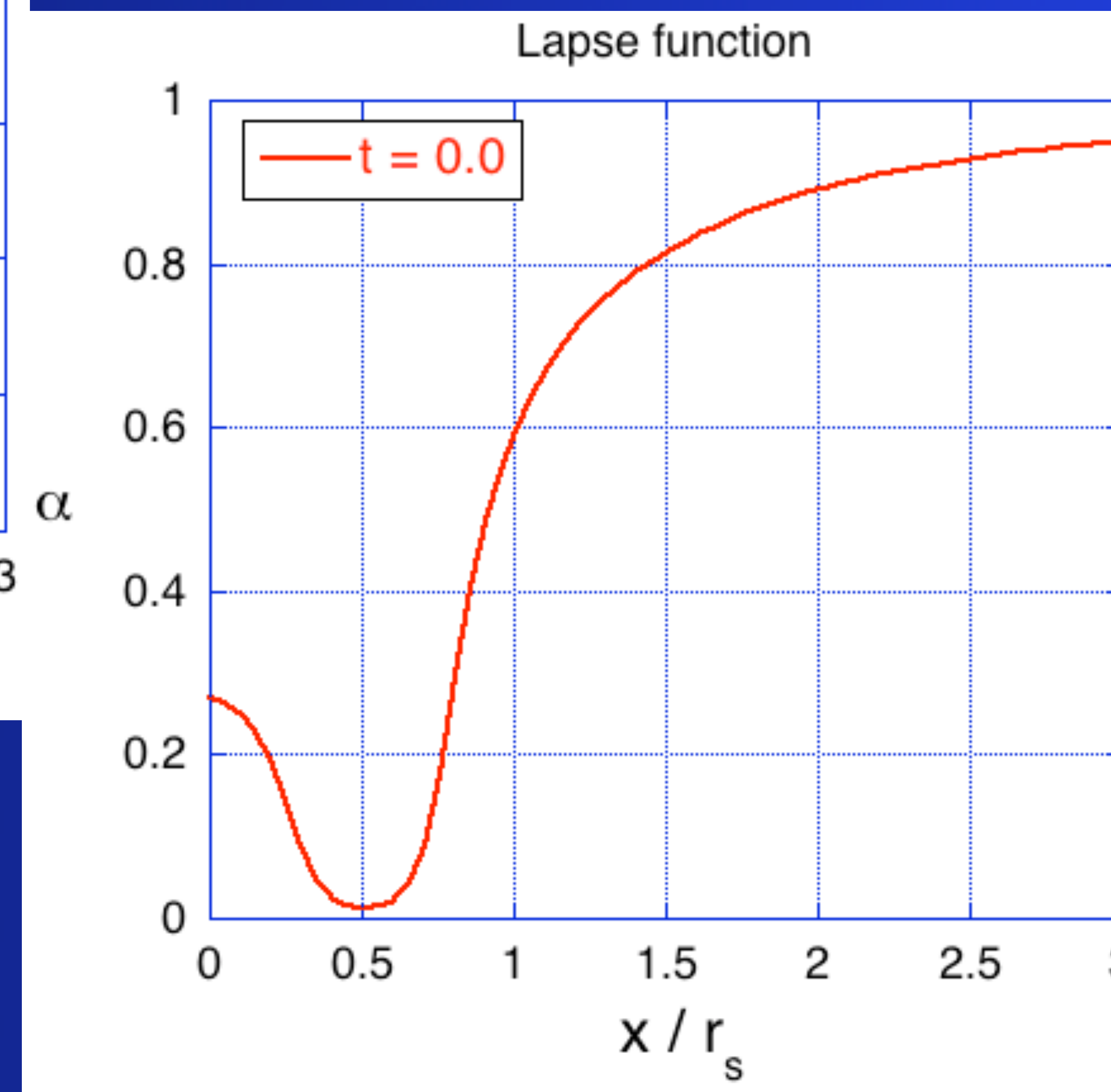


3. Evolution (case I)

t=0 No Horizon

t=0.2 Common Horizon

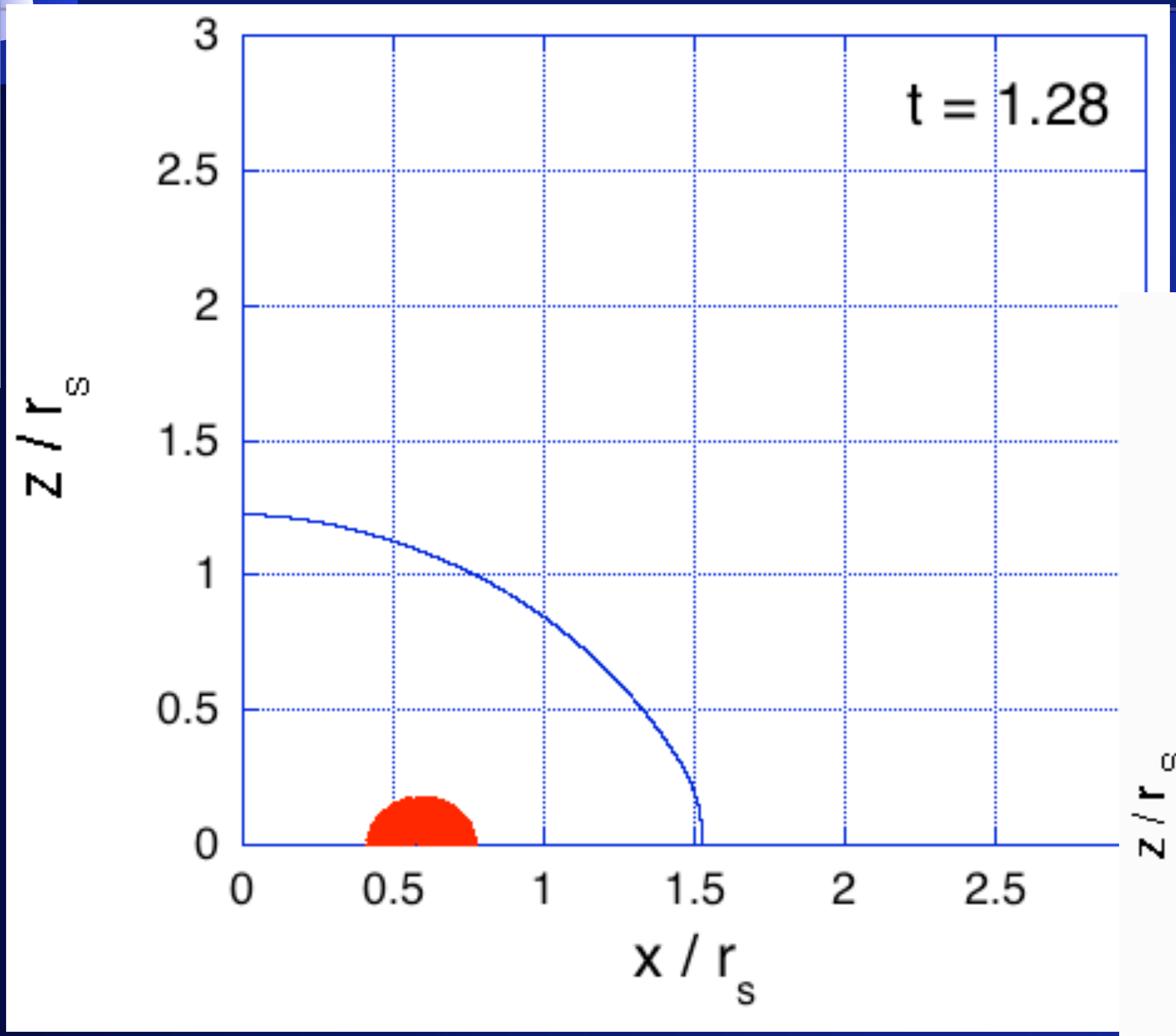


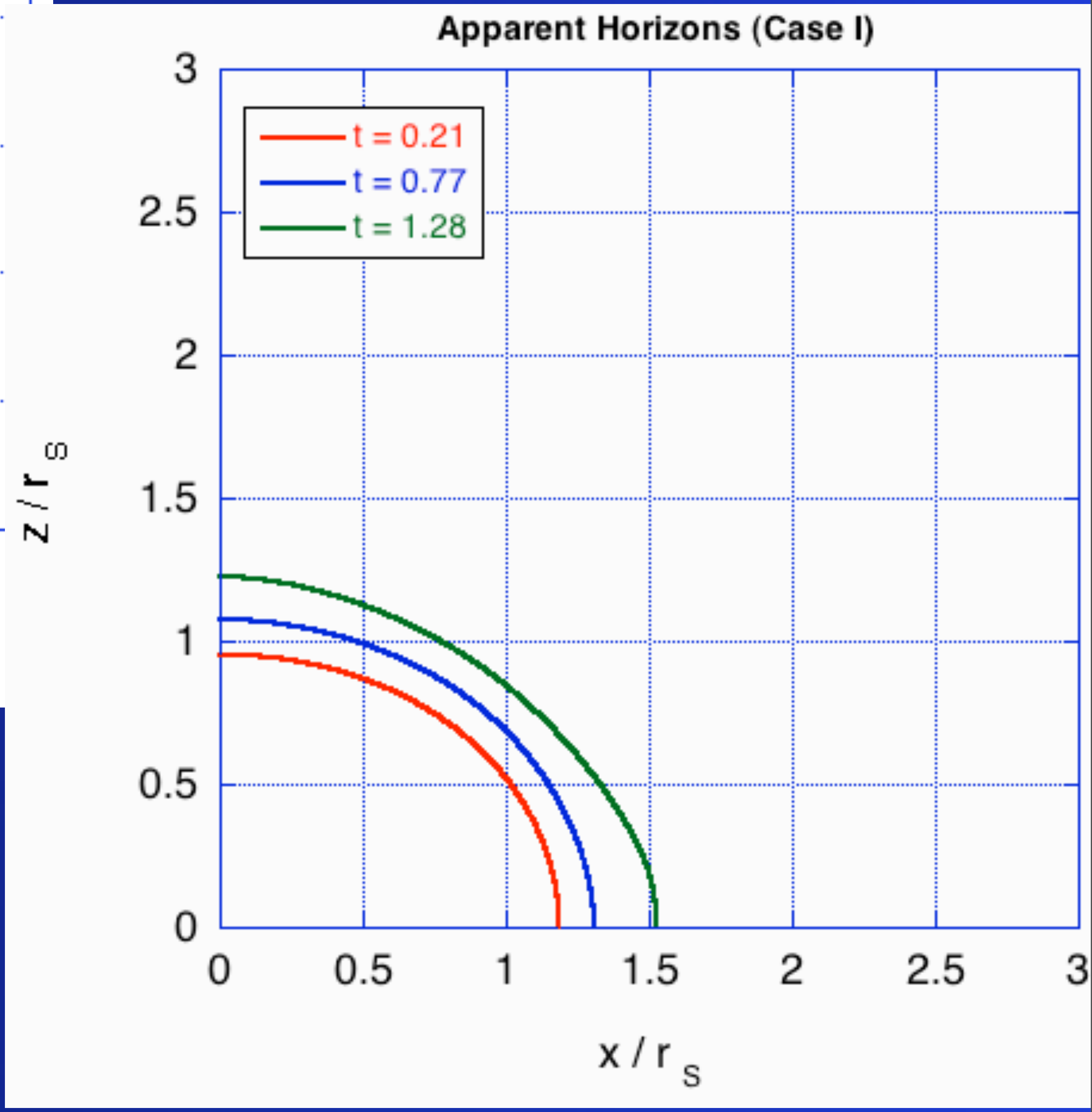


