Nonlinear dynamics in the Einstein-Gauss-Bonnet gravity

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- 4-dim, 5-dim, 6-dim,
  ··· how dimensionality affects to dynamics?
- Gauss-Bonnet terms
  ··· how higher-order curvature terms affects to dynamics?
- 2 models
  Colliding scalar pulses / Fate of wormholes
- Ref: HS & Torii , arXiv:1706.02070

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Dynamics in Gauss-Bonnet gravity?

- Action

$$S = \int_{\mathcal{M}} d^n x \sqrt{-g} \left[\frac{1}{2\kappa^2} (\alpha_{\rm GR} \mathcal{R} + \alpha_{\rm GB} \mathcal{L}_{\rm GB}) + \mathcal{L}_{\rm matter} \right]$$

where $\mathcal{L}_{GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$

- Field equation

$$\alpha_1 G_{\mu\nu} + \alpha_2 H_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa^2 T_{\mu\nu}$$

where $H_{\mu\nu} = 2[\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}] - \frac{1}{2} g_{\mu\nu}\mathcal{L}_{GB}$

- **has the simplest leading terms from String Theory**
- **has two solution branches (GR/non-GR).**
- **has minimum mass for static spherical BH solution**
  T Torii & H Maeda, PRD 71 (2005) 124002
  W-K Ahn, B Gwak, B-H Lee, W Lee, Eur. Phys. J. C75 (2015) 372
- **is expected to have singularity avoidance feature.**
  (but has never been demonstrated in full gravity.)
- **new topic in numerical relativity.**
  S Golod & T Piran, PRD 85 (2012) 104015
  N Deppe+, PRD 86 (2012) 104011
  F Izaurieta & E Rodriguez, 1207.1496
- **much attentions in WH community**
  H Maeda & M Nozawa, PRD 78 (2008) 024005
  P Kanti, B Kleihaus & J Kunz, PRL 107 (2011) 271101
  P Kanti, B Kleihaus & J Kunz, PRD 85 (2012) 044007

Formulation for evolution [dual null]

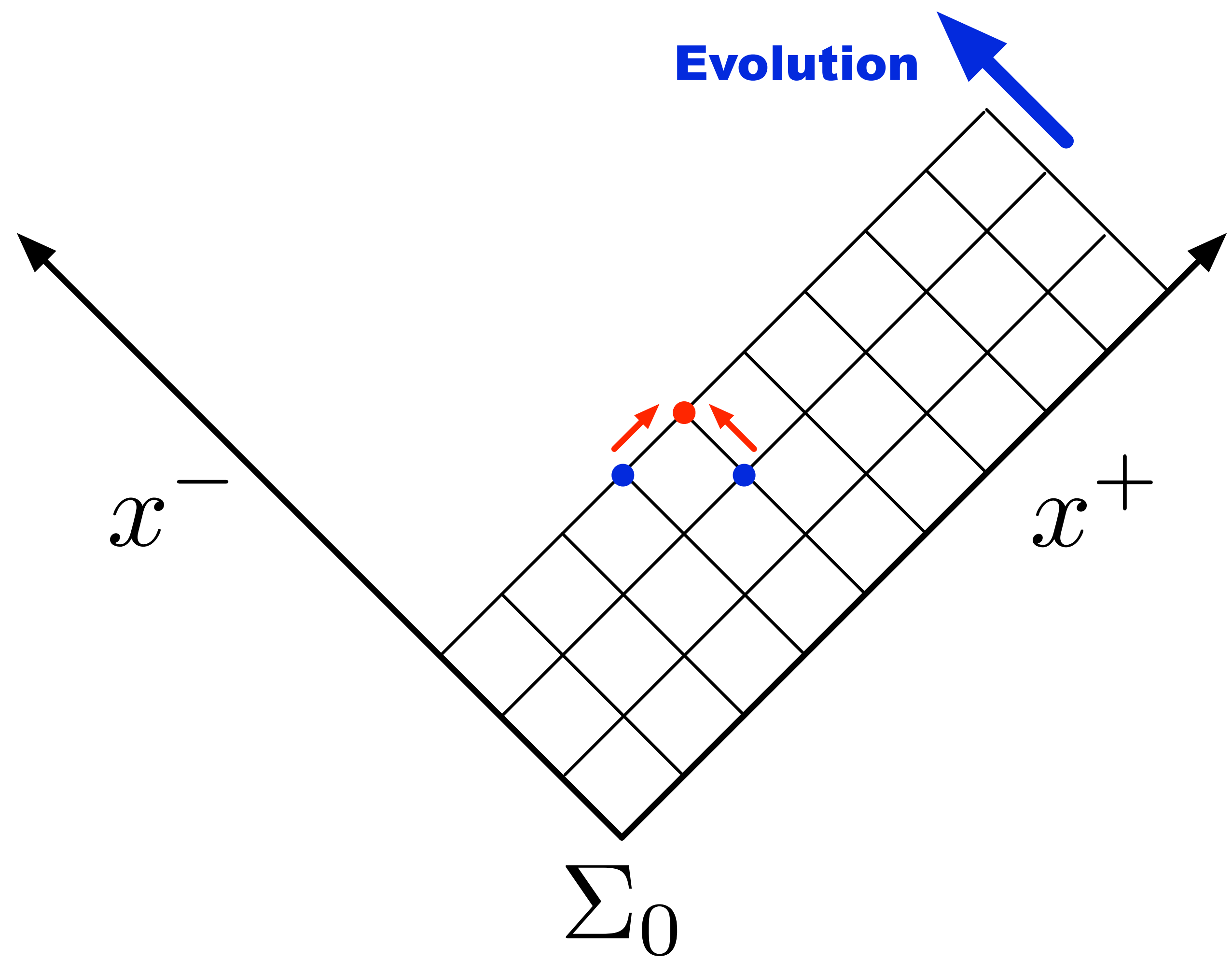
Metric n-dimensional, dual-null coordinate, $2+(n-2)$ decomposition

$$ds^2 = -2e^{-f(x^+,x^-)}\,dx^+\,dx^- + r^2(x^+,x^-)\gamma_{ij}dx^i dx^j \tag{1}$$

Variables

$\Omega = \dfrac{1}{r}$	Conformal factor
$\vartheta_\pm = (n-2)\partial_\pm r$	expansion
f	lapse function
$\nu_\pm = \partial_\pm f$	inaffinity (shift)

ψ	scalar field (normal)
$\pi_\pm = r\partial_\pm\psi$	scalar momentum
ϕ	scalar field (ghost)
$p_\pm = r\partial_\pm\phi$	scalar momentum



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Parameters

n	dimension
k	curvature
Λ	cosmological constant

For simplicity, we define

$$\tilde{\alpha} = (n-3)(n-4)\alpha_2, \tag{2}$$

$$A = \alpha_1 + 2\tilde{\alpha}\Omega^2 Z, \tag{3}$$

$$W = \frac{2e^f}{(n-2)^2}\vartheta_+\vartheta_-, \tag{4}$$

$$Z = k + W, \tag{5}$$

$$\eta = \Omega^2\frac{(n-2)(n-3)}{2}e^{-f}Z, \tag{6}$$

matter variables

normal field $\psi(u,v)$ and/or ghost field $\phi(u,v)$

$$\begin{aligned} T_{\mu\nu} &= T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi) \\ &= \left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] + \left[-\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(-\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right] \end{aligned}$$

this derives Klein-Gordon equations

$$\Box\psi = \frac{dV_1}{d\psi}, \qquad \Box\phi = \frac{dV_2}{d\phi}.$$

Scalar field variables

$$\begin{aligned} \pi_\pm &\equiv r\partial_\pm\psi = \frac{1}{\Omega}\partial_\pm\psi \\ p_\pm &\equiv r\partial_\pm\phi = \frac{1}{\Omega}\partial_\pm\phi \end{aligned}$$

Klein-Gordon eqs.

$$\begin{aligned} \Box\phi &= -\frac{e^f}{r}(2r\phi_{uv} + (n-2)r_u\phi_v + (n-2)r_v\phi_u) \\ &= -2e^f\phi_{uv} - e^f\Omega^2(\vartheta_- p_+ + \vartheta_+ p_-) \end{aligned}$$

Energy-momentum tensor

$$\begin{aligned} T_{++} &= \Omega^2(\pi_+^2 - p_+^2) \\ T_{--} &= \Omega^2(\pi_-^2 - p_-^2) \\ T_{+-} &= -e^{-f}(V_1(\psi) + V_2(\phi)) \\ T_{zz} &= e^f(\pi_+\pi_- - p_+p_-) - \frac{1}{\Omega^2}(V_1(\psi) - V_2(\phi)) \end{aligned}$$

evolution equations (1)

Equations for x^+ direction

$$\partial_+\Omega = -\frac{1}{n-2}\vartheta_+\Omega^2 \tag{7}$$

$$\partial_+\vartheta_+ = -\vartheta_+\nu_+ - \frac{1}{\Omega A}\kappa^2 T_{++} = -\vartheta_+\nu_+ - \frac{1}{A}\kappa^2\Omega(\pi_+^2 - p_+^2) \tag{8}$$

$$\partial_+\vartheta_- = \frac{1}{A}\frac{e^{-f}}{\Omega}\left[-\alpha_1\Omega^2\frac{(n-2)(n-3)}{2}Z + \Lambda + \kappa^2(V_1+V_2)\right] - \frac{\tilde{\alpha}}{A}\Omega^3 e^{-f}\frac{(n-2)(n-5)}{2}\left[Z^2+W\right] \tag{9}$$

$$\partial_+ f = \nu_+ \tag{10}$$

$\partial_+\nu_+ =$ no evolution eq. exists

$$\begin{aligned}\partial_+\nu_- = {}& \frac{\alpha_1}{A}Ze^{-f}\Omega^2\frac{(n-3)}{2}\left\{-\frac{\alpha_1}{A}2(n-3)+n-4\right\} \\ &+\frac{1}{A}\Omega^2 e^{-f}\kappa^2(\pi_+\pi_- - p_+p_-) + \frac{1}{A}e^{-f}\left\{\frac{\alpha_1}{A}\frac{2(n-3)}{(n-2)} - 1\right\}\{\Lambda+\kappa^2(V_1+V_2)\} \\ &-\frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\left[\frac{\alpha_1}{A}\Omega^2(n-3)\left\{k^2+2WZ+2Z^2\right\} + \frac{\tilde{\alpha}}{A}\Omega^4 2(n-5)\left\{k^2+2WZ\right\}Z\right] \\ &+\frac{\tilde{\alpha}}{A}e^{-f}\Omega^2(n-5)\times\left[\frac{1}{2}\Omega^2\left\{(n-2)k^2+2WZ-4Z^2\right\} + \frac{1}{A}\frac{4}{n-2}Z\left\{\Lambda+\kappa^2(V_1+V_2)\right\}\right] \\ &-\frac{\tilde{\alpha}}{A}e^{f}\Omega^2\frac{4}{(n-2)^2}\left\{\nu_+\vartheta_+(\partial_-\vartheta_-) + \nu_-\vartheta_-(\partial_+\vartheta_+) + (\partial_+\vartheta_+)(\partial_-\vartheta_-) + \nu_+\nu_-\vartheta_+\vartheta_- - (\partial_-\vartheta_+)^2\right\}\end{aligned} \tag{11}$$

$$\partial_+\psi = \Omega\pi_+ \tag{12}$$

$$\partial_+\phi = \Omega p_+ \tag{13}$$

$\partial_+\pi_+ =$ no evolution eq. exists

$$\partial_+\pi_- = \left(\frac{1}{n-2}-\frac{1}{2}\right)\Omega\vartheta_+\pi_- - \frac{1}{2}\Omega\vartheta_-\pi_+ - \frac{1}{2e^f\Omega}\frac{dV_1}{d\psi} \tag{14}$$

$\partial_+p_+ =$ no evolution eq. exists

$$\partial_+p_- = \left(\frac{1}{n-2}-\frac{1}{2}\right)\Omega\vartheta_+p_- - \frac{1}{2}\Omega\vartheta_-p_+ - \frac{1}{2e^f\Omega}\frac{dV_2}{d\phi} \tag{15}$$

evolution equations (2)

Equations for x^- direction

$$
\begin{aligned}
\partial_- \Omega &= -\frac{1}{n-2}\vartheta_- \Omega^2 && (16)\\
\partial_- \vartheta_+ &= (9) && (17)\\
\partial_- \vartheta_- &= -\vartheta_- \nu_- - \frac{1}{\Omega A}\kappa^2 T_{--} = -\vartheta_- \nu_- - \frac{1}{A}\Omega\kappa^2(\pi_-^2 - p_-^2) && (18)\\
\partial_- f &= \nu_- && (19)\\
\partial_- \nu_+ &= (11) && (20)\\
\partial_- \nu_- &= \text{no evolution eq. exists} && \\
\partial_- \psi &= \Omega \pi_- && (21)\\
\partial_- \phi &= \Omega p_- && (22)\\
\partial_- \pi_+ &= -\frac{1}{2}\Omega \vartheta_+ \pi_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega \vartheta_- \pi_+ - \frac{1}{2e^f \Omega}\frac{dV_1}{d\psi} && (23)\\
\partial_- \pi_- &= \text{no evolution eq. exists} && \\
\partial_- p_+ &= -\frac{1}{2}\Omega \vartheta_+ p_- + \left(\frac{1}{n-2} - \frac{1}{2}\right)\Omega \vartheta_- p_+ - \frac{1}{2e^f \Omega}\frac{dV_2}{d\phi} && (24)\\
\partial_- p_- &= \text{no evolution eq. exists} &&
\end{aligned}
$$

This constitutes the first-order dual-null form, suitable for numerical coding.

