# Gravitational Collapse in Five-dimensional Space-time


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## abstract

We numerically investigate black-hole (and black-ring) formation in five-dimensional spacetime. We model the initial matter distribution in non-rotating homogeneous spheroidal and toroidal configurations under the momentarily static assumption, and express the matter with collisionless particles. We evolve the spacetime using the maximal time slicing condition for the lapse function and the minimal strain condition for shift vector. We search apparent horizons both of $S^3$ and $S^1 \times S^2$ topology during time evolutions. For toroidal matter cases, we observe the topology of apparent horizons depends on the ring radius, and also we find the topology change of horizon from ring to spherical shape in time evolution. For spheroidal configurations, we can not find apparent horizon for highly prolate matter, which indicates also the formation of naked singularity. We discuss when black-hole or black-ring is formed and also report the possibility of a naked singularity formation.


## 1. Motivation

### Formation of naked singularities

formation of naked singulatities in 4D

・Gravitational collapse of collisionless particles with spheroidal configuration. (Shapiro, Teukolsky, PRL66, 994, 1991)

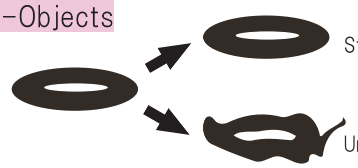
**possibility of naked singularities in 5D**

・Suggest the formation of naked singularities by spindle collapse. (Yoo, Ida, Nakao, PRD71, 104014, 2005)

### Dynamics of Black-Objects

・Formation process ?

・Dynamical features ?



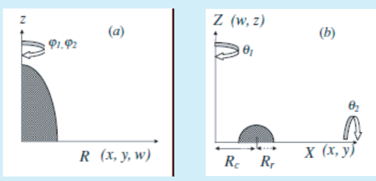

## 2. Our Numerical Approach

・Evolution of two kinds of non-rotating matter configurations.

・Using the (4 + 1) ADM formalism.

・Express the matter with collisionless particles.

・Search apparent horizons.




## 3. Initial data (Yamada, Shinkai, CQG27, 045012)

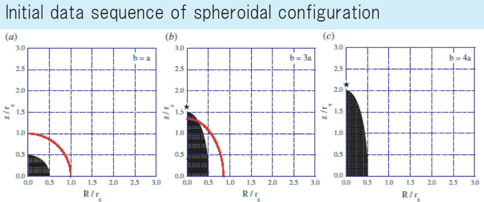
We construct sequences of initial data with
・conformally flat, moment of time symmetry, asymptotically flat

・Conformal transformation $\gamma_{ij} = \psi^2 \hat{\gamma}_{ij}$

・The Hamiltonian constraint equation $\hat{\Delta}\psi = -4\pi^2 G_5 \rho$

*boundary condition $\psi = 1 + \dfrac{M_{ADM}}{r^2}$

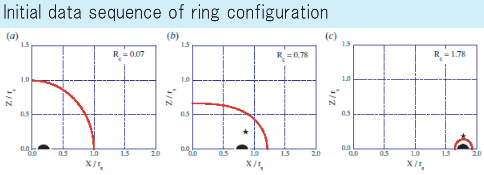
Initial data sequence of spheroidal configuration

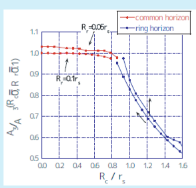


・The horizon is not formed when the matter is highly prolate shape.

・The asterisk indicates the location of the maximum Kretschmann invariant.

Initial data sequence of ring configuration






・Rc > 0.78rs → only the ring horizon
・Rc < 0.78rs → only the common horizon

・Both horizon's area are smoothly connected.
・scenario of gravitational collapse
collapse → black ring ?
collapse → black ring → black hole ?

