Wormhole Dynamics in Gauss-Bonnet gravity

Hisaaki Shinkai & Takashi Torii (Osaka Inst. Technology)

真貝寿明 & 鳥居隆 (大阪工業大学)

Outline & Summary

(a) "Fate of Morris-Thorne (Ellis) wormhole" was investigated in 2002. [HS & Hayward, PRD66, 044005].
Dynamics was followed numerically, using dual-null formulation.
The fate is either black-hole collapse or inflationary expansion, depending on the excessed energy.

(b) The higher-dimensional Ellis wormholes are constructed, and evolved.
The same features as 4-dim are observed.

(c) The same configuration is also evolved with Gauss-Bonnet field equations.
A preliminal result suggests that GB correction term prevents black hole collapse.

Motivations

Why wormholes?

- They make great science fiction – short cuts between otherwise distant regions. Morris & Thorne 1988, Sagan "Contact" etc
- They increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects (Casimir effect, Hawking radiation) or alternative gravity theories, brane-world models etc.
- They are very similar to black holes –both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH).

Wormhole ≡ Hypersurface foliated by marginally trapped surfaces

- BH and WH are interconvertible? New duality?

[1] How the stability changes in 5-d GR?
[2] How the stability changes in Gauss-Bonnet gravity?

Morris-Thorne's "Traversable" wormhole

M.S. Morris and K.S. Thorne, Am. J. Phys. 56 (1988) 395
M.S. Morris, K.S. Thorne, and U. Yurtsever, PRL 61 (1988) 3182
H.G. Ellis, J. Math. Phys. 14 (1973) 104
(G. Clément, Am. J. Phys. 57 (1989) 967)

Desired properties of traversable WHs

1. Spherically symmetric and Static ⇒ M. Visser, PRD 39(89) 3182 & NPB 328 (89) 203
2. Einstein gravity
3. Asymptotically flat
4. No horizon for travel through
5. Tidal gravitational forces should be small for traveler
6. Traveler should cross it in a finite and reasonably small proper time
7. Must have a physically reasonable stress-energy tensor
   ⇒ Weak Energy Condition is violated at the WH throat.
   ⇒ (Null EC is also violated in general cases.)
8. Should be perturbatively stable
9. Should be possible to assemble

BH and WH are interconvertible ? (New Duality?)

S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

- They are very similar – both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)
- Only the causal nature of the THs differs, whether THs evolve in plus / minus density.

	Black Hole	Wormhole
Locally defined by	Achronal(spatial/null) outer TH ⇒ 1-way traversable	Temporal (timelike) outer THs ⇒ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appearance	occur naturally	Unlikely to occur naturally. but constructible ???

Field Eqs.

Gauss-Bonnet gravity

$$S=\int_{\mathcal M} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R}+\alpha_2(\mathcal{R}^2-4\mathcal{R}_{\alpha\beta}\mathcal{R}^{\alpha\beta}+\mathcal{R}_{\alpha\beta\gamma\delta}\mathcal{R}^{\alpha\beta\gamma\delta})\}+\mathcal{L}_{\rm matter}\Big]$$

- has GR correction terms from String Theory.
- has two solution branches (GR/non-GR).
- is expected to have singularity avoidance feature. (but has never been demonstrated.)

- new topic in numerical relativity.
  (S Golod & T Piran, PRD 85 (2012) 104015;
  F Izaurieta & E Rodriguez, 1207.1496; N Deppe+ 1208.5250)

Field Equations

- Action

$$S=\int_{\mathcal M} d^{N+1}x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\{\alpha_1\mathcal{R}+\alpha_2\mathcal{L}_{GB}\}+\mathcal{L}_{\rm matter}\Big] \quad (1)$$

where $\mathcal{L}_{GB}=\mathcal{R}^2-4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}+\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation

$$\alpha_1 G_{\mu\nu}+\alpha_2 H_{\mu\nu}+g_{\mu\nu}\Lambda=\kappa^2 T_{\mu\nu} \quad (2)$$

where $H_{\mu\nu}=2[\mathcal{R}\mathcal{R}_{\mu\nu}-2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu}-2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta}+\mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}]-\frac{1}{2}g_{\mu\nu}\mathcal{L}_{GB}$