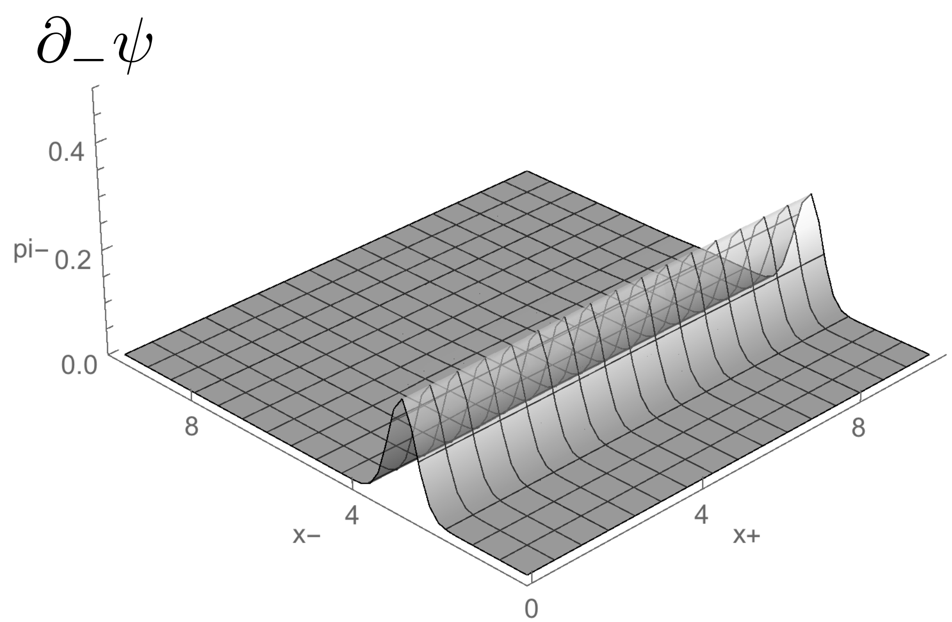
GR 5d: small amplitude waves

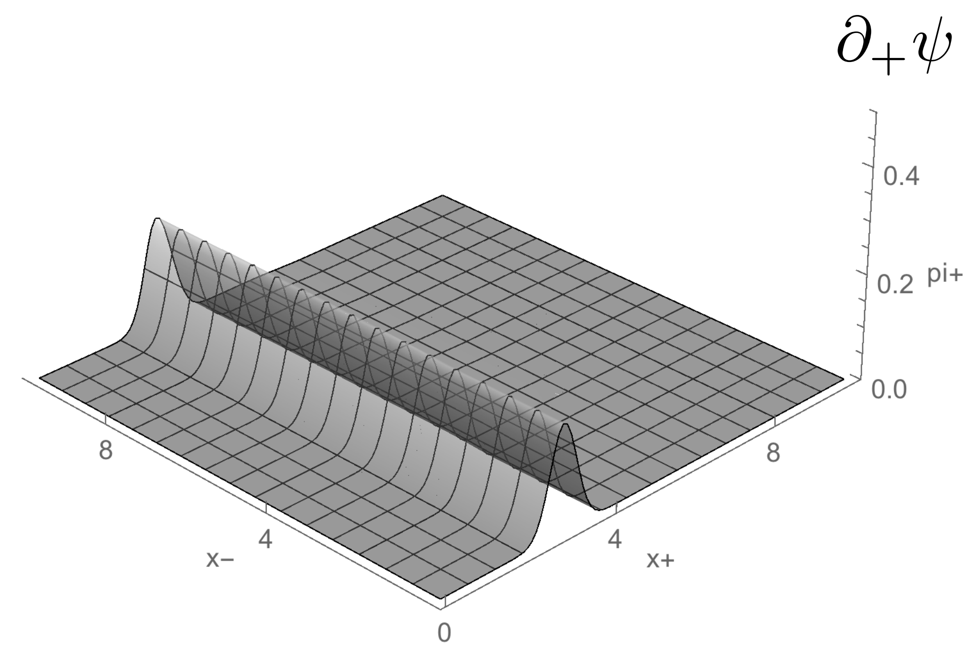
flat background, normal scalar field

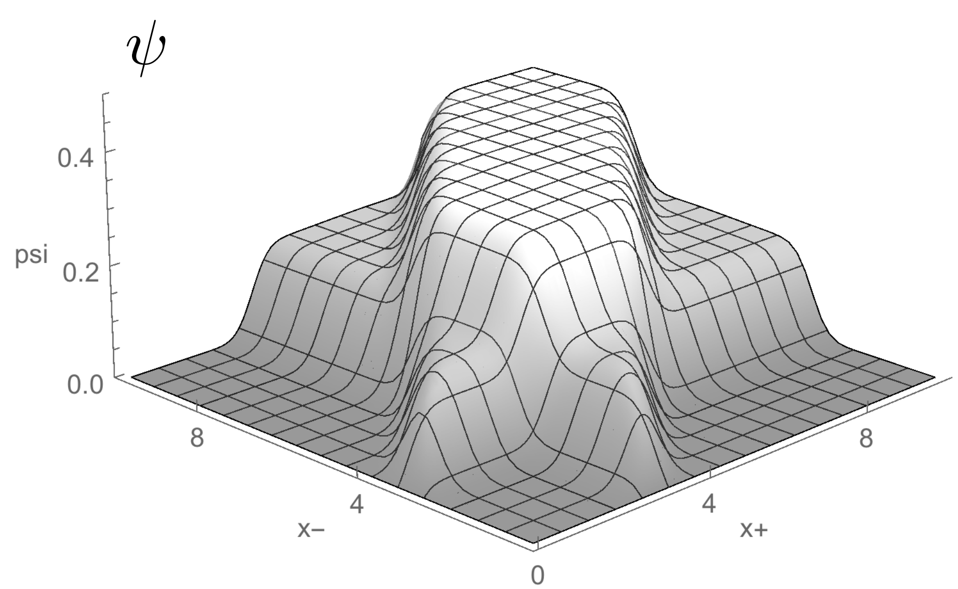
Initial data:

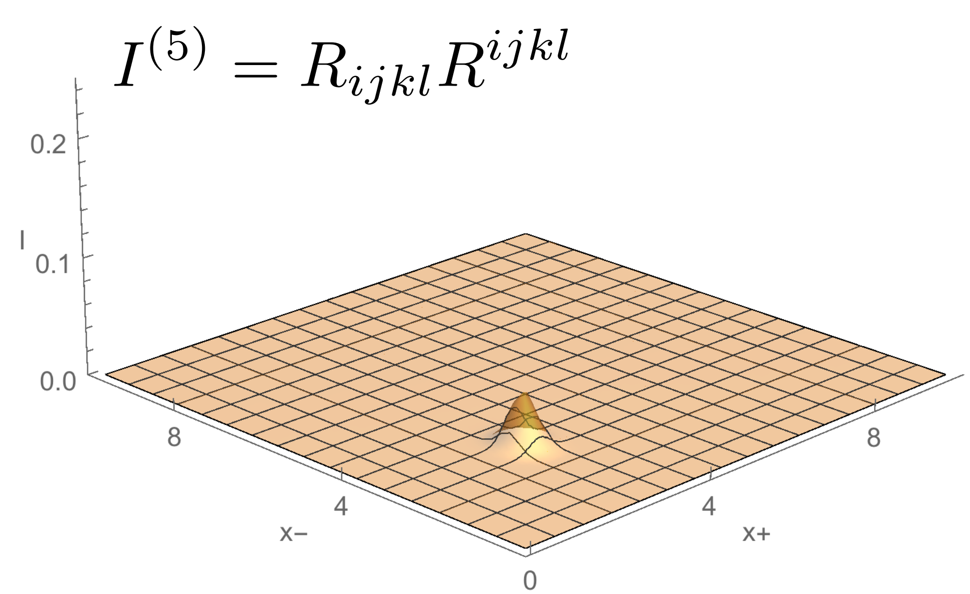
$\psi = 0,\ \pi_+ = a\exp(-b(z-c)^2)$ on $x_- = 0$ surface, where $z = x^+/\sqrt{2}$

$\psi = 0,\ \pi_- = a\exp(-b(z-c)^2)$ on $x_+ = 0$ surface, where $z = x^-/\sqrt{2}$

