

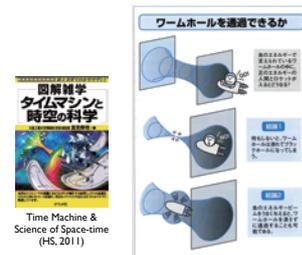
ワームホールの不安定性

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Outline & Summary

タイムマシンの原理としても有名な Morris-Thorne のワームホール解 (Ellis WH 解) はゴーストスカラー場で構成される仮想的な一般相対論の解である。静的な仮定で導かれた解だが、著者らは動的に不安定であることを以前報告した【Shinkai & Hayward, Phys. Rev. D 66 (2002) 044005】。今回は、高次元に拡張したり、宇宙項を入れたとしても同様に不安定であることを報告する。

我々は、 n 次元に拡張した時空中で Ellis WH 解を求め、摂動に対して不安定であることを見つけた【Torii & Shinkai, PRD88 (2013) 064023】。数値計算により、不安定であることを確かめ、WH の喉 (throat) は、与えるゆらぎのエネルギーの正負によって、ブラックホール (BH) に転じるか、あるいはインフレーション的に膨張することがわかった。BH に転じる場合、与えるゆらぎは小さくても形成される BH 質量には最小値が存在し、次元が上がれば最小値は大きくなる。宇宙項の正負によっても WH は BH または拡大する。Gauss-Bonnet 重力理論では、BH 形成は抑えられるようだが、定常解ではない。総じて、このような単純なワームホールは不安定であると結論でき、観測するのは難しそうである。



Motivations

Why wormholes?

- They make great science fiction – short cuts between otherwise distant regions. Morris & Thorne 1988, Sagan "Contact" etc
- They increase our understanding of gravity when the usual energy conditions are not satisfied, due to quantum effects (Casimir effect, Hawking radiation) or alternative gravity theories, brane-world models etc.
- They are very similar to black holes – both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH).

Wormhole \equiv Hypersurface foliated by marginally trapped surfaces

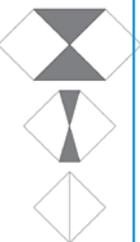
- BH and WH are interconvertible?
- New duality?

- How the stability changes in 5-d GR?
- How the stability changes in Gauss-Bonnet gravity?

BH and WH are interconvertible? (New Duality?)

S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

- They are very similar – both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)
- Only the causal nature of the THs differs, whether THs evolve in plus / minus density.

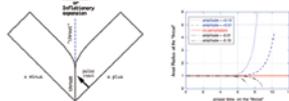


	Black Hole	Wormhole
Locally defined by	Achronal (spatial/null) outer TH \Rightarrow 1-way traversable	Temporal (timelike) outer TH \Rightarrow 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appearance	occur naturally	Unlikely to occur naturally, but constructible ???

Results in 4-dim. GR

PRD66 (2002) 044005

Bifurcation of the horizons – go to a Black Hole or inflationary expansion



Ghost pulse input – Bifurcation of the horizons

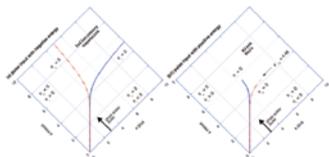


Figure 3: Bifurcation diagrams for ghost pulse input. The graphs show the evolution of the apparent horizon (red) and event horizon (blue) for various initial conditions and pulse amplitudes.

Travel through a Wormhole – with Maintenance Operations

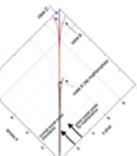


Figure 4: A total of wormhole solutions. The diagram shows a wormhole with a maintenance operation (vertical line) that keeps the throat open. The graphs show the evolution of the apparent horizon (red) and event horizon (blue) for various initial conditions and pulse amplitudes.

N-dim. Ellis Wormhole sol.

A Wormhole Solution (n-Dim, massless ghost scalar)

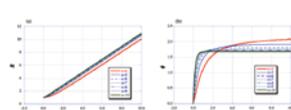
- spherical symmetry.

$$ds^2 = -f(t,r)c^{2(n-1)}dt^2 + f(t,r)^{-1}dr^2 + R(t,r)^2d\Omega^2 \quad (1)$$

- with massless ghost scalar field ϕ .
- static, $f \equiv 1$, and throat radius $R(0) = a$; Just solve

$$\frac{d^2R}{dr^2} = \frac{(n-3)2^{n-3}}{R^{2n-3}} \quad (2)$$

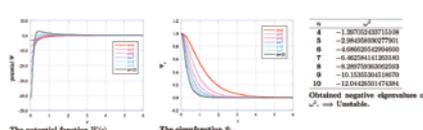
$$\frac{d\phi}{dr} = \sqrt{(n-2)(n-3)} \frac{1}{R^{n-2}} \quad (3)$$



The n-dimensional wormhole solutions. The circumference radius R (a) and the scalar field ϕ (b) are plotted as a function of radial coordinate r . The cases of $n = 4-6$ are shown.