3. Evolution (case I)

t=0 No Horizon

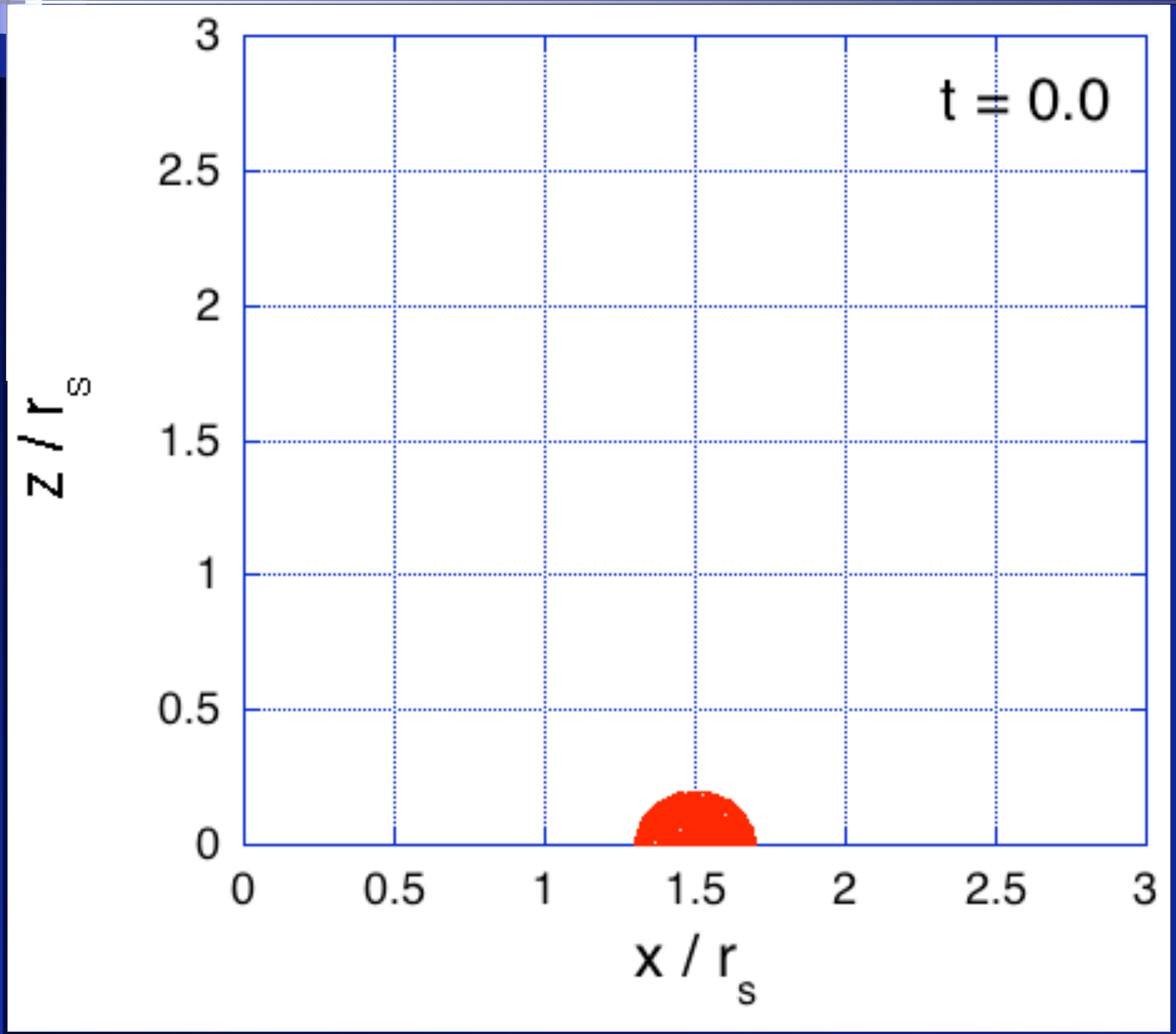
t=0.2 Common Horizon





3. Evolution (case II)

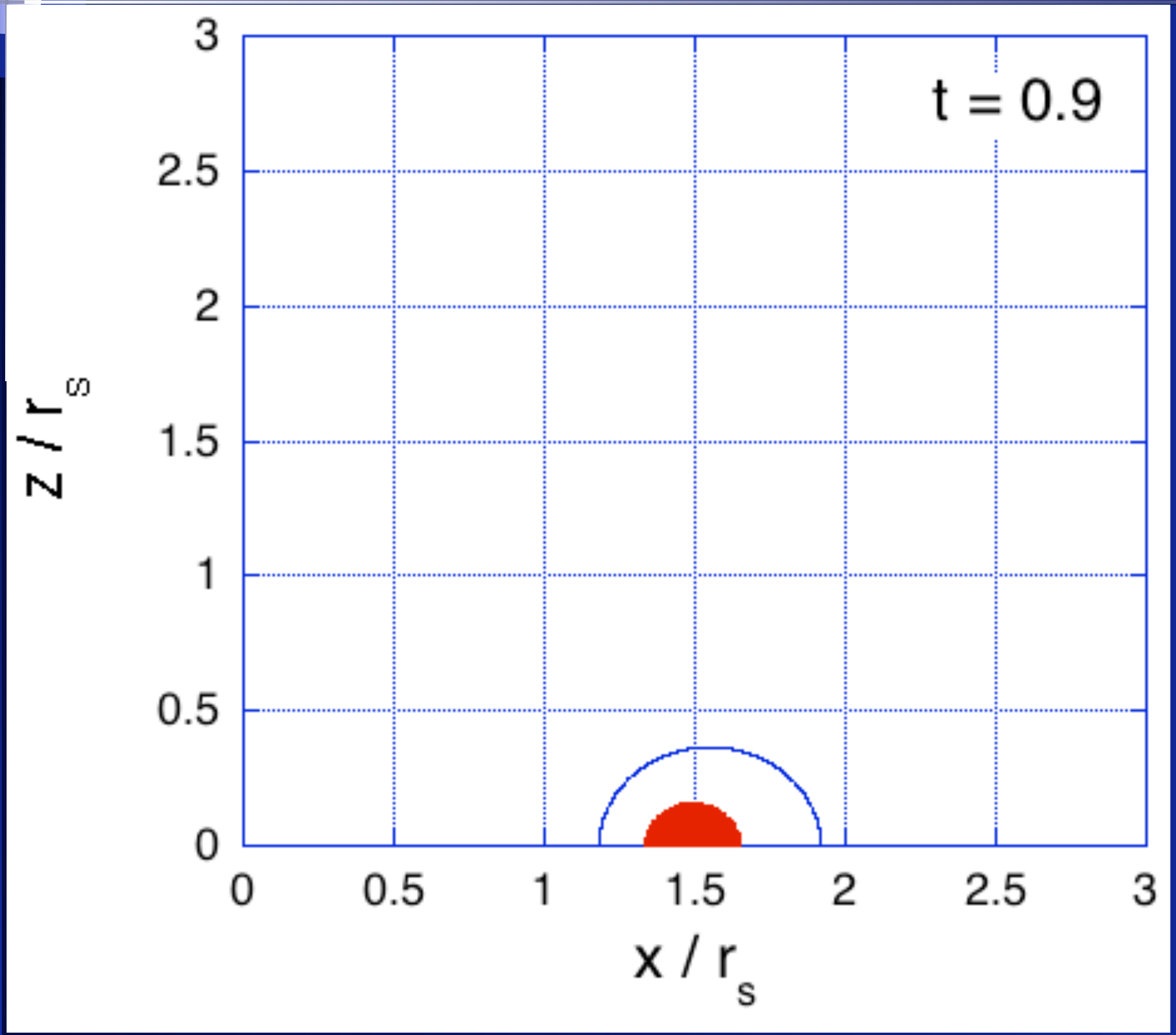