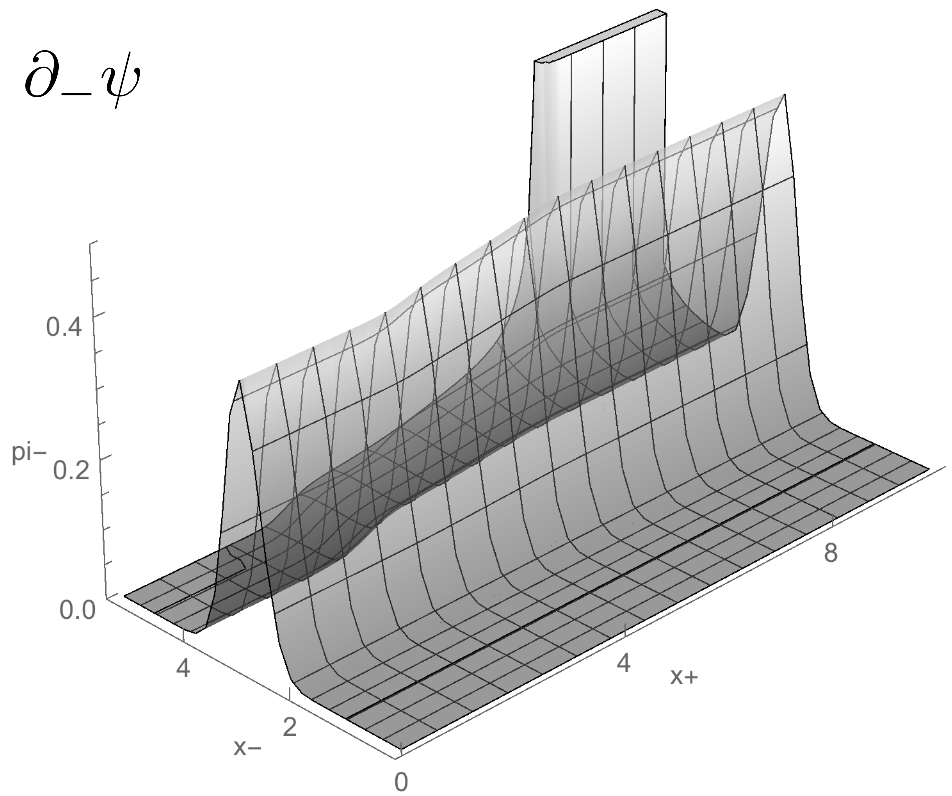


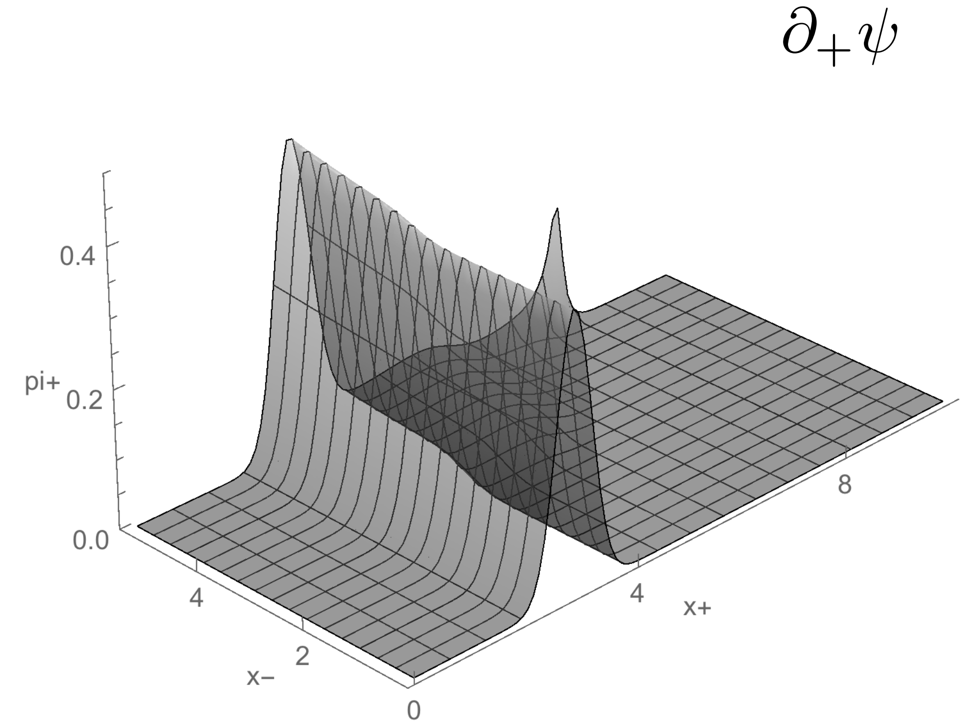


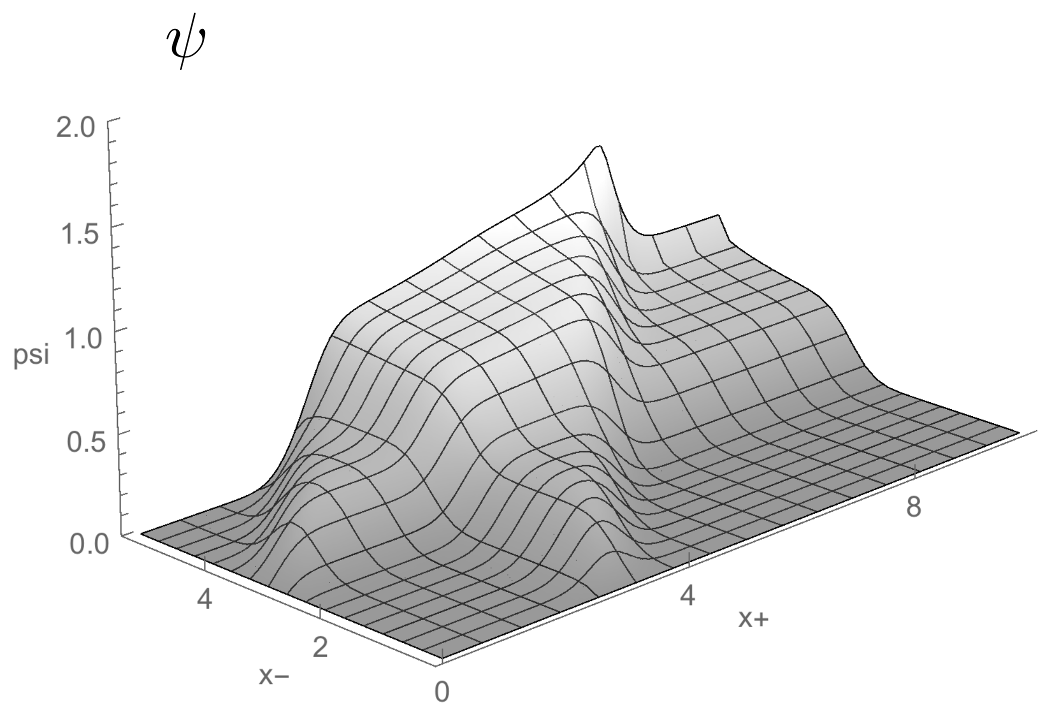


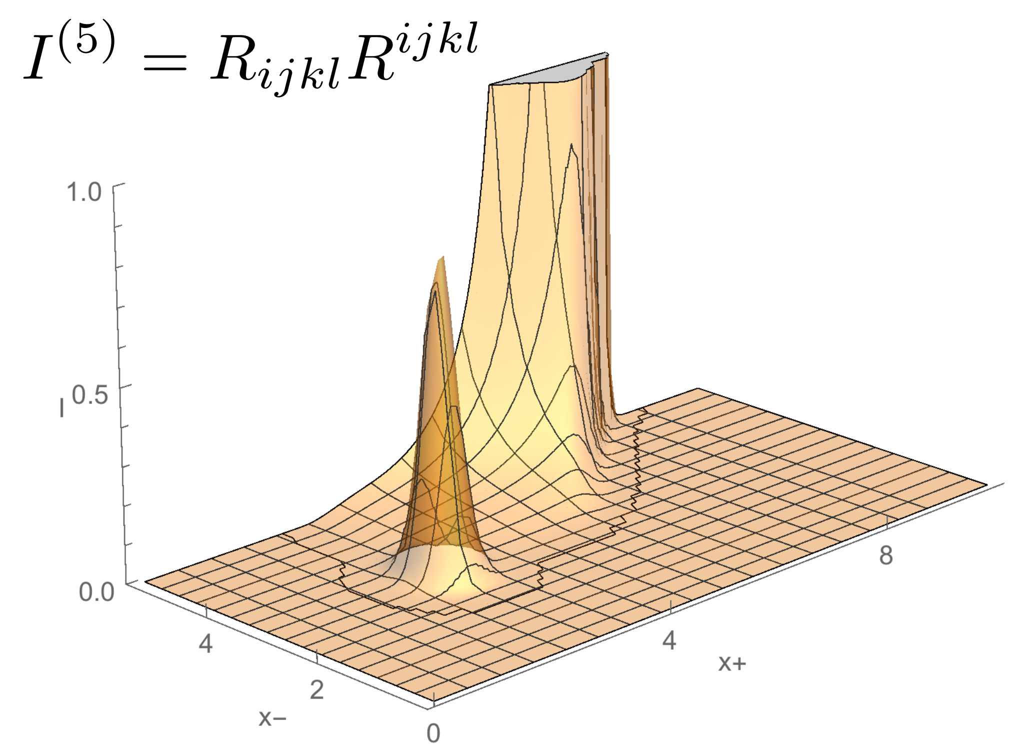


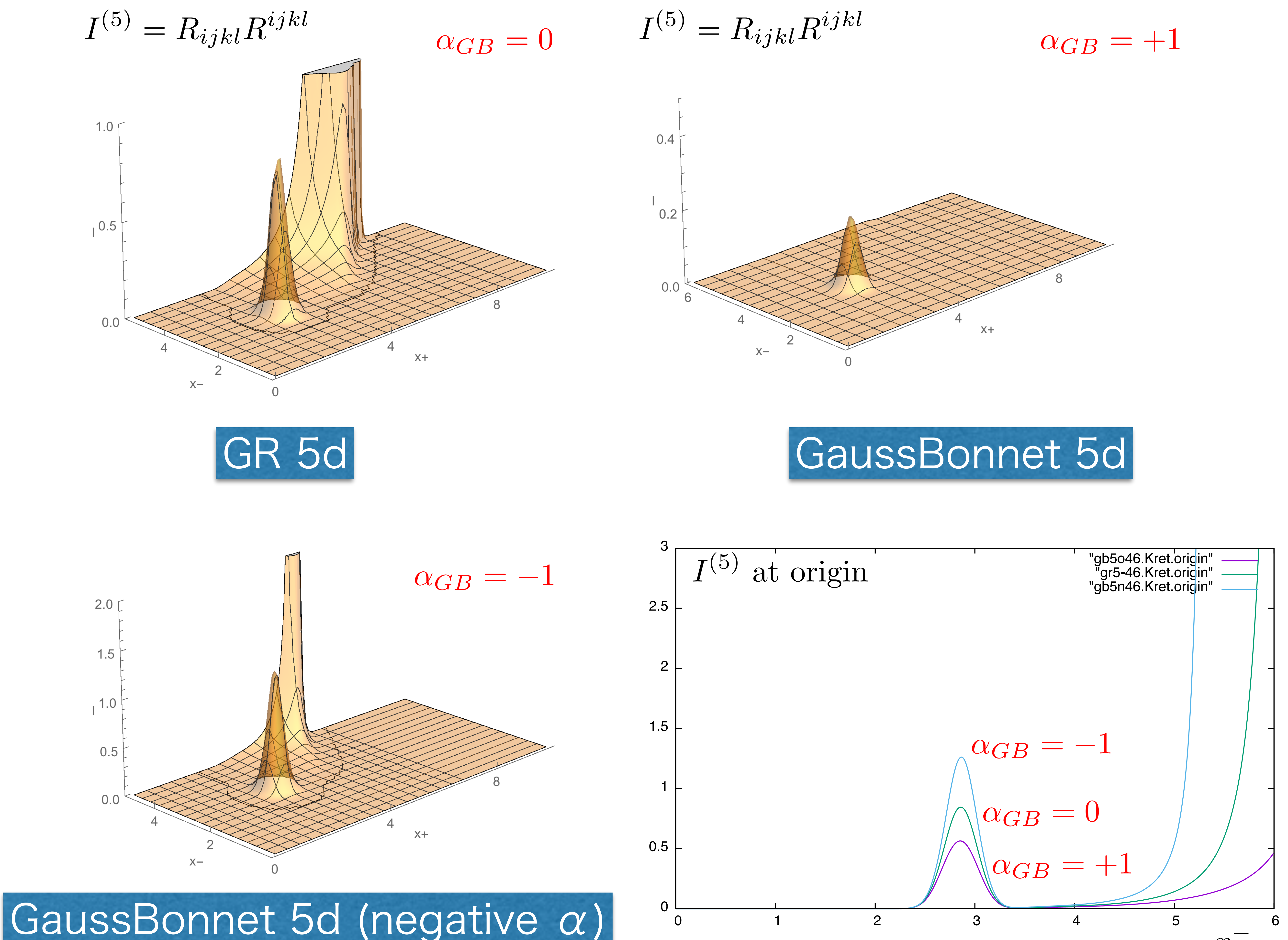
GR 5d: large amplitude waves











$I^{(5)} = R_{ijkl}R^{ijkl}$
$\alpha_{GB} = 0$
1.0
0.5
0.0
I
x−
x+
0
2
4
4
8
GR 5d
$I^{(5)} = R_{ijkl}R^{ijkl}$
$\alpha_{GB} = +1$
0.4
0.2
0.0
I
6
4
2
0
4
8
x−
x+
GaussBonnet 5d
$\alpha_{GB} = -1$
2.0
1.5
1.0
0.5
0.0
I
x−
x+
GaussBonnet 5d (negative α)
$I^{(5)}$ at origin
"gb5o46.Kret.origin"
"gr5-46.Kret.origin"
"gb5n46.Kret.origin"
$\alpha_{GB} = -1$
$\alpha_{GB} = 0$
$\alpha_{GB} = +1$
0
0.5
1
1.5
2
2.5
3
0
1
2
3
4
5
6
x^-