- matter

normal field $\psi(u,v)$ and/or ghost field $\phi(u,v)$

$$T_{\mu\nu}=T^{\psi}_{\mu\nu}+T^{\phi}_{\mu\nu}$$
$$=\Big[\psi_{,\mu}\psi_{,\nu}-g_{\mu\nu}\Big(\frac{1}{2}(\nabla\psi)^2+V_1(\psi)\Big)\Big]-\Big[\phi_{,\mu}\phi_{,\nu}-g_{\mu\nu}\Big(\frac{1}{2}(\nabla\phi)^2+V_2(\phi)\Big)\Big] \quad (3)$$

this derives Klein-Gordon equations

$$\Box\psi=\frac{dV_1}{d\psi},\qquad \Box\phi=\frac{dV_2}{d\phi}. \quad (4)$$

Equations in 5-D with Gauss-Bonnet corrections

$$\alpha_1 G_{\mu\nu}+\alpha_2 H_{\mu\nu}+g_{\mu\nu}\Lambda=\kappa^2 T_{\mu\nu}$$

$$\eta=\Omega^2\Big(e^{-f}+\frac{2}{9}\vartheta_+\vartheta_-\Big),\quad \tilde{A}=(\alpha_1+4\alpha_2\eta e^f),\quad B=\kappa^2T_{+-}+e^{-f}\Lambda$$

x^+-direction ∂_+ | x^--direction ∂_-

$$\partial_\pm\Omega=-\frac{1}{3}\vartheta_\pm\Omega^2 \quad (1)$$
$$\partial_\pm\vartheta_\pm=-\nu_\pm\vartheta_\pm-\frac{1}{\tilde{A}\Omega}\kappa^2T_{\pm\pm} \quad (2)$$
$$\partial_\pm\vartheta_\mp=\frac{1}{\tilde{A}\Omega}(-3\alpha_1\eta+B) \quad (3)$$
$$\partial_\pm f=\nu_\pm \quad (4)$$
$$\partial_\pm\nu_\mp=\frac{\alpha_1}{\tilde{A}}\Big\{\eta-\frac{4(3\alpha_1\eta-B)}{3\tilde{A}}\Big\}+\frac{(\kappa^2T_{zz}\Omega^2-\Lambda)}{\tilde{A}e^f}+\frac{8\alpha_2}{9\tilde{A}^3}\{e^f(3\alpha_1\eta-B)^2-\kappa^4T_{++}T_{--}\} \quad (5)$$
$$\partial_\pm\psi=\Omega\pi_\pm \quad (6)$$
$$\partial_\pm\phi=\Omega p_\pm \quad (7)$$
$$\partial_\pm\pi_\mp=-\frac{1}{6}\Omega\vartheta_\pm\pi_\mp-\frac{1}{2}\Omega\vartheta_\mp\pi_\pm-\frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} \quad (8)$$
$$\partial_\pm p_\mp=-\frac{1}{6}\Omega\vartheta_\pm p_\mp-\frac{1}{2}\Omega\vartheta_\mp p_\pm-\frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} \quad (9)$$

Energy-momentum tensor

$$T_{++}=\Omega^2(\pi_+^2-p_+^2) \quad (10)$$
$$T_{--}=\Omega^2(\pi_-^2-p_-^2) \quad (11)$$
$$T_{+-}=-e^{-f}(V_1(\psi)+V_2(\phi)) \quad (12)$$
$$T_{zz}=e^f(\pi_+\pi_--p_+p_-)-\frac{1}{\Omega^2}(V_1(\psi)-V_2(\phi)) \quad (13)$$

Results in 4-dim. GR

PRD66 (2002) 044005

Bifurcation of the horizons
-- go to a Black Hole or Inflationary expansion

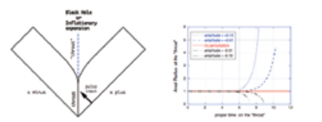


Figure 4: Partial Penrose diagram of the evolved space-time.
Figure 6: Areal radius r of the "throat" $x^+=x^-$, plotted as a function of proper time. Additional negative energy causes inflationary expansion, while reduced negative energy causes collapse to a black hole and central singularity.