t=0 No Horizon



3. Evolution (case II)

t=0 No Horizon

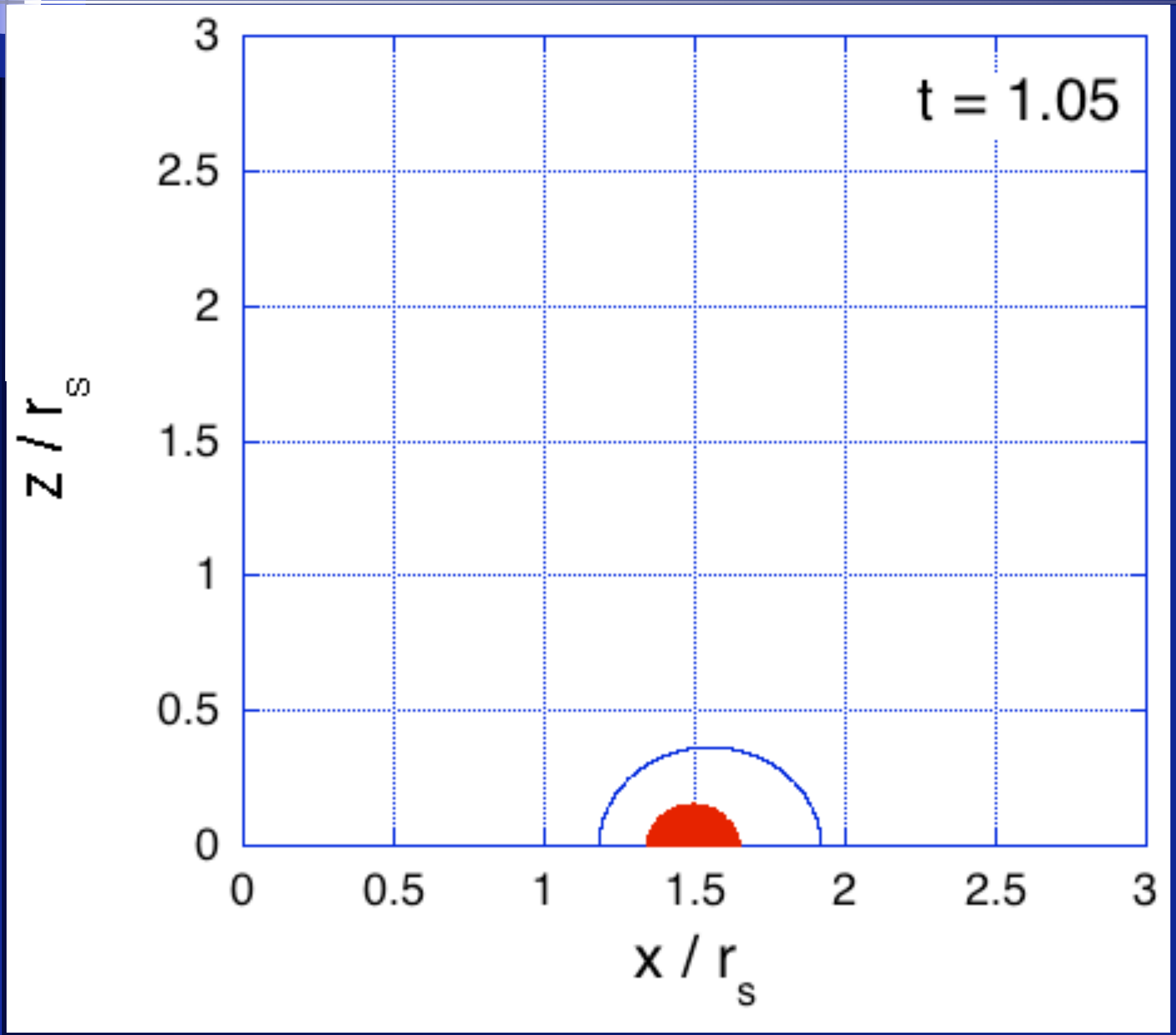
t=0.9 Ring Horizon



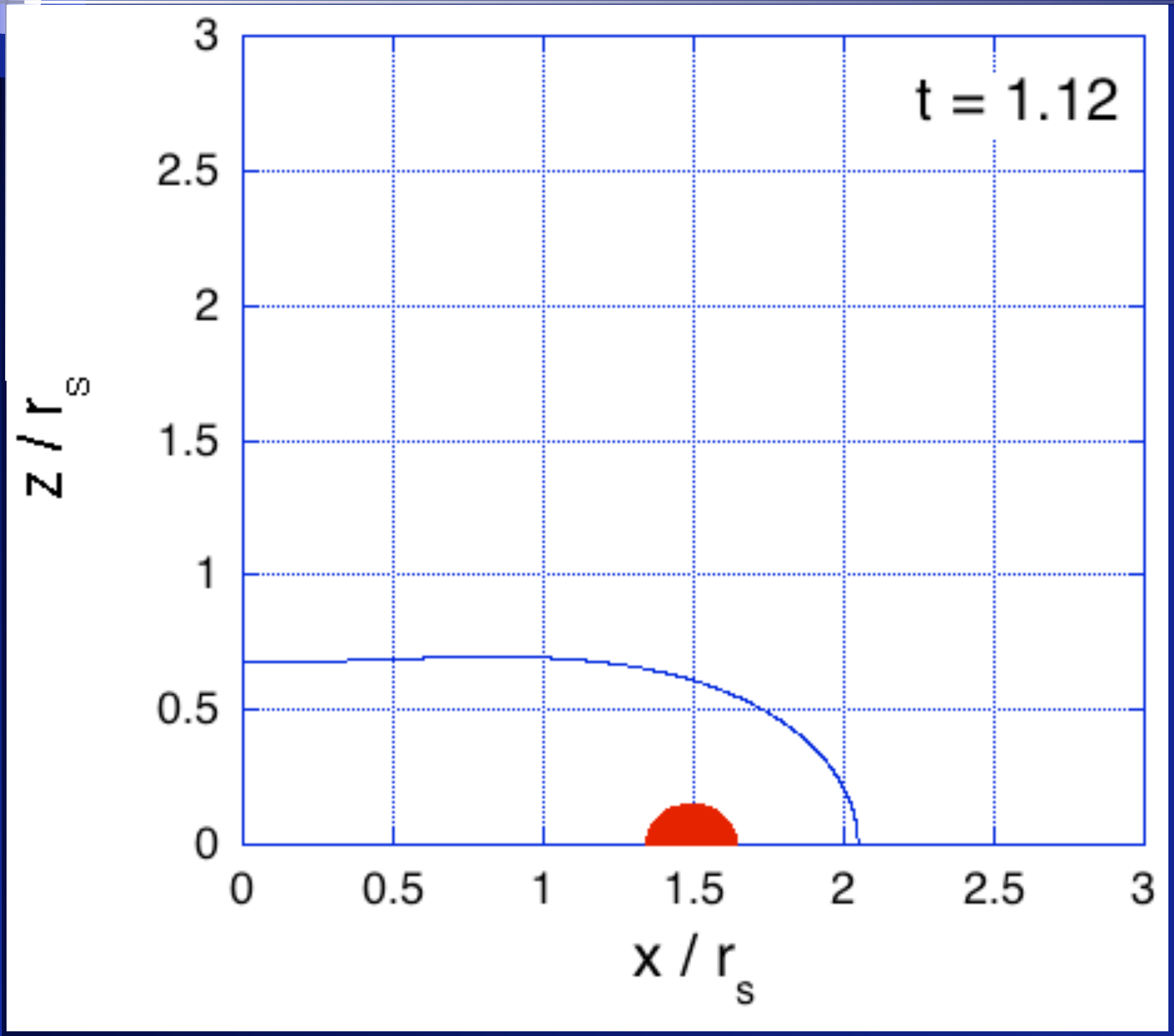
3. Evolution (case II)