Colliding Scalar Waves

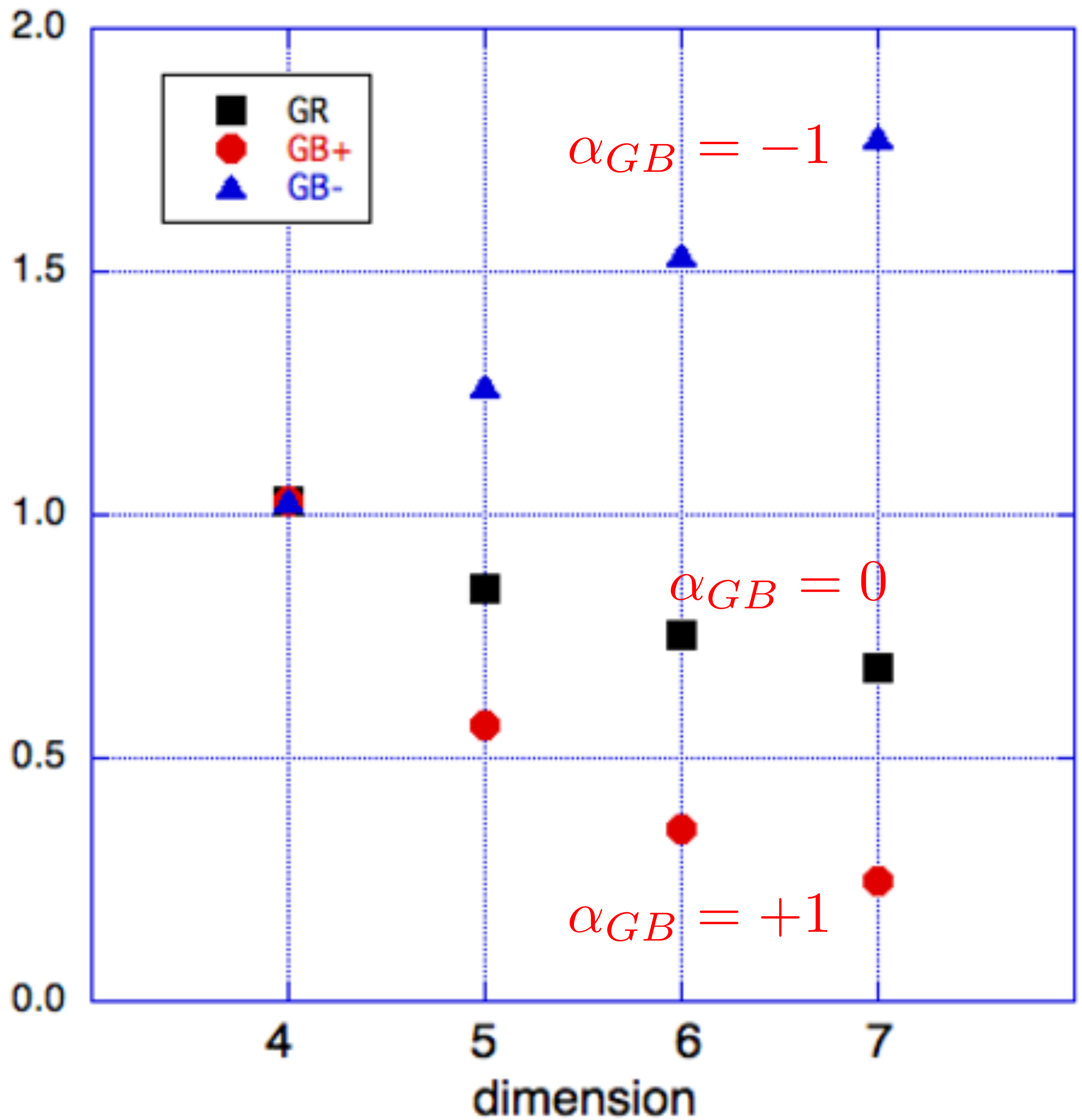
∗4dim, 5dim, 6dim,⋯ higher dim

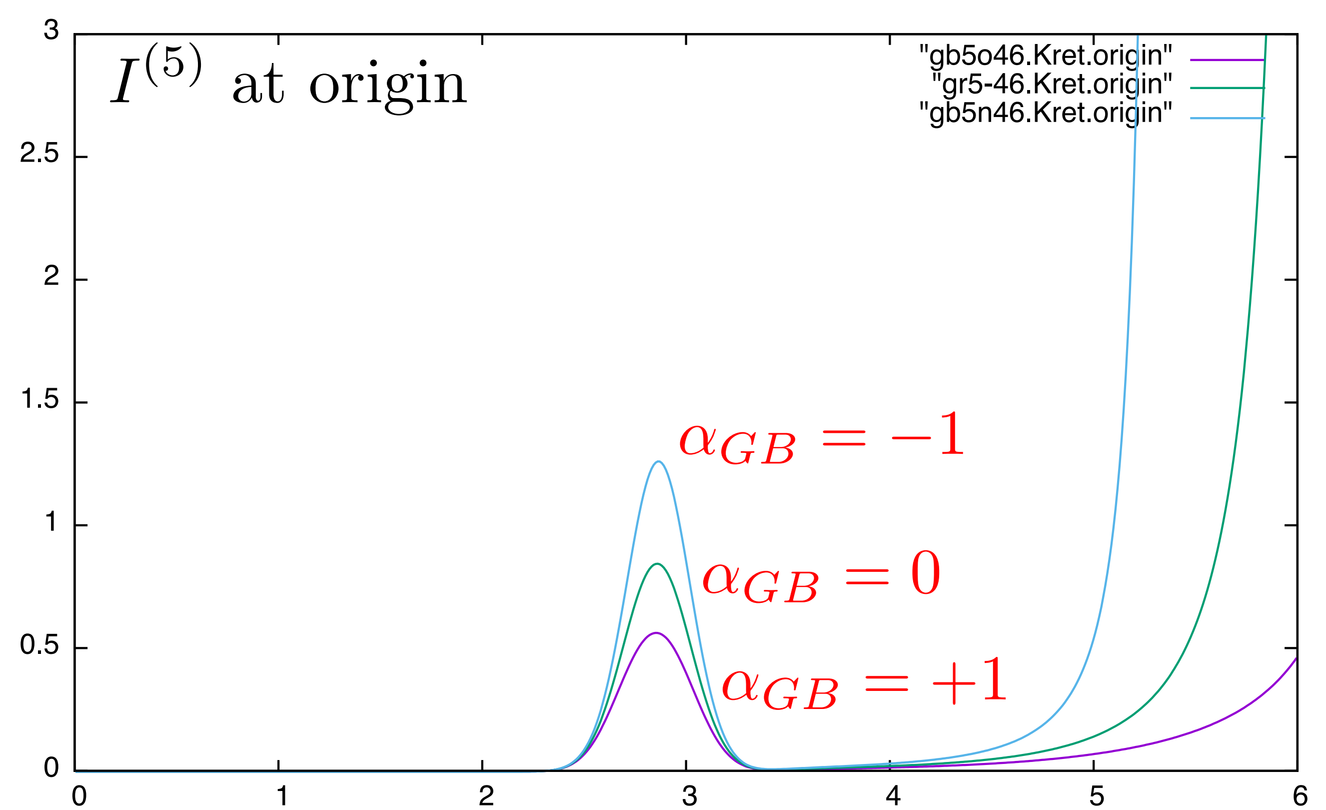
∗Gauss-Bonnet coupling (α>0)

—> less growth of curvature

$\max\ (R_{ijkl}R^{ijkl})$

maximum of Kretschmann invariant



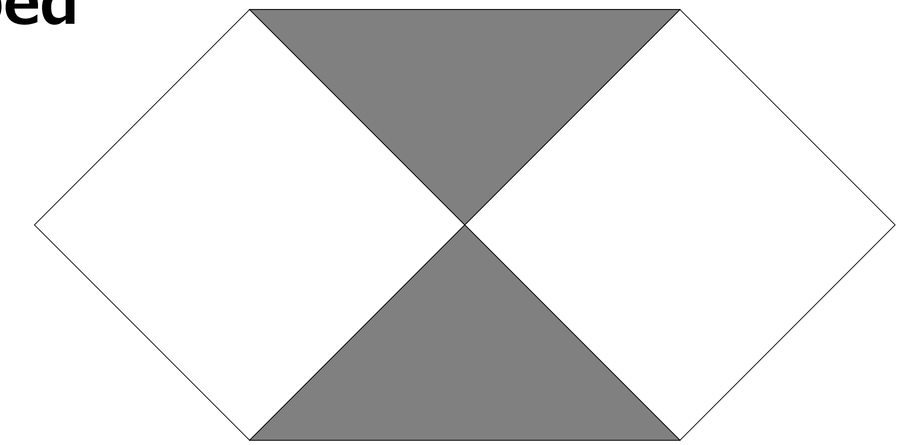


BH & WH are interconvertible?

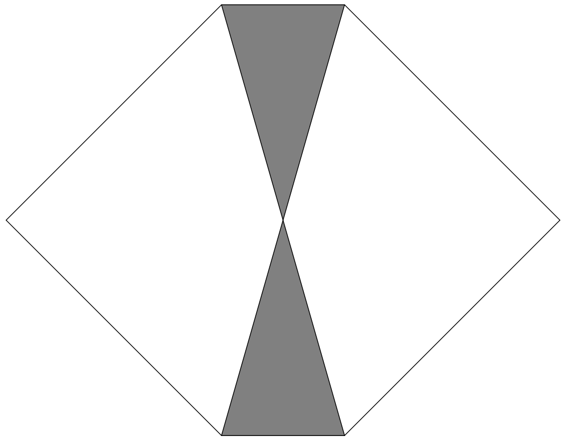
S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

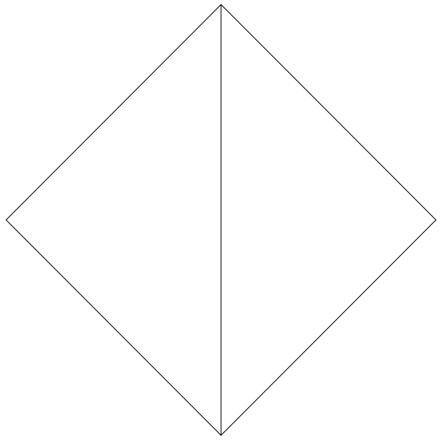
They are very similar -- both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)

Only the causal nature of the THs differs, whether THs evolve in plus / minus density which is given locally.



	Black Hole	Wormhole
Locally defined by	Achronal (spatial/null) outer TH ⇒ 1-way traversable	Temporal (timelike) outer THs ⇒ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appear-ance	occur naturally	Unlikely to occur naturally. but constructible??





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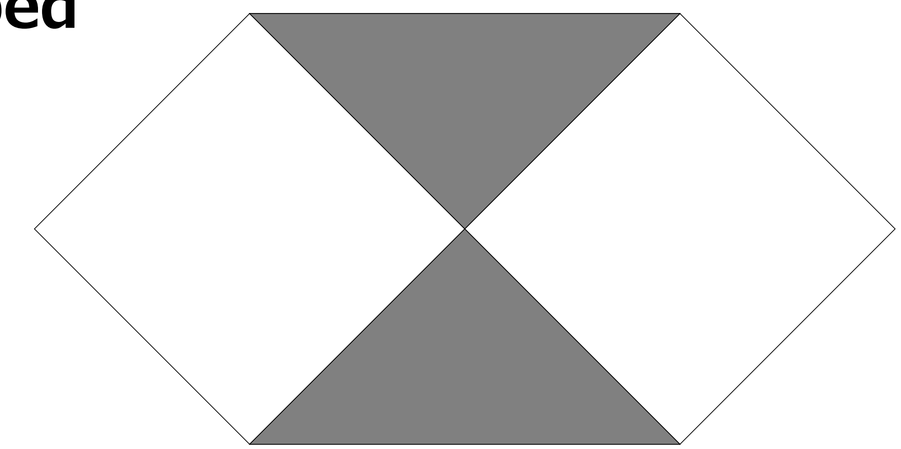