## 4. Evolution

・Evolution equations

$$\frac{\partial \gamma_{ij}}{\partial t} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\frac{\partial K_{ij}}{\partial t} = \alpha({}^{(4)}R_{ij} + KK_{ij}) - 2\alpha K_{il}K^{lj} - 12\pi^2\alpha(S_{ij} + \frac{1}{3}\gamma_{ij}(\rho - S)) - D_iD_j\alpha + D_i\beta^m K_{mj} + D_j\beta^m K_{mi} + \beta^m D_m K_{ij}$$

・Maximal time slicing condition for lapse function

$$\Delta\alpha = \alpha(K_{ij}K^{ij} + \frac{2}{3}\kappa\rho + \frac{1}{3}\kappa S)$$

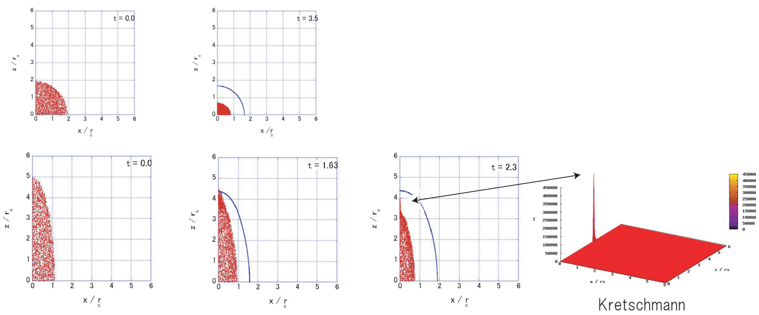
・Minimal strain condition for shift vector

$$\Delta\beta^i + D^iD_j\beta^j + R_{ij}\beta^j = 2D^j(\alpha K_{ij})$$

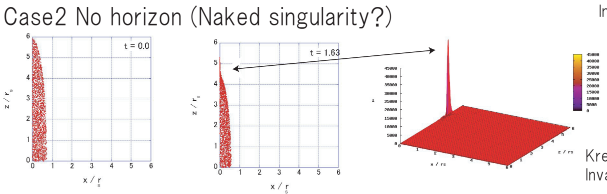
・We assume axi-symmetric space-time using the Cartoon method.

## 5. Time evolution of spheroidal configuration

Case1 horizon forms



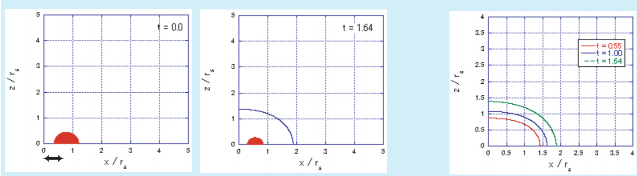

Case2 No horizon (Naked singularity?)



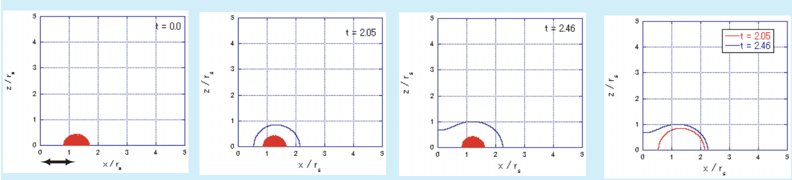

## 6. Time evolution of ring configuration

・We find three different cases for apparent horizon formation depending the ring radius Rc at t = 0.

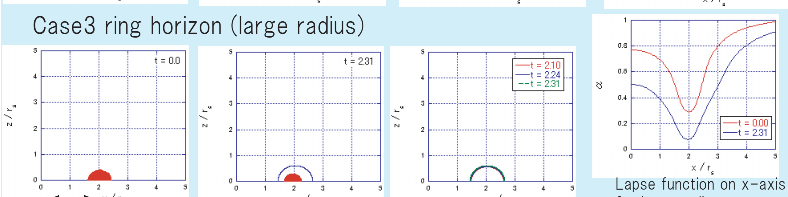
Case1 common horizon (small radius)




Case2 ring horizon → common horizon




Case3 ring horizon (large radius)




Lapse function on x-axis for large radius case