t=0 No Horizon

t=0.9 Ring Horizon



3. Evolution (case II)



t=0 No Horizon

t=0.9 Ring Horizon

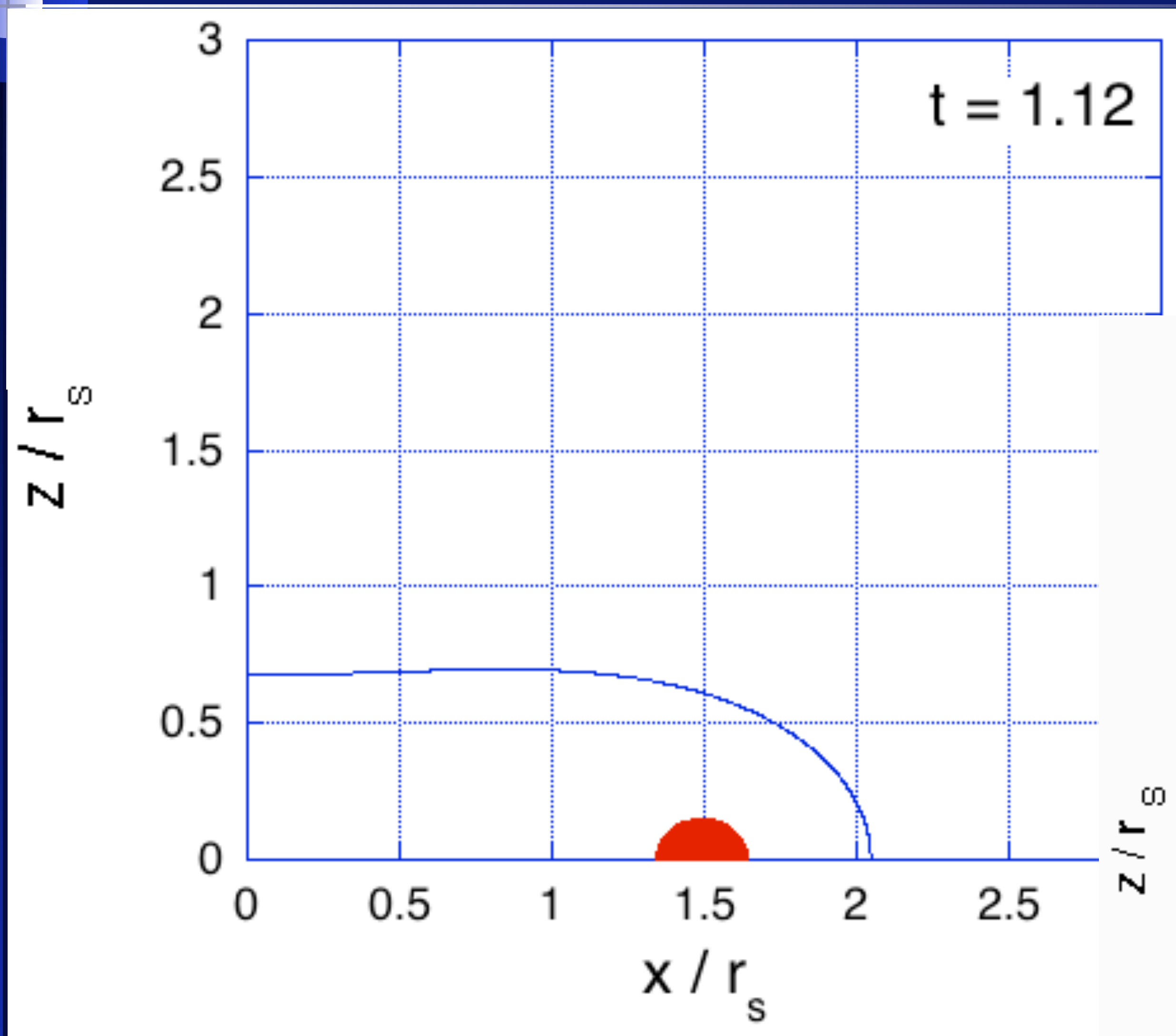
t=1.1 Common Horizon

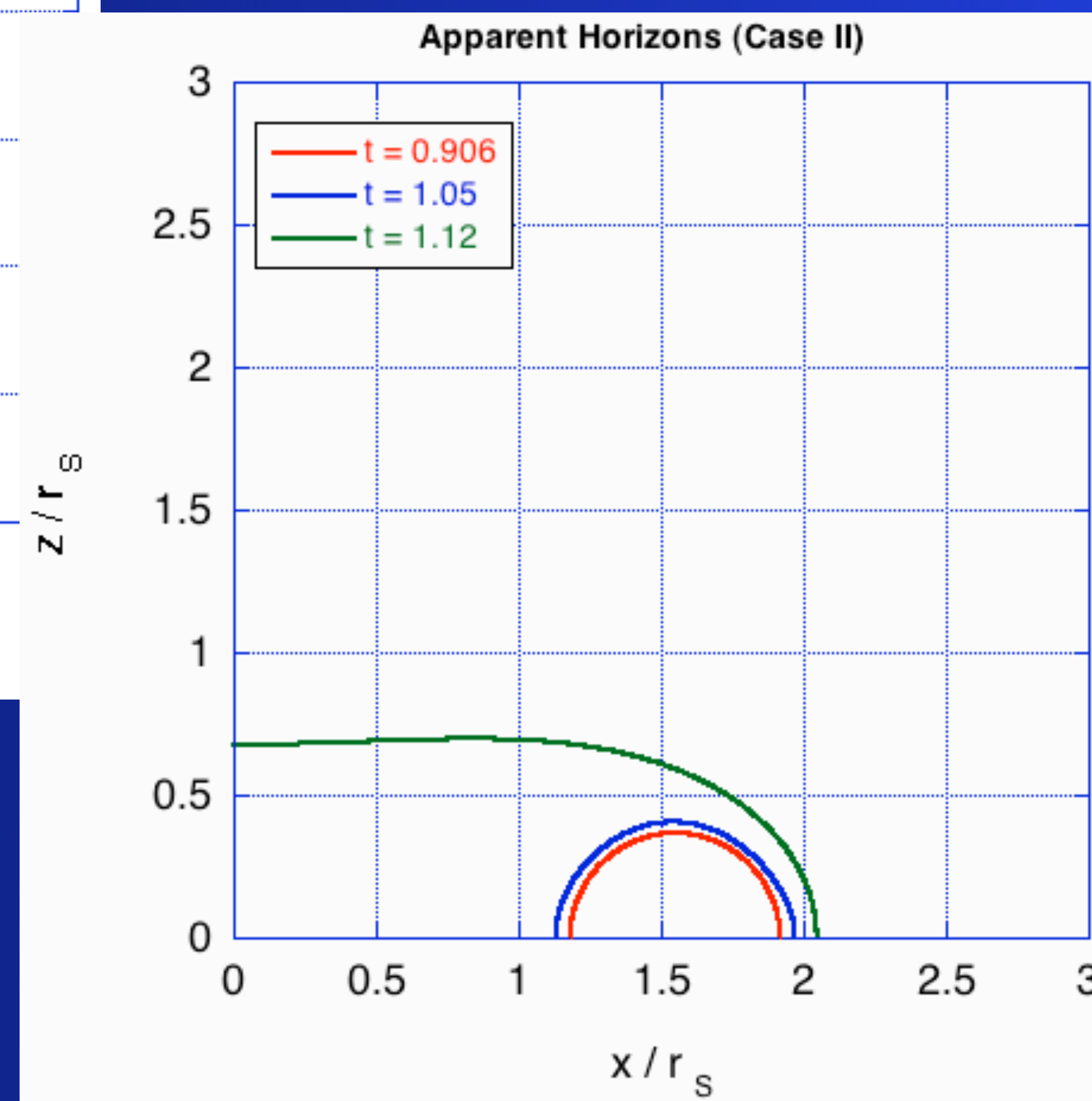
3. Evolution (case II)

t=0 No Horizon

t=0.9 Ring Horizon

t=1.1 Common Horizon



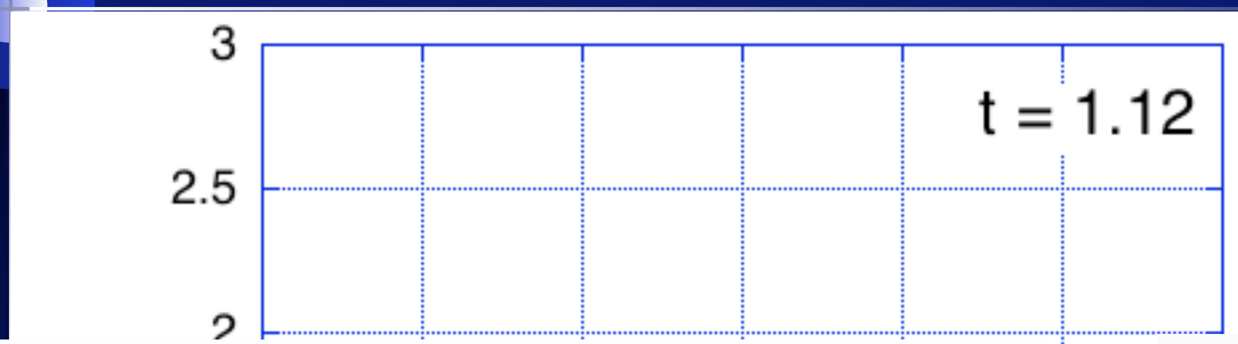


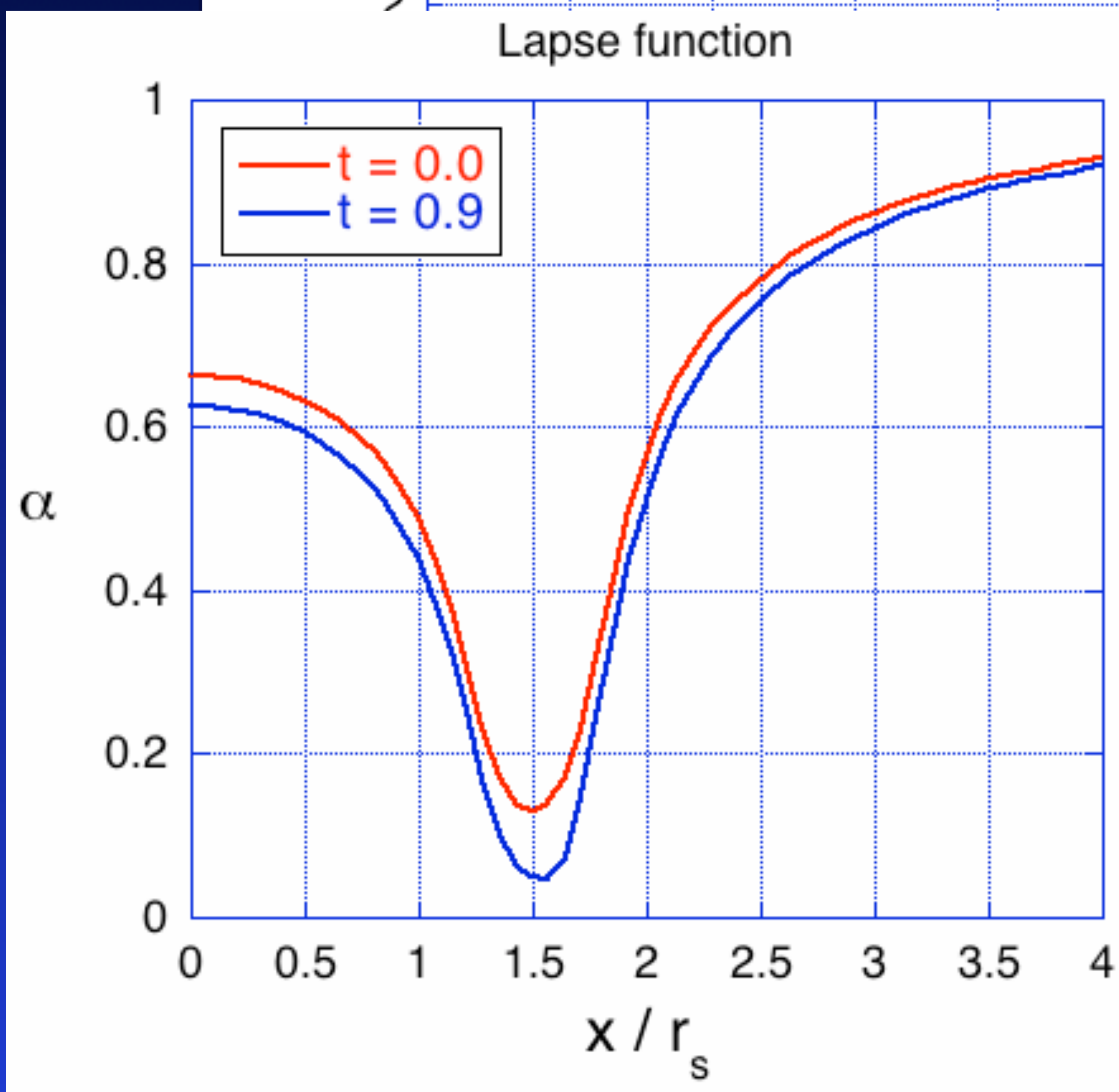
3. Evolution (case II)