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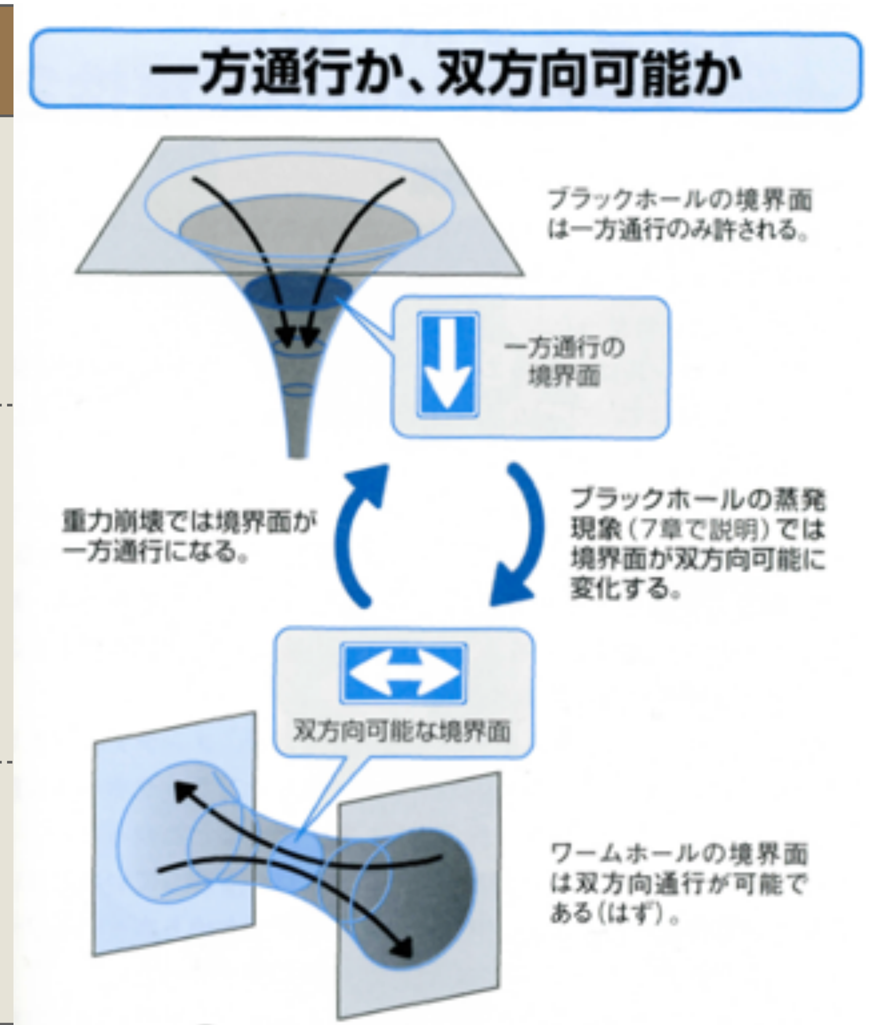
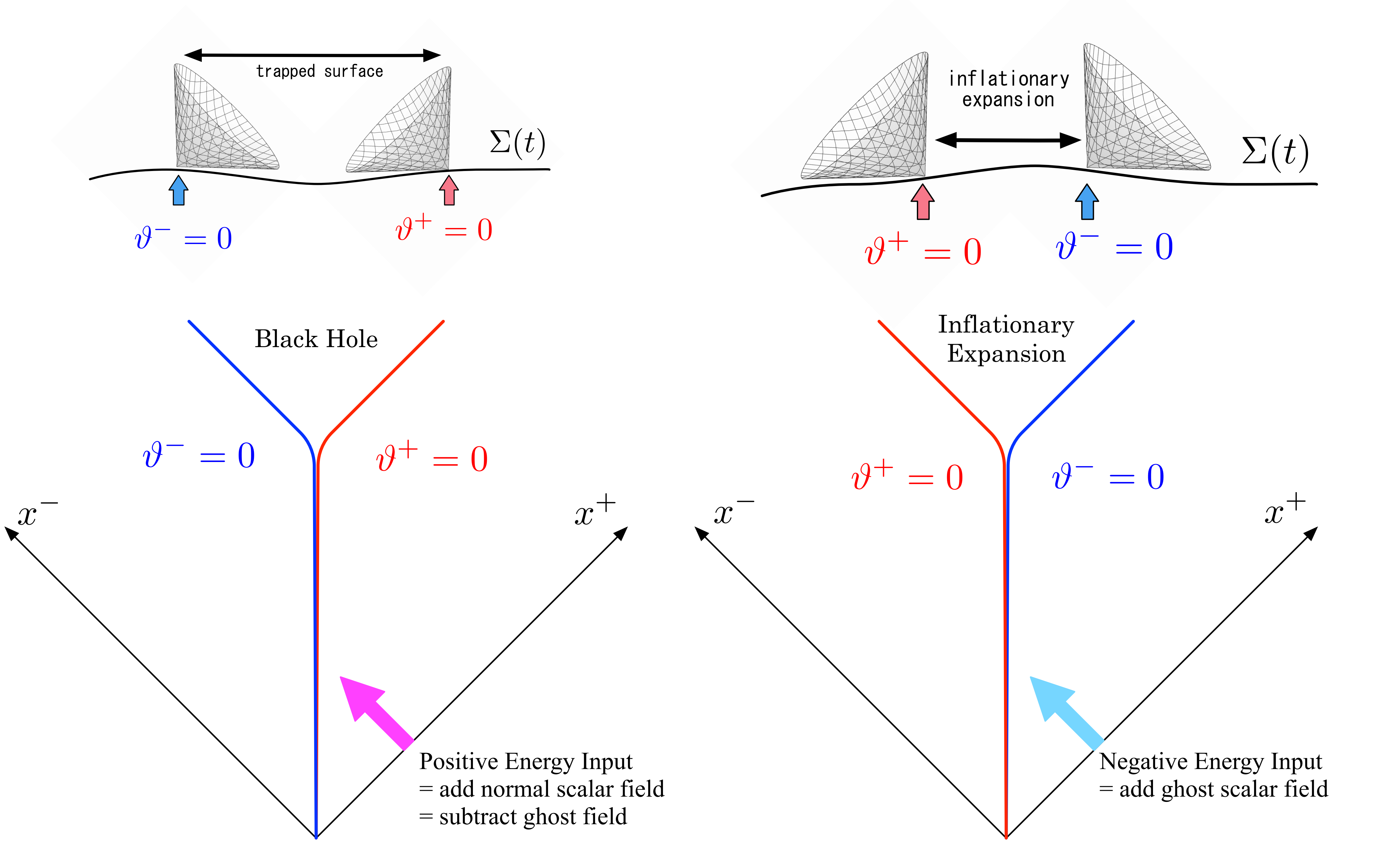
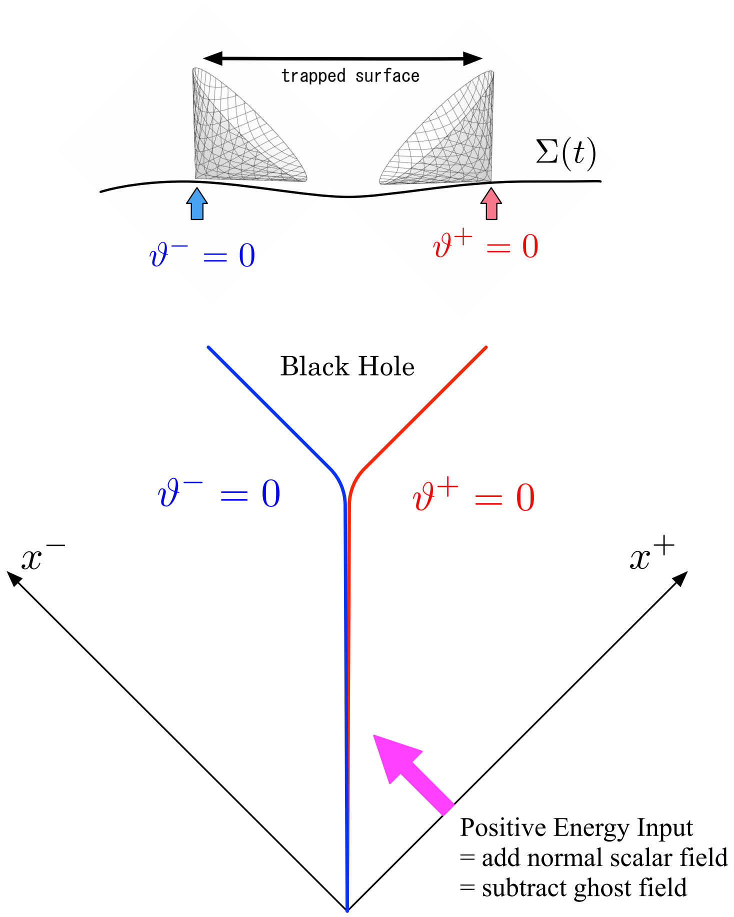


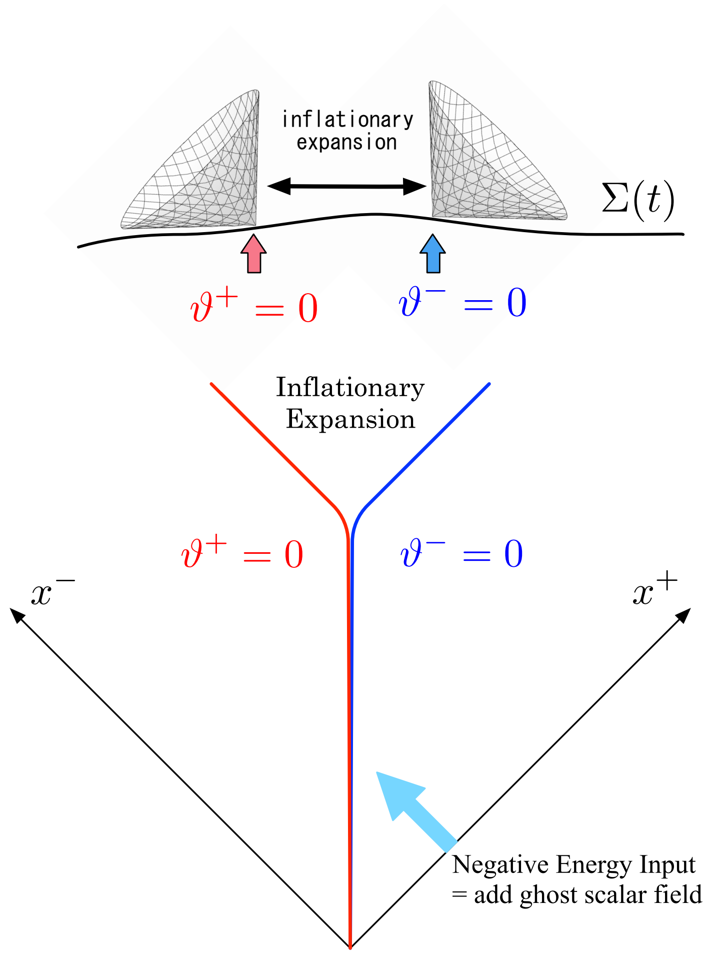
Wormhole evolution (known fact)

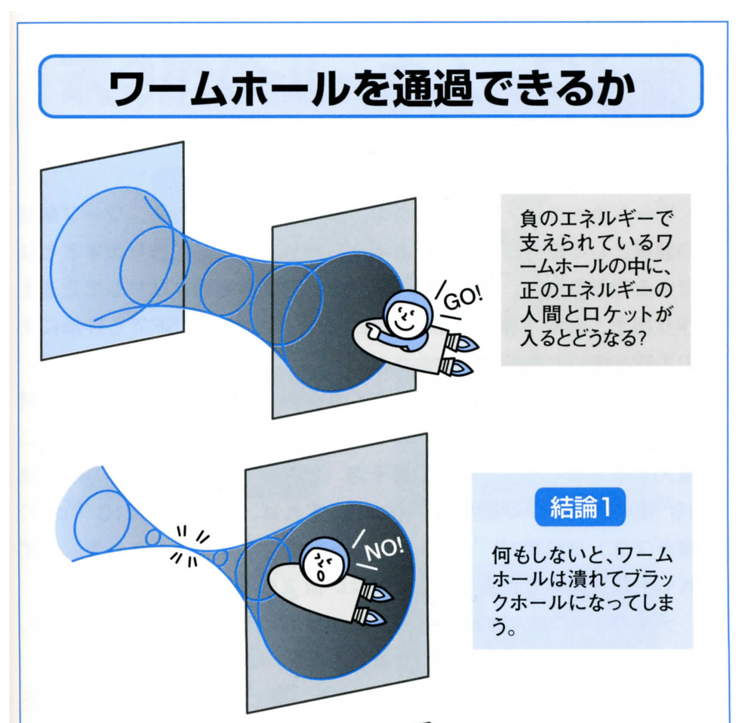


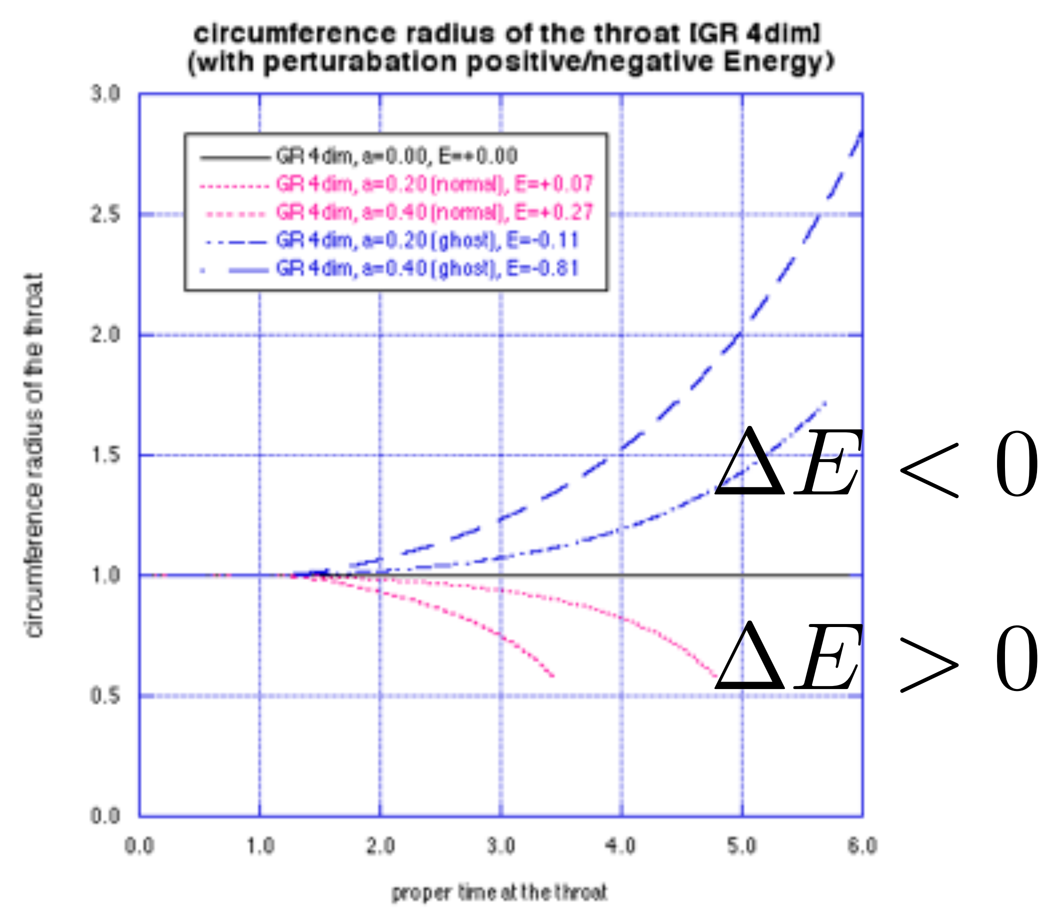
4d GR $\alpha_{\rm GB} = 0$ HS-Hayward PRD 66(2002) 044005