Torii & HS, PRD88 (2013), 064023

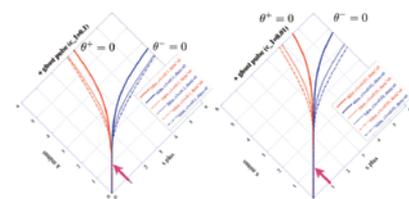
Perturbation Analysis



摂動に対して不安定なモードが必ず存在する

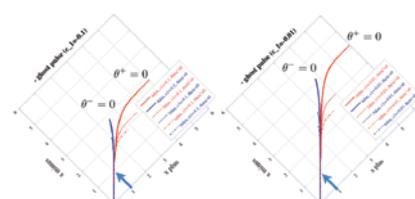
WH evolution in 4, 5, 6-dim. GR in prep.

4d 5d 6d GR ghost pulse (additional amp.) input



negative energy input \rightarrow throat inflates

4d 5d 6d GR ghost pulse (subtract amp.) input

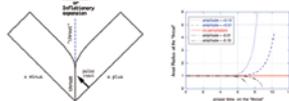


positive energy input \rightarrow BH formation

Results in 4-dim. GR

PRD66 (2002) 044005

Bifurcation of the horizons – go to a Black Hole or inflationary expansion



Ghost pulse input – Bifurcation of the horizons

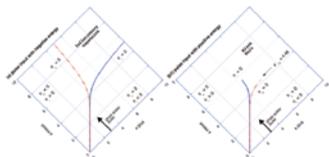


Figure 3: Bifurcation diagrams for ghost pulse input. The graphs show the evolution of the apparent horizon (red) and event horizon (blue) for various initial conditions and pulse amplitudes.

Travel through a Wormhole – with Maintenance Operations

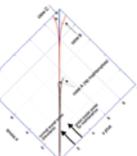


Figure 4: A total of wormhole solutions. The diagram shows a wormhole with a maintenance operation (vertical line) that keeps the throat open. The graphs show the evolution of the apparent horizon (red) and event horizon (blue) for various initial conditions and pulse amplitudes.

Gauss-Bonnet gravity

- Action

$$S = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} (R + \alpha_1 C_{ab}C^{ab}) + \mathcal{L}_{matter} \right] \quad (1)$$

$$\text{where } \mathcal{L}_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

- Field eqs.

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu} \Lambda = \kappa^2 T_{\mu\nu} \quad (2)$$

$$\text{where } H_{\mu\nu} = 2[R_{\mu\nu} - 2R_{\mu\nu}R_{\alpha\beta} - 2R^{\alpha\beta}R_{\mu\nu\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}] - \frac{1}{2}g_{\mu\nu}C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$$

- Assumptions

- 5-dim. Spherical Symmetry
- Dual-null coordinate

Field Eqs.

$$ds^2 = -2e^{-f(u,r)} du^2 + e^{2f(u,r)} dr^2 + r^2 d\Omega_3^2 \quad (3)$$

$$\text{conformal factor } \Omega = \frac{1}{r} \quad (4)$$

$$\text{expansions } \theta_{\pm} = 3\partial_r r = -3\Omega^{-1}\partial_u \Omega \quad (5)$$

$$\text{infinities } v_{\pm} = \partial_t f \quad (6)$$

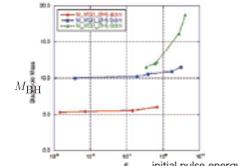
$$\text{momenta of } \phi \quad p_{\pm} = r\partial_r \phi = \Omega^{-1}\partial_u \phi \quad (7)$$

$$\text{momenta of } \psi \quad \pi_{\pm} = r\partial_r \psi = \Omega^{-1}\partial_u \psi \quad (8)$$

- matter = normal field $\psi(u,r)$ and/or ghost field $\phi(u,r)$

$$T_{\mu\nu} = T_{\mu\nu}^{\psi} + T_{\mu\nu}^{\phi} = \left[\frac{1}{2}(\nabla_{\mu}\psi)^2 + V_{\psi}(\psi) \right] + \left[-\frac{1}{2}(\nabla_{\mu}\phi)^2 + V_{\phi}(\phi) \right] \quad (9)$$

Minimum Mass of Black Hole



Quasi-Local Energy (Misner-Sharp mass in N-dim.)

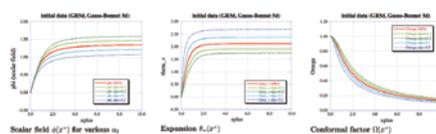
Using the area of the 3th-dimensional space of constant coordinates, A , (or the associated area $A_s = A/r^2$, E_s may be defined as (cf Misner & Newman, 2008)

$$E_s = \frac{(n-2)A_s}{2\kappa^2} \left[\frac{1}{2} \dot{A}_s^2 + \left(\frac{2}{n-2} \right) \dot{r}_s^2 \right]$$

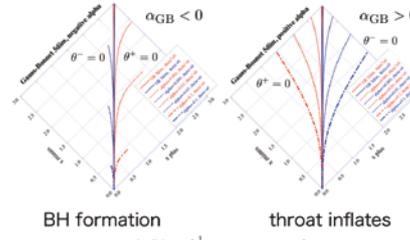
WH evolution in 5-dim. GB

in prep.

Wormholes in Gauss-Bonnet gravity (initial data on x^+)



5d GR vs Gauss-Bonnet instability appears



BH formation throat inflates

$$S = \int_{\mathcal{M}} d^5x \sqrt{-g} \left[\frac{1}{2\kappa^2} (R + \alpha_1 C_{ab}C^{ab}) + \mathcal{L}_{matter} \right] \quad (10)$$

where $\mathcal{L}_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$