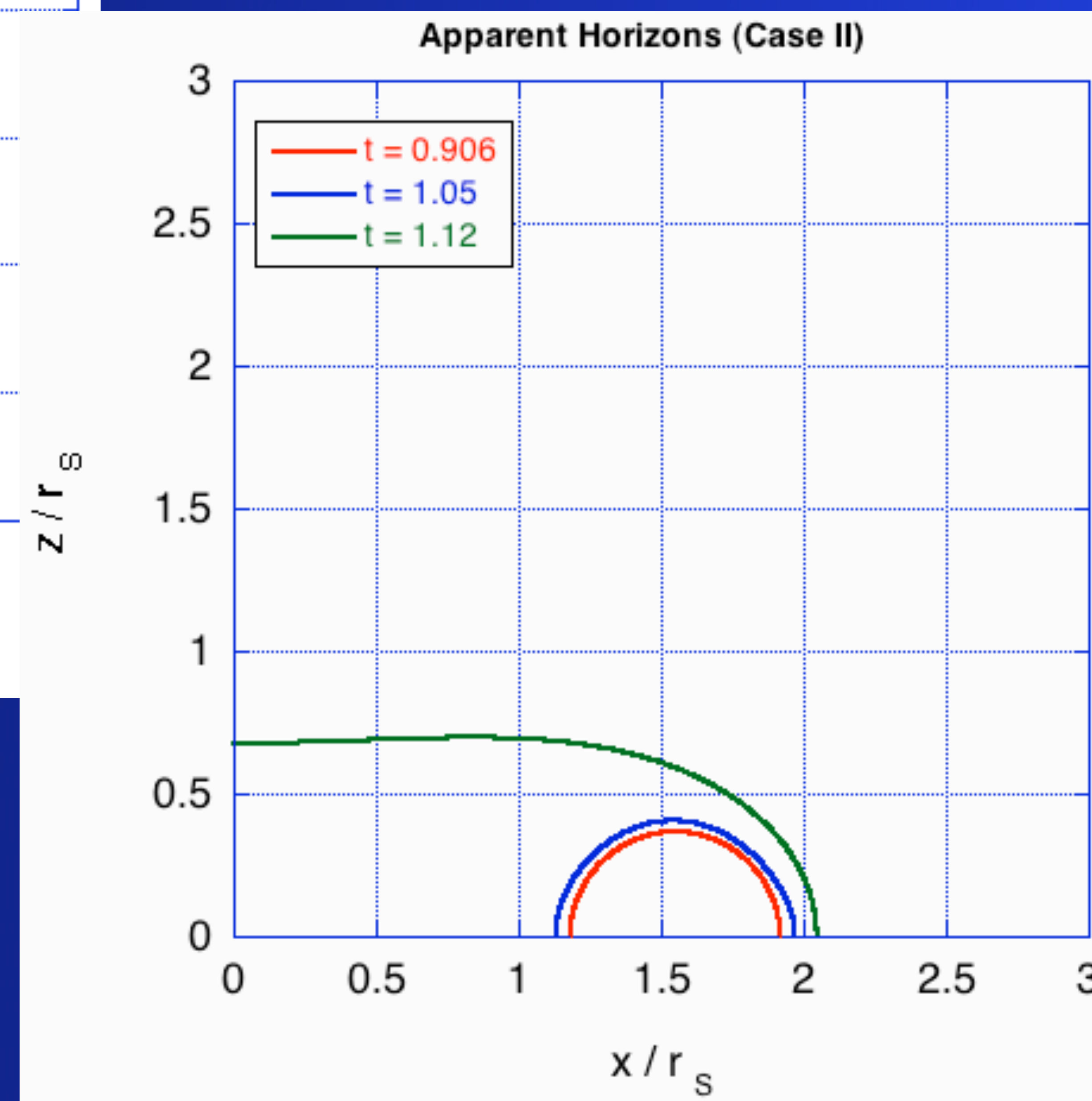
t=0 No Horizon

t=0.9 Ring Horizon

t=1.1 Common Horizon

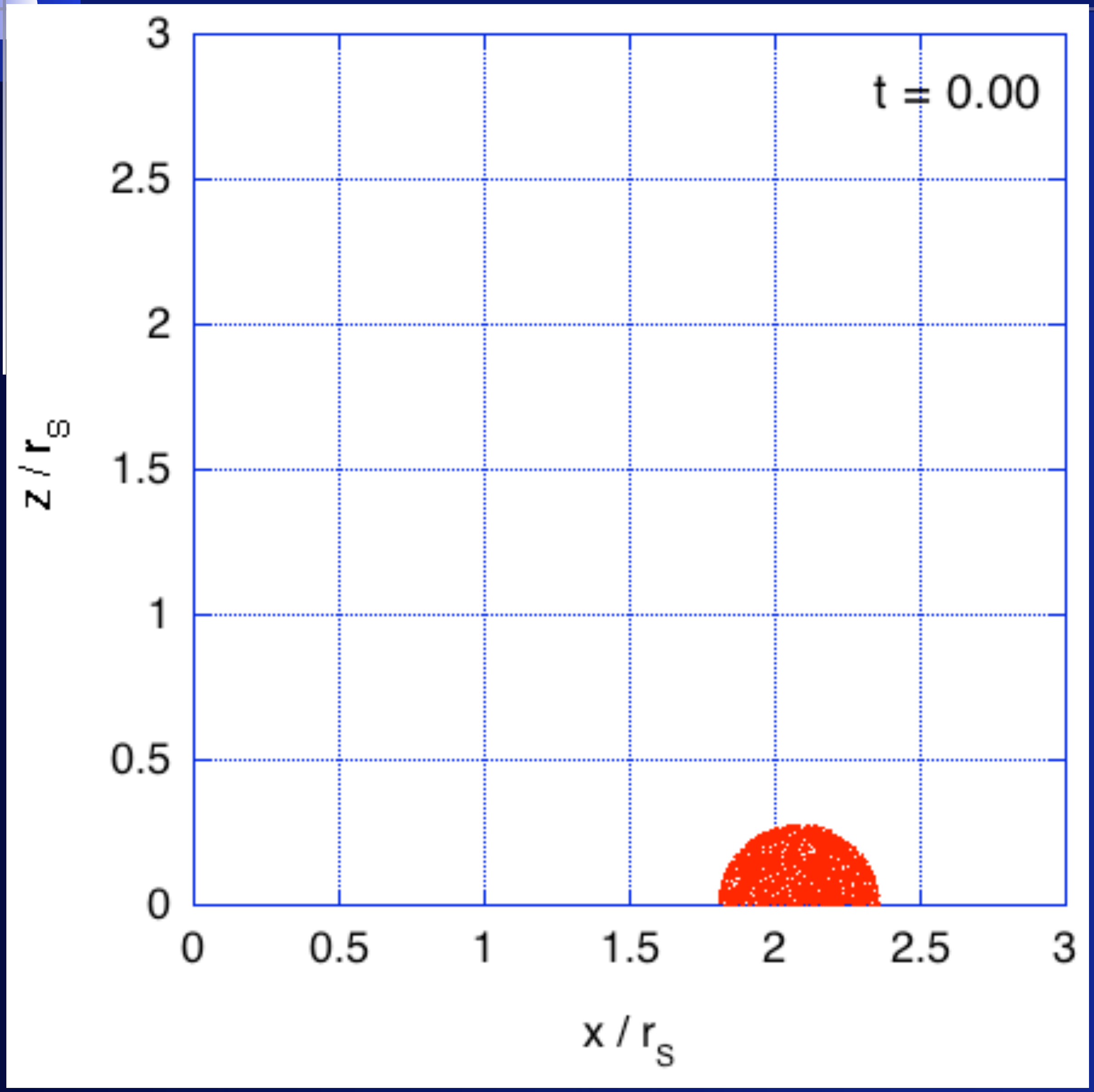






3. Evolution (case III)

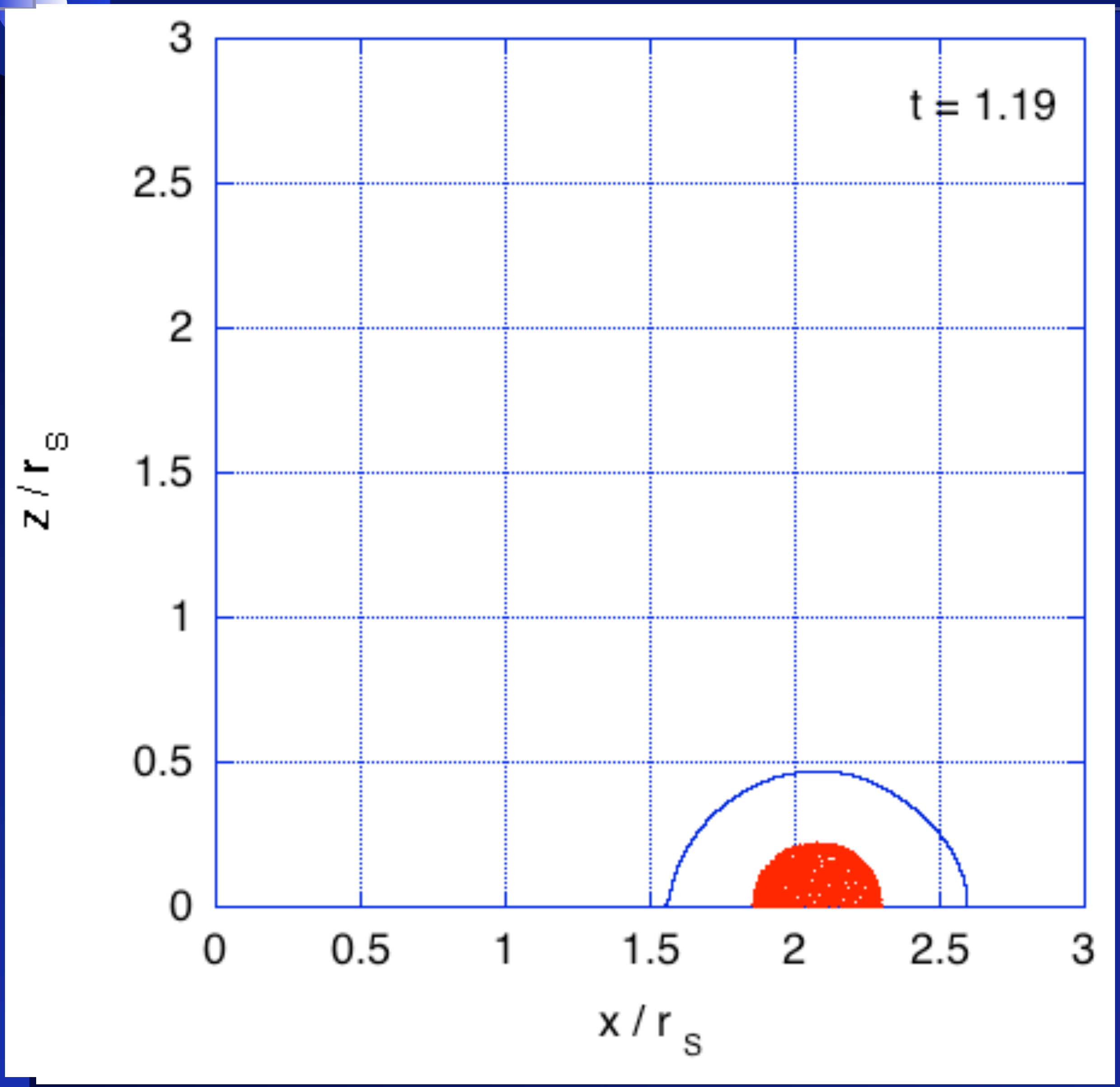