Wormhole evolution (known fact)









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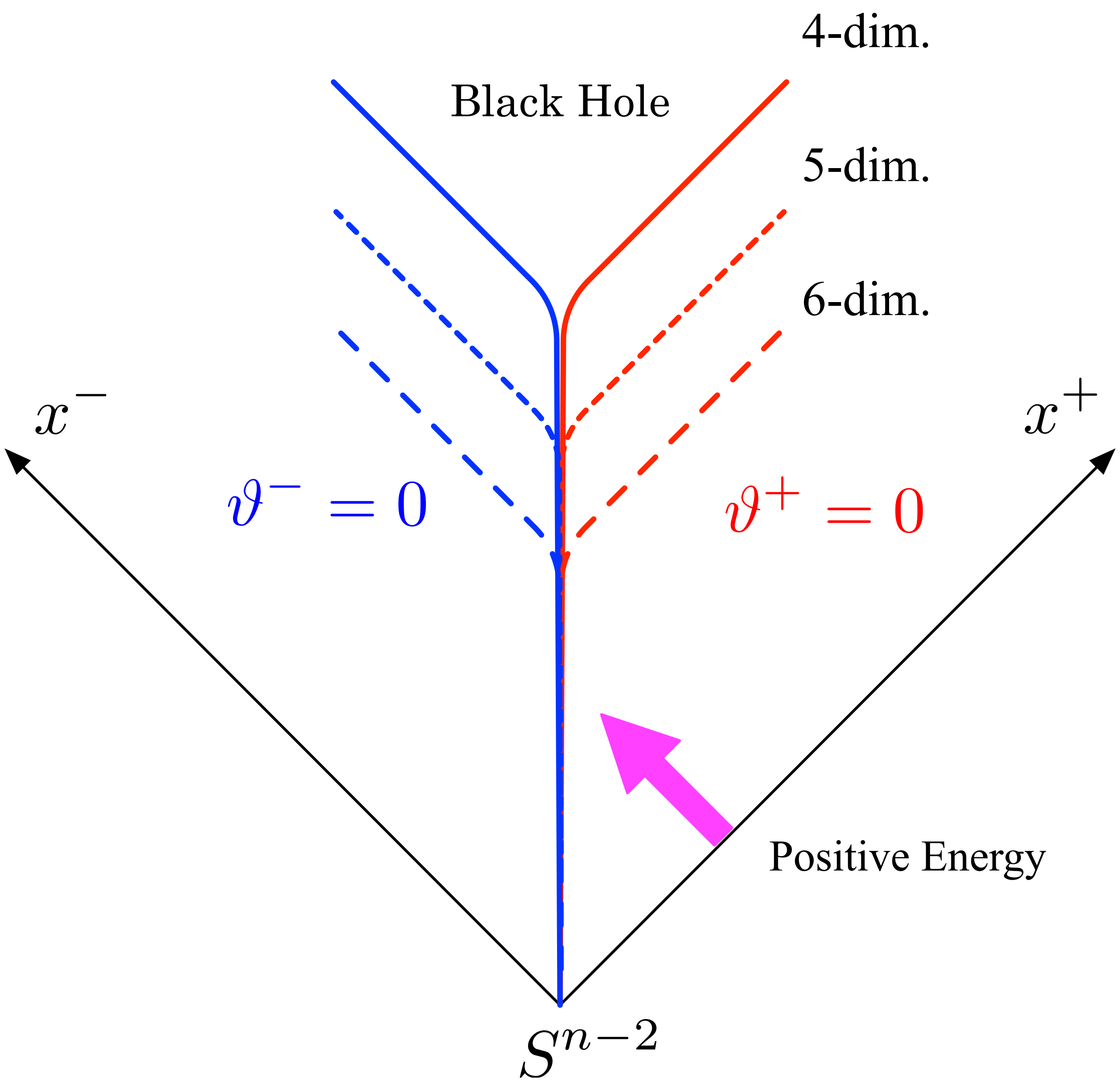
Wormhole evolution in n-dim (known fact)



PHYSICAL REVIEW D **88**, 064027 (2013)

TABLE I. The negative eigenvalues ω^2.

n	ω^2
4	−1.39705243371511
5	−2.98495893027790
6	−4.68662054299460
7	−6.46258414126318
8	−8.28975936306259
9	−10.1535530451867
10	−12.0442650147438
11	−13.9552091676647
20	−31.5751101285105
50	−91.3457759137153
100	−191.283017729717

$$f(t, r) = f_0(r) + \varepsilon f_1(r)e^{i\omega t}, \quad (3.1)$$

$$\delta(t, r) = \delta_0(r) + \varepsilon \delta_1(r)e^{i\omega t}, \quad (3.2)$$

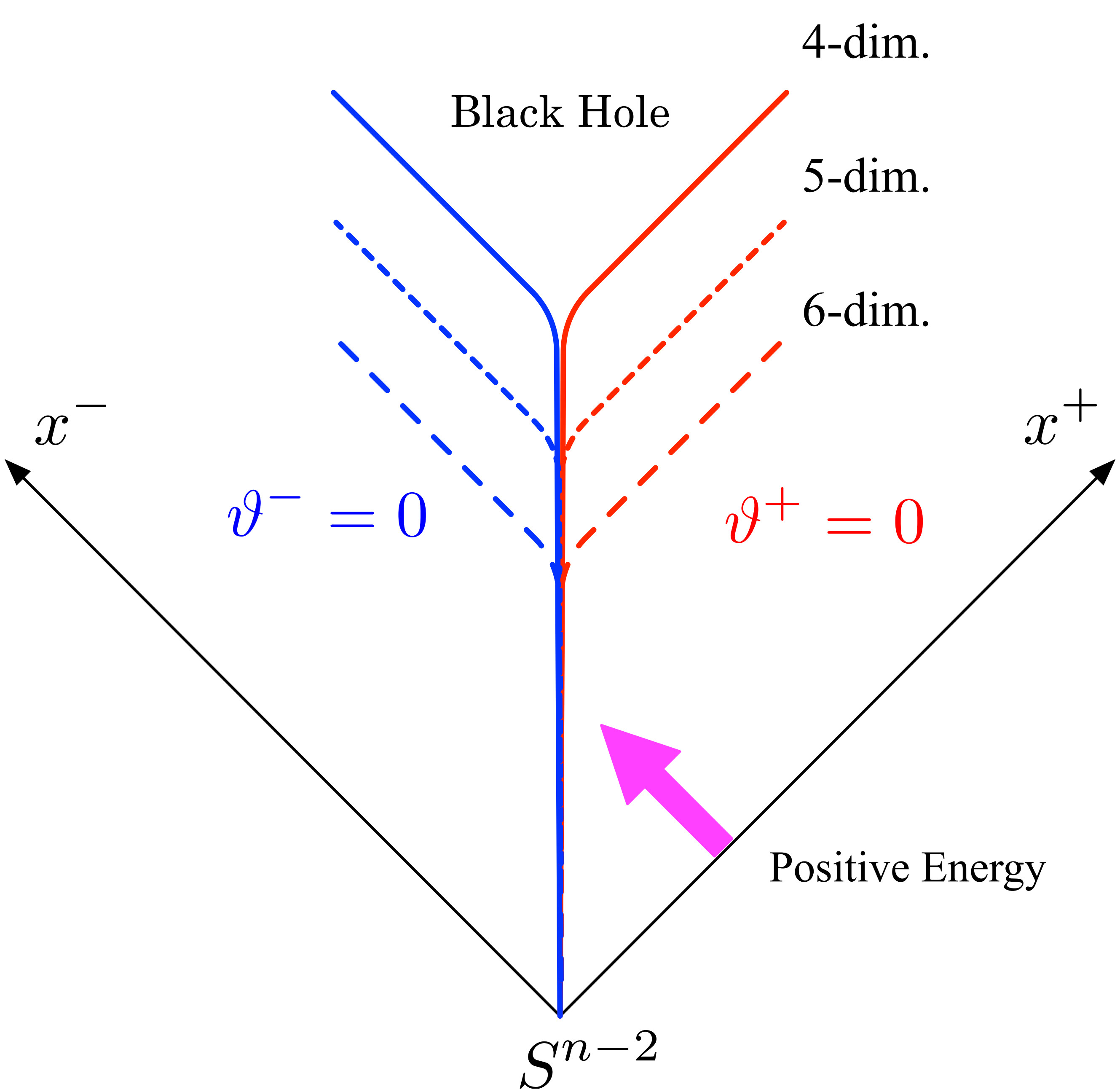
$$R(t, r) = R_0(r) + \varepsilon R_1(r)e^{i\omega t}, \quad (3.3)$$

$$\phi(t, r) = \phi_0(r) + \varepsilon \phi_1(r)e^{i\omega t}. \quad (3.4)$$

In higher dim, large instability.
(linear perturbation analysis)

n-dim GR $\alpha_{\rm GB} = 0$ Torii-HS PRD 88 (2013) 064027

Wormhole evolution in n-dim (known fact)



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$$f(t, r) = f_0(r) + \varepsilon f_1(r) e^{i\omega t}, \quad (3.1)$$

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$$\phi(t, r) = \phi_0(r) + \varepsilon \phi_1(r) e^{i\omega t}. \quad (3.4)$$

In higher dim, large instability.
(linear perturbation analysis)