Ghost pulse input -- Bifurcation of the horizons

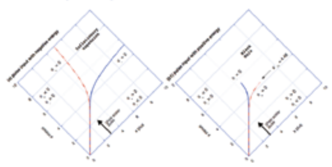
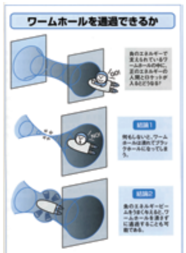


Figure 3: Horizon locations, $\vartheta_\pm=0$, for perturbed wormhole. Fig.(a) is the case we supplement the ghost field, $c_a=0.1$, and (b1) and (b2) are where we reduce the field, $c_a=-0.1$ and -0.01. Dashed lines and solid lines are $\vartheta_+=0$ and $\vartheta_-=0$ respectively. In all cases, the pulse hits the wormhole throat at $(x^+,x^-)=(3,3)$. A 45° counterclockwise rotation of the figure corresponds to a partial Penrose diagram.

Travel through a Wormhole
-- with Maintenance Operations!

Figure 11: A trial of wormhole maintenance. After a normal scalar pulse, we signalled a ghost scalar pulse to extend the life of wormhole throat. The travellers pulse are commonly expressed with a normal scalar field pulse.
(A) no maintenance case (results in a black hole),
(B) with maintenance pulse (results in an inflationary expansion),
(C) with maintenance pulse (keep stationary structure upto the end of this range).

Time Machine & Science of Space-time (HS, 2011)



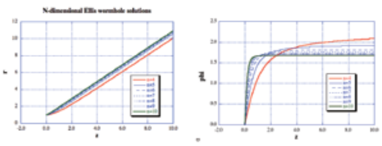
N-dim. Ellis Wormhole sol.

in prep.

A Wormhole Solution (n-Dim, massless ghost scalar)

- massless ghost scalar field ϕ,
- static, spherical symmetry.

$$ds^2=-dt^2+dz^2+r^2(z)d\Omega^2$$
$$\frac{d^2r}{dz^2}=\frac{(n-3)a^{2(n-3)}}{r^{2n-5}} \quad (1)$$
$$\frac{d\phi}{dz}=\sqrt{(n-2)(n-3)}\frac{a^{n-3}}{r^{n-2}} \quad (2)$$



A Wormhole Solution (5-Dim, massive ghost scalar)

- massive ghost scalar field ϕ.
- static, spherical symmetry.

$$ds^2=-2e^{-f(x^+,x^-)}dx^+dx^-+r^2(x^+,x^-)d\Omega^2$$

metric ($z=\frac{x^+-x^-}{\sqrt{2}}$, a: throat radius)

$$r=\sqrt{a^2+z^2}\qquad \vartheta_\pm=\pm\frac{3}{\sqrt{2}}\frac{z}{r} \quad (1)$$
$$e^{-f}=\frac{r^2+z^2}{2r^2} \quad (2)$$

scalar field

$$\phi=-\sqrt{3}\tanh^{-1}\frac{-z}{\sqrt{r^2+z^2}} \quad (3)$$
$$\frac{dV_2}{d\phi}=-\frac{\sqrt{3}}{a^2}\sinh\frac{2\phi}{\sqrt{3}}\Big(1-2\tanh^2\frac{\phi}{\sqrt{3}}\Big)^3 \quad (4)$$

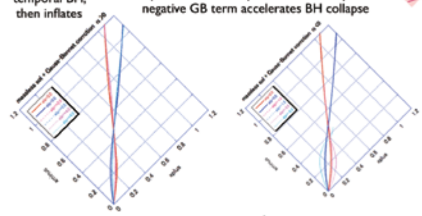
WH evolution in 5-dim. GB

in prep.

WH evolution in 5D Gauss-Bonnet gravity Preliminary

temporal BH, then inflates

positive GB term prevents BH collapse
negative GB term accelerates BH collapse



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where $\mathcal{L}_{GB}=\mathcal{R}^2-4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}+\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

shinkai@is.oit.ac.jp
http://www.is.oit.ac.jp/~shinkai/

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