t=0 No Horizon



3. Evolution (case III)

t=0 No Horizon

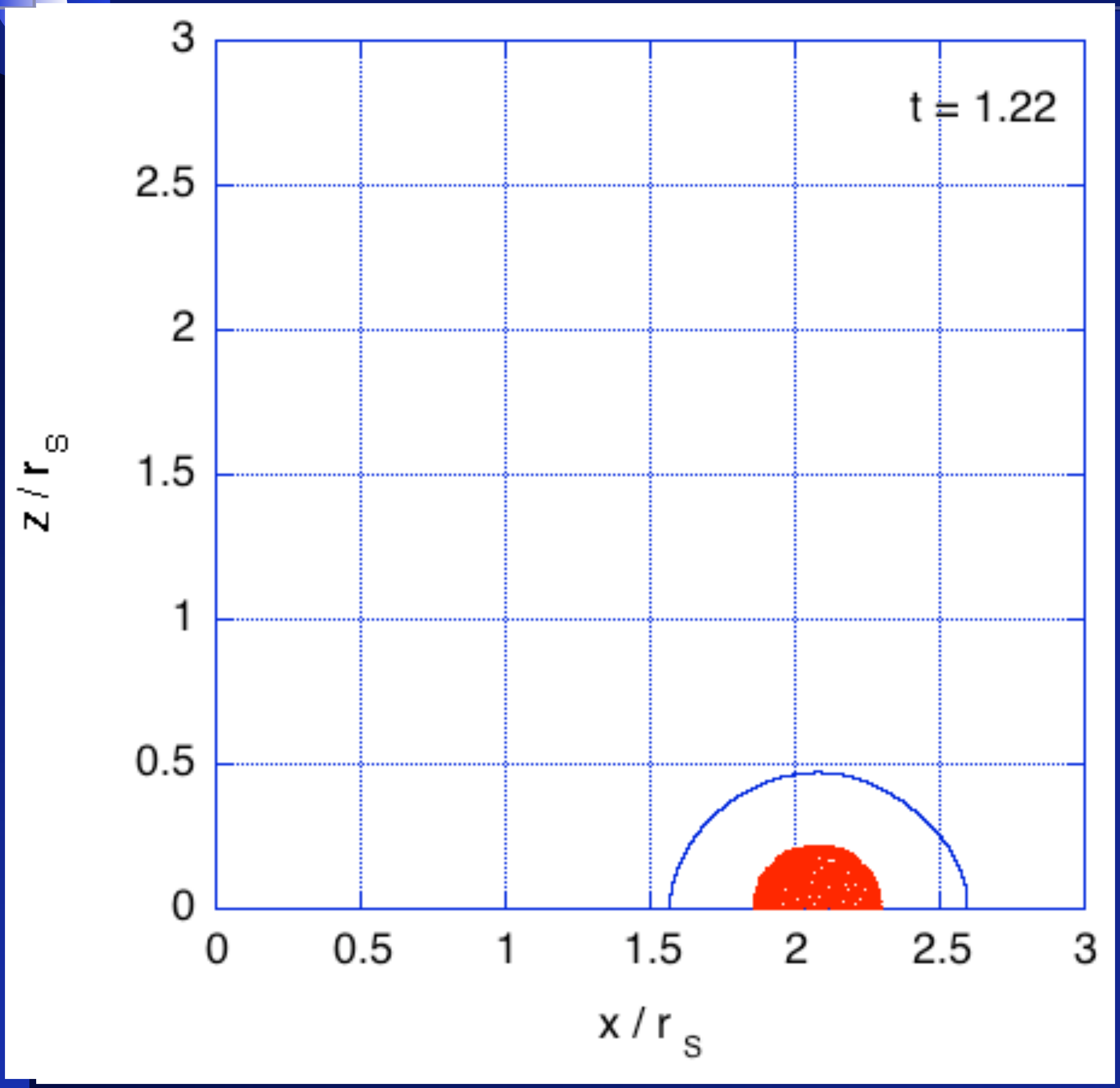
t=1.19 Ring Horizon



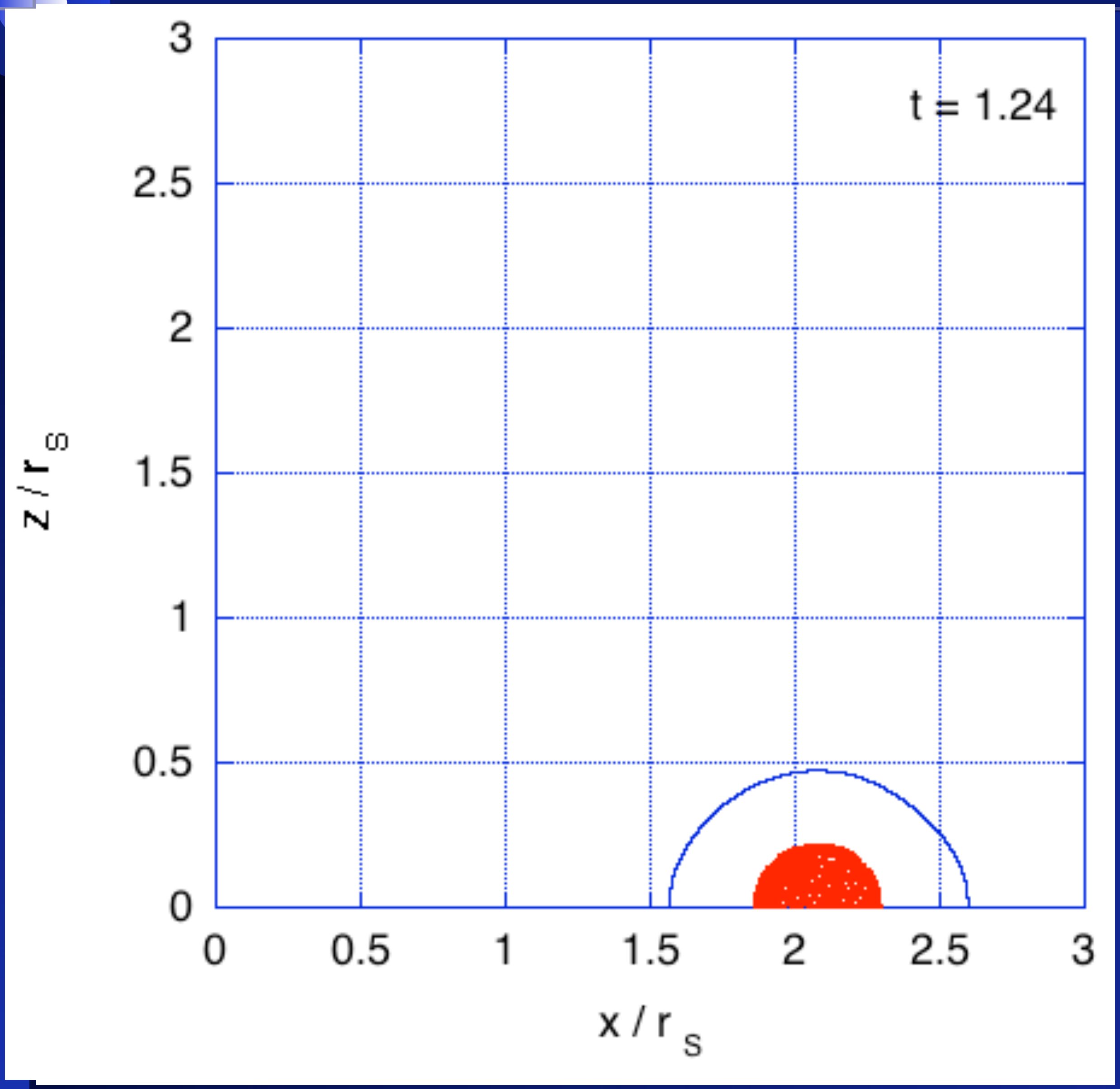
3. Evolution (case III)