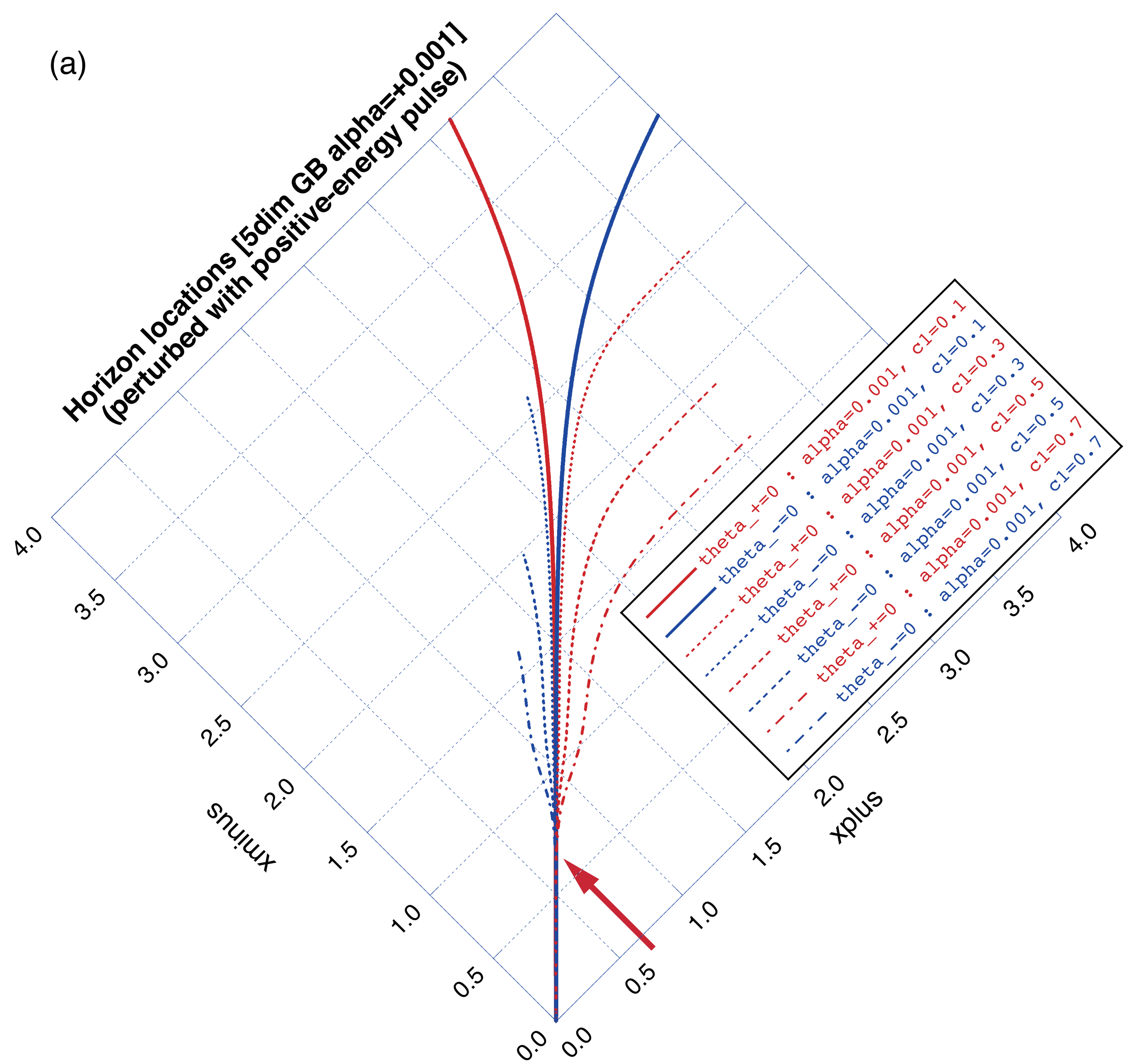
Confirmed Numerically

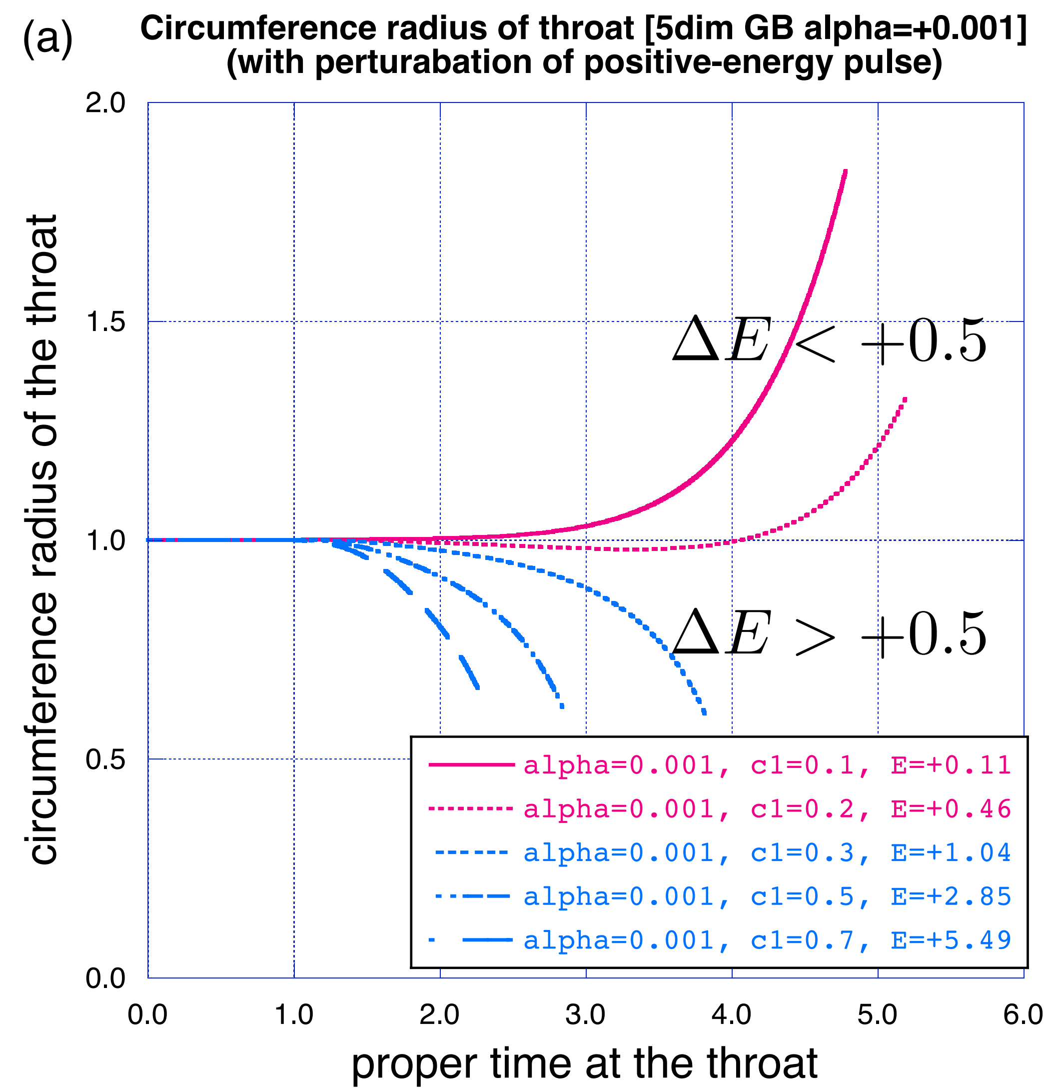
5d Gauss-Bonnet WH : positive energy injection

$$\alpha_{\rm GB} = +0.001$$

$$m = \frac{(n-2)V_{n-2}^k}{2\kappa_n^2} r^{n-3} \left[-\tilde{\Lambda} r^2 + \left(k + \frac{2}{(n-2)^2} r^2 e^f \theta_+ \theta_- \right) + \tilde{\alpha} r^{-2} \left(k + \frac{2}{(n-2)^2} r^2 e^f \theta_+ \theta_- \right)^2 \right]$$

MSmass, H.Maeda-Nozawa, PRD77 (2008) 063031





$\Delta E > \Delta E_5 > 0$ --> BH collapse

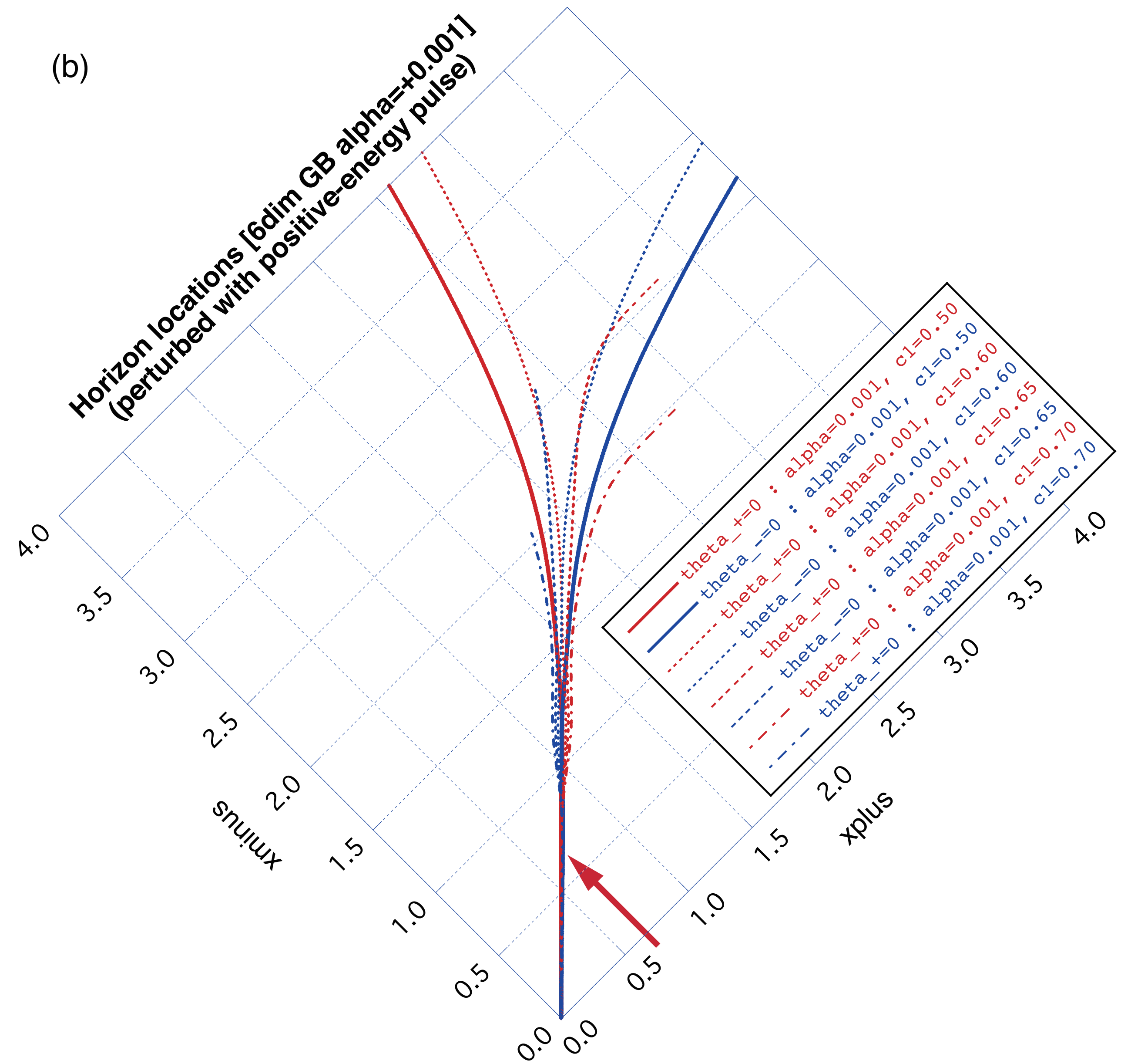
$\Delta E < \Delta E_5$ --> Inflationary expansion

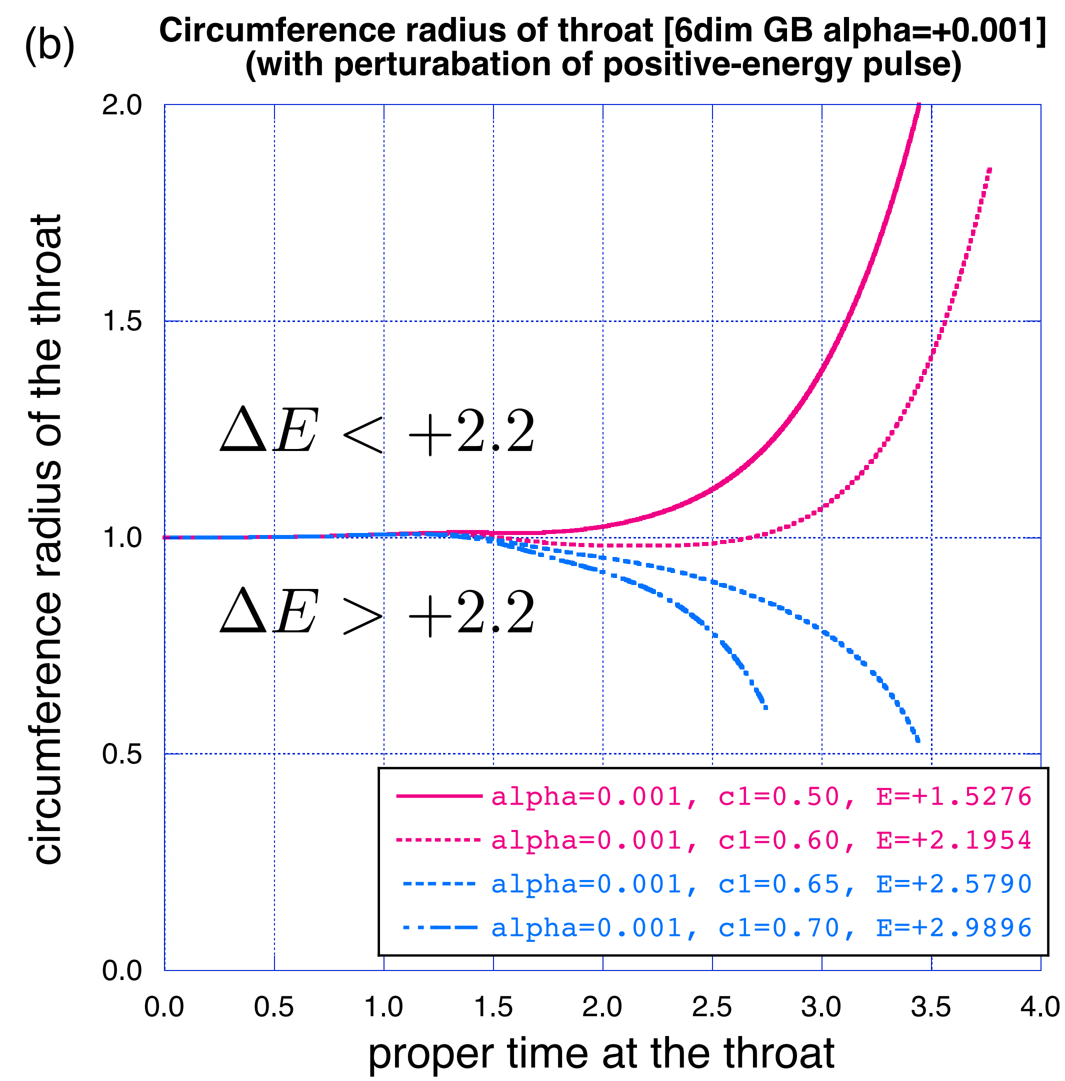
6d Gauss-Bonnet WH : positive energy injection

