Autoregressive Approach to Extract Ring-down Gravitational Wave of Black-hole Merger

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