t=0 No Horizon

t=1.19 Ring Horizon



3. Evolution (case III)



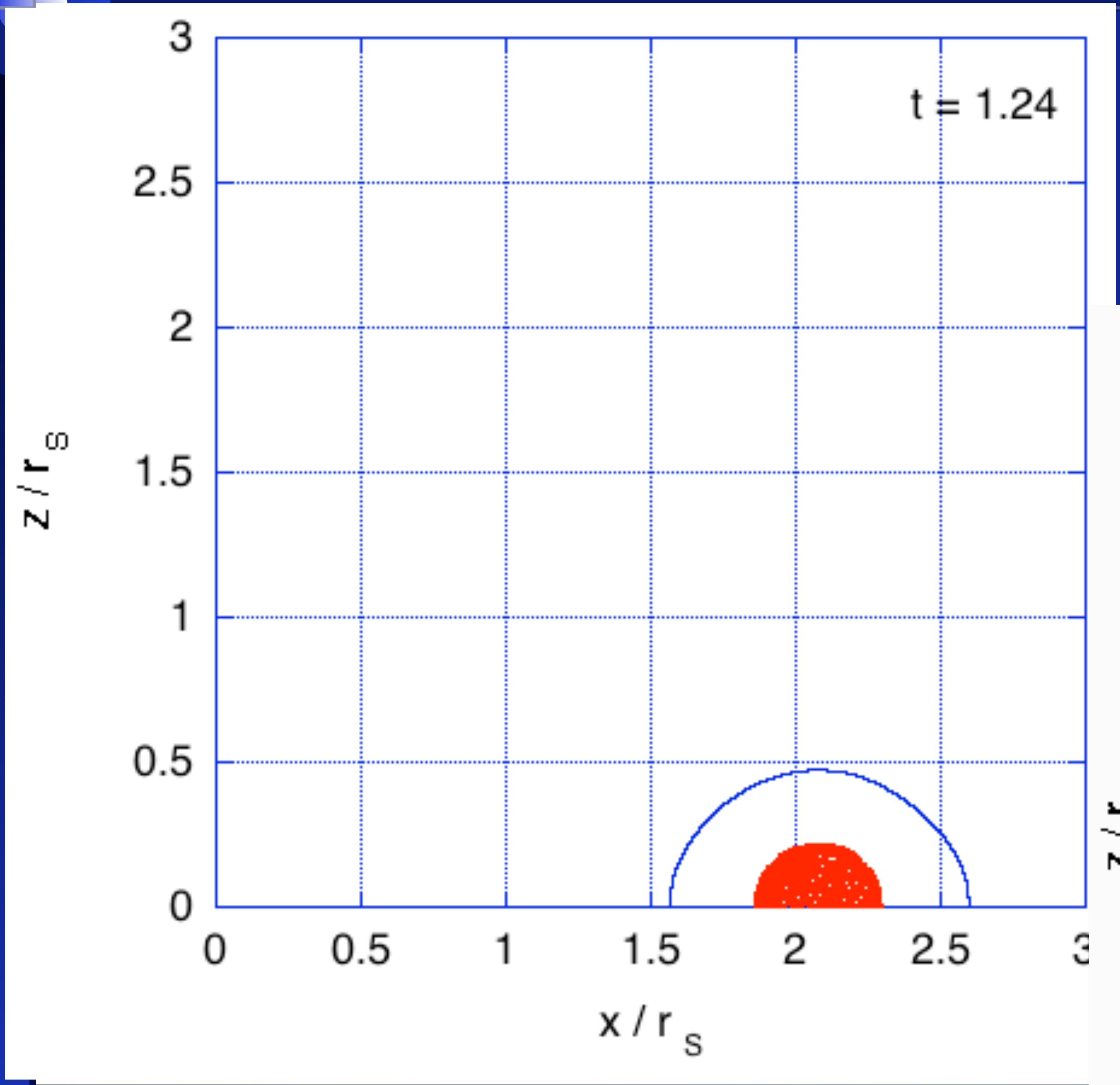
t=0 No Horizon

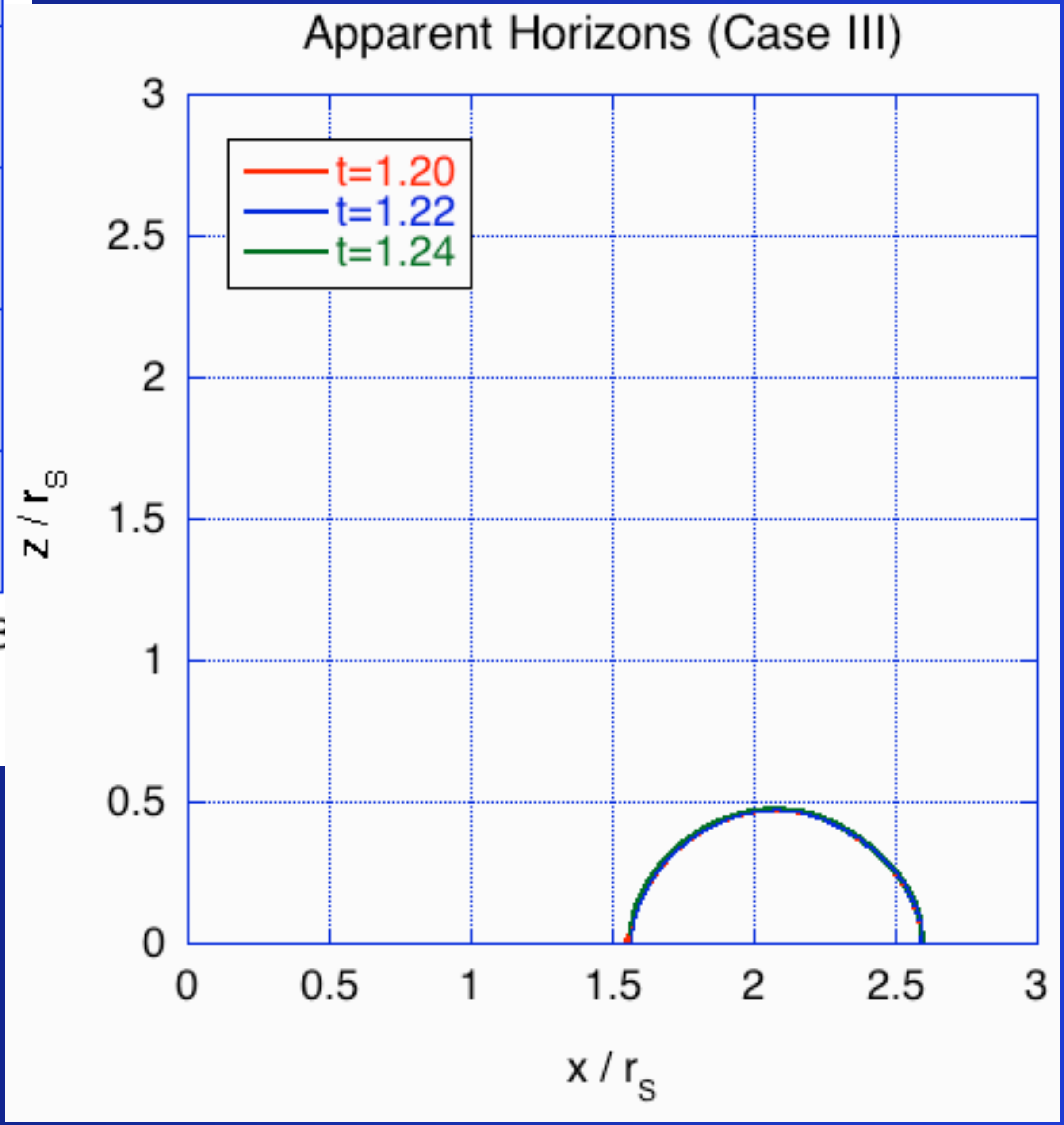
t=1.19 Ring Horizon

3. Evolution (case III)

t=0 No Horizon

t=1.19 Ring Horizon

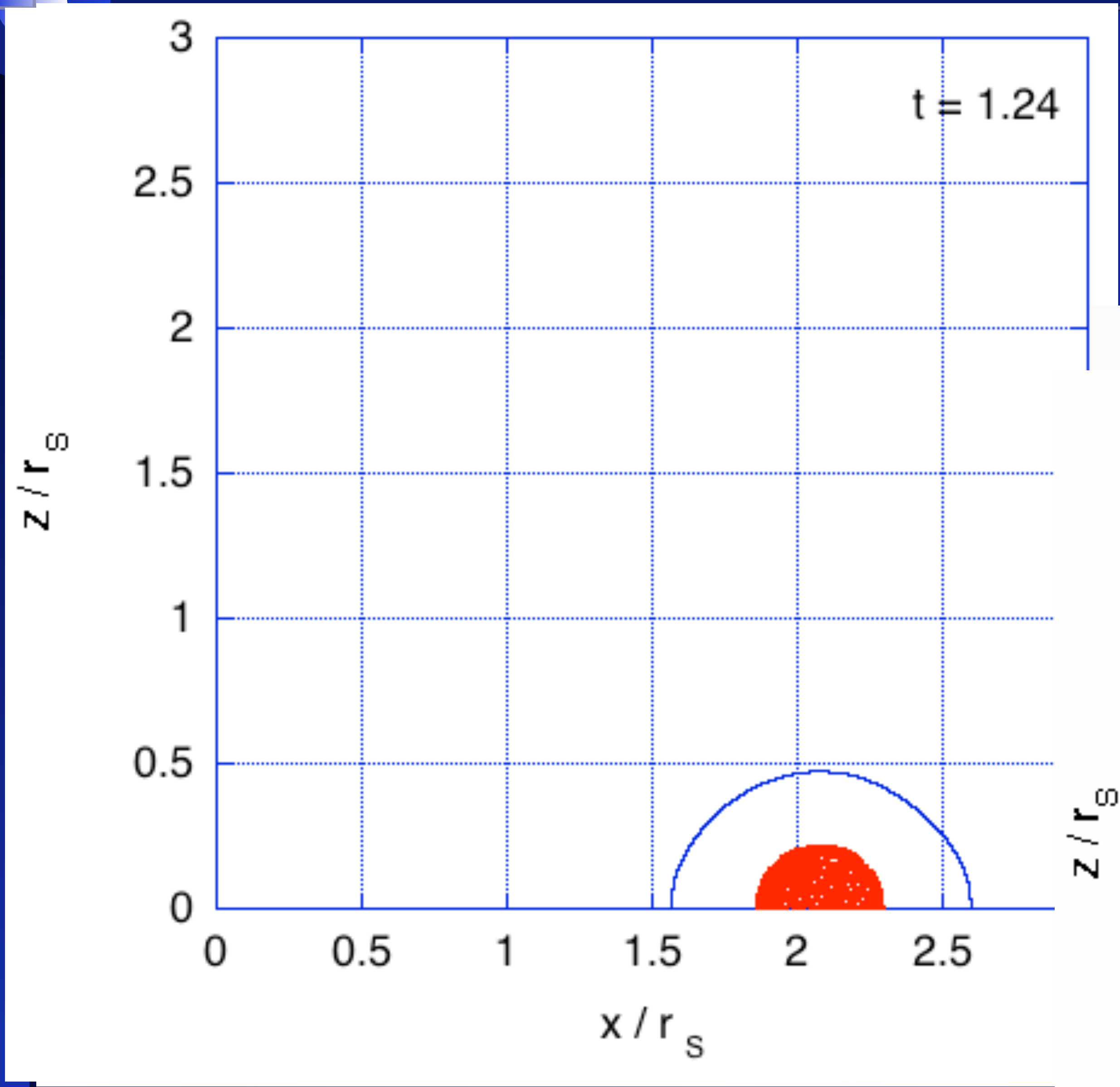


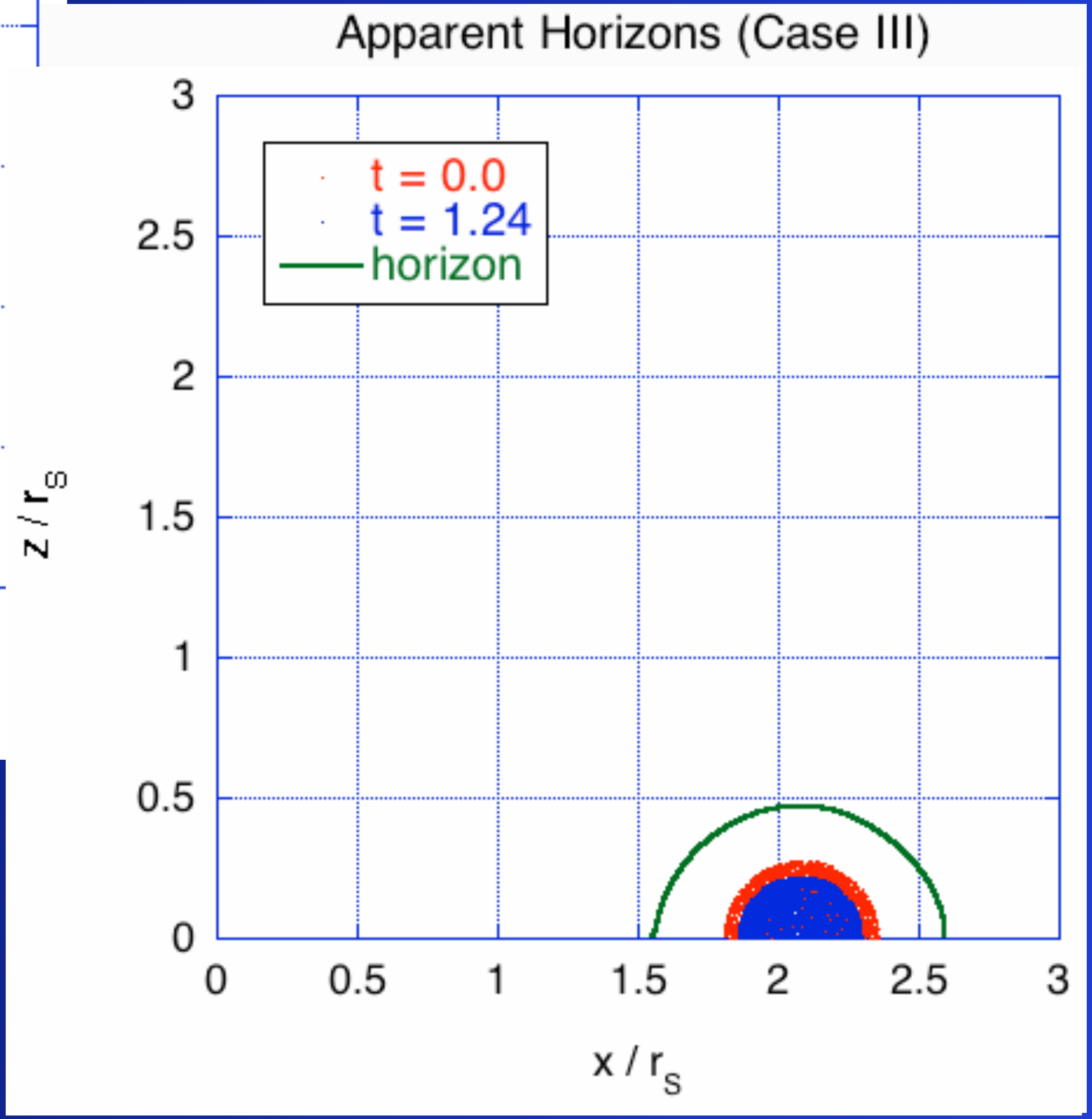


3. Evolution (case III)

t=0 No Horizon

t=1.19 Ring Horizon

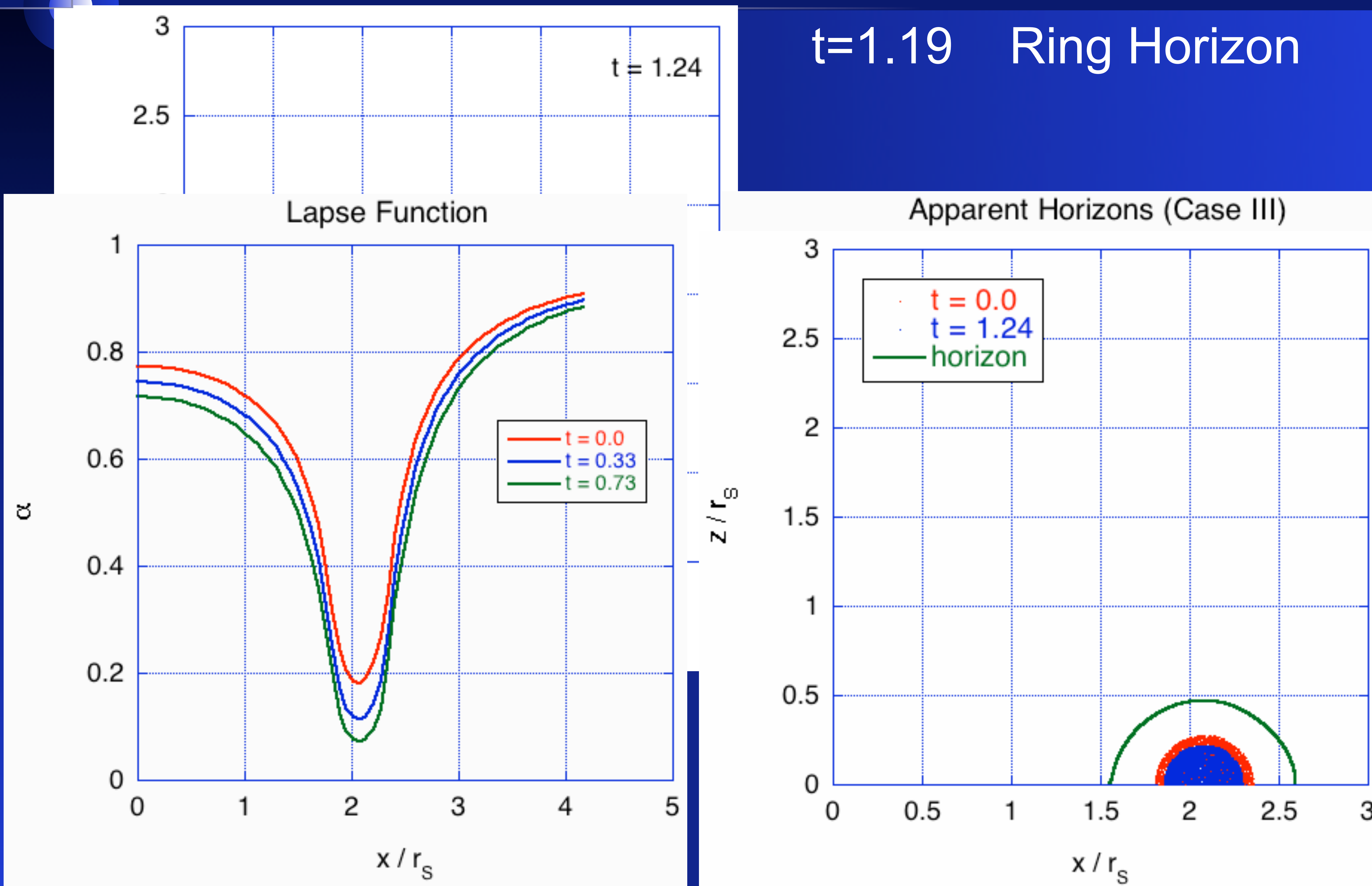




3. Evolution (case III)

t=0 No Horizon

t=1.19 Ring Horizon



4. Summary and Future Plans

Towards Dynamics of 5-dim Black Objects

Initial Data:

Topology of horizon changes with matter configurations

Hyper-Hoop prediction

works well for formations of spheroidal black holes

but not for rings.

Evolution:

no horizon → common horizon

no horizon ↘ ring horizon ⇢ common horizon

Future Plans:

include rotation, change slicing conditions

search event horizon,

investigate the stability, formation/decay process,....