$$\alpha_{\rm GB} = +0.001$$

$$m = \frac{(n-2)V^k_{n-2}}{2\kappa_n^2} r^{n-3}\left[-\tilde{\Lambda} r^2 + \left(k+\frac{2}{(n-2)^2} r^2 e^f \theta_+\theta_-\right) + \tilde{\alpha} r^{-2}\left(k + \frac{2}{(n-2)^2} r^2 e^f \theta_+\theta_-\right)^2\right]$$

MSmass, H.Maeda-Nozawa, PRD77 (2008) 063031





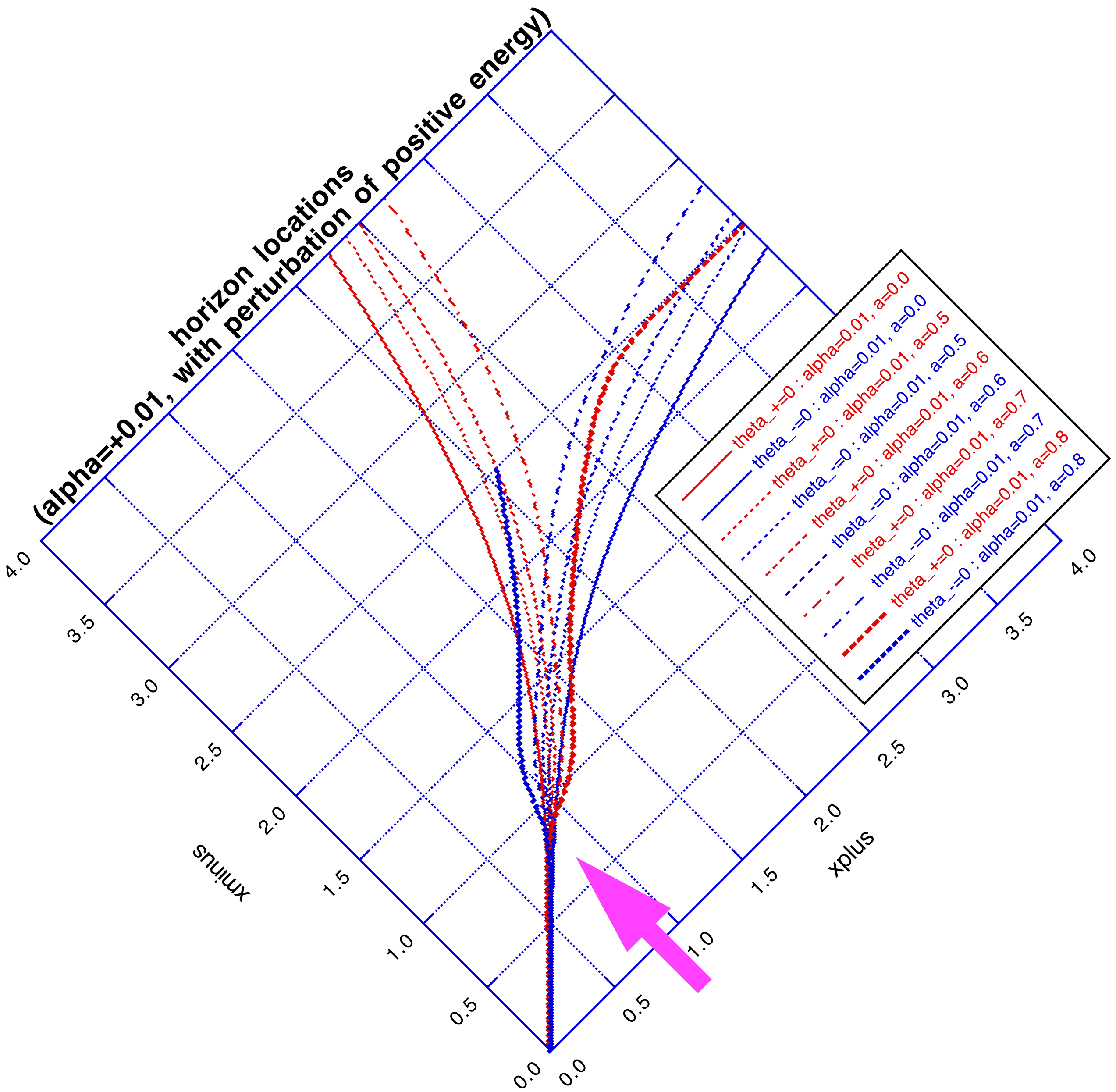
$\Delta E > \Delta E_6 > \Delta E_5 > 0$ --> BH collapse

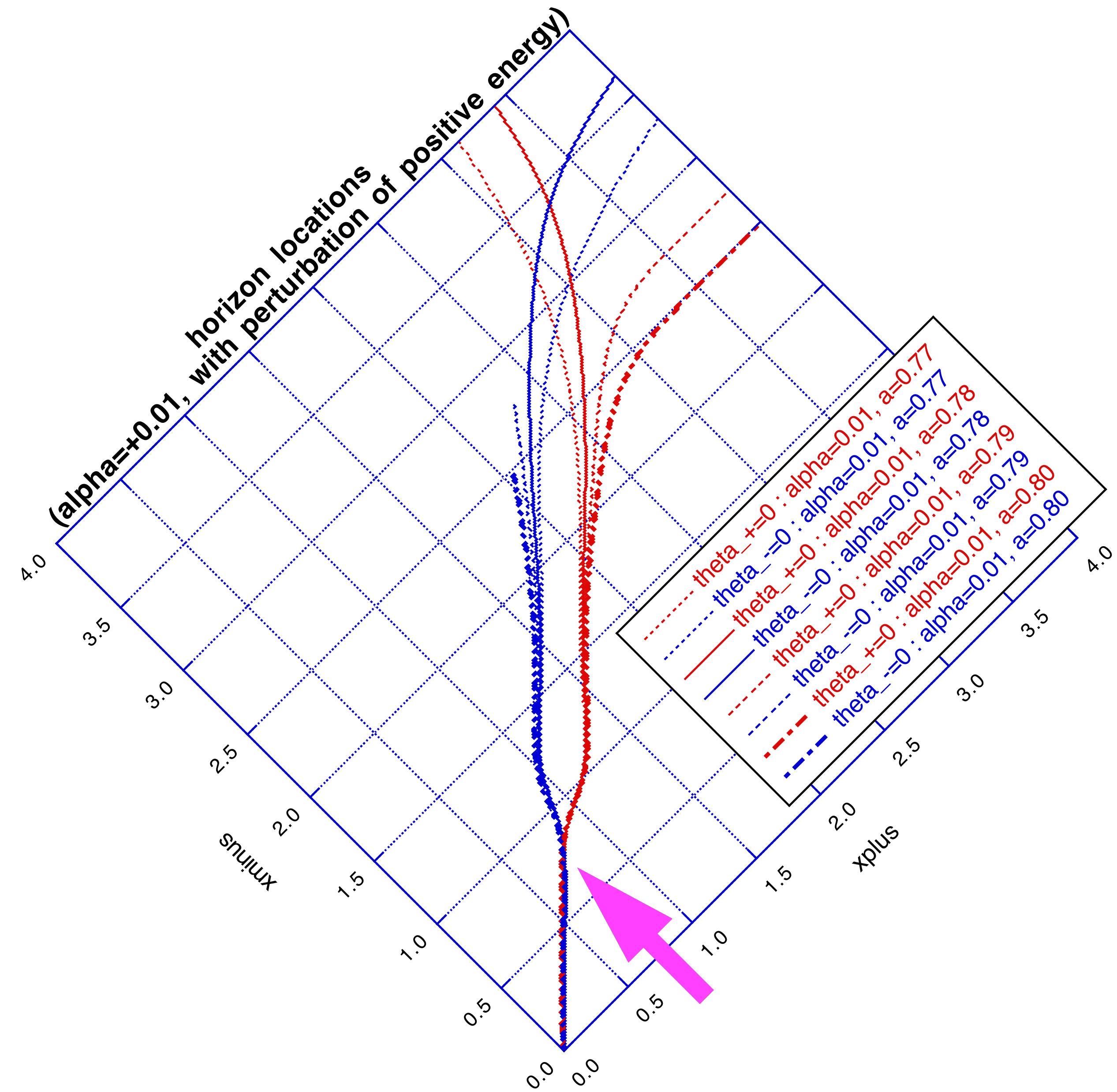
$\Delta E < \Delta E_5 < \Delta E_6$ --> Inflationary expansion

5d Gauss-Bonnet WH : trapped surface

$\alpha_{\rm GB} = 0.01$

critical behavior





existence of trapped surface

—> not necessary to form a BH

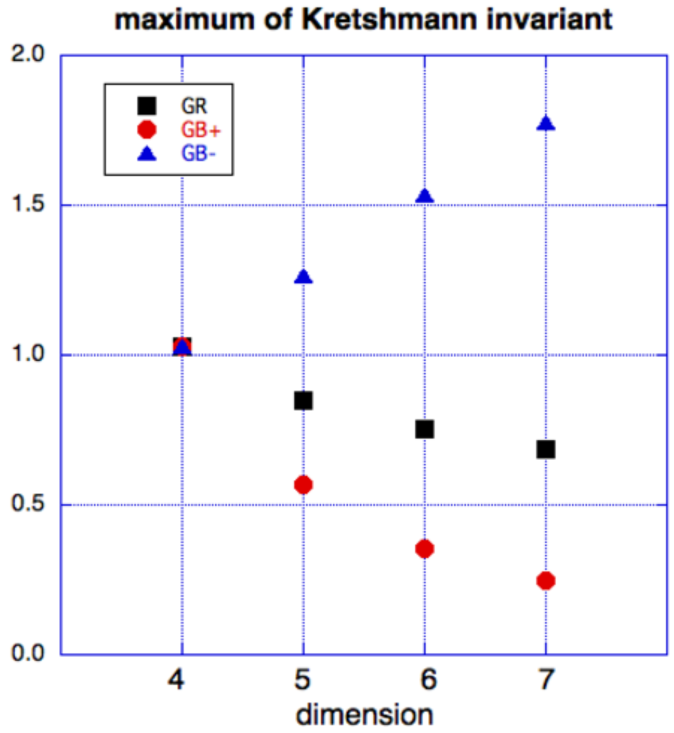
Summary

$$S = \int_{\mathcal{M}} d^n x \sqrt{-g}\left[\frac{1}{2\kappa^2}(\alpha_{\rm GR}\mathcal{R} + \alpha_{\rm GB}\mathcal{L}_{\rm GB}) + \mathcal{L}_{\rm matter}\right]$$

$$\mathcal{L}_{\rm GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$$

Colliding Scalar Waves

$\max\ (R_{ijkl}R^{ijkl})$



Wormhole Evolution

$\Delta E > \Delta E_6 > \Delta E_5 > 0$ --> BH collapse

$\Delta E < \Delta E_5 < \Delta E_6$ --> Inflationary expansion

We found that in the critical situation for forming a BH, the existence of the trapped region in the Einstein-GB gravity does not directly indicate a formation of a BH.

For both models, the normal corrections ($\alpha_{\rm GB} > 0$) work for avoiding the appearance of singularity, although it is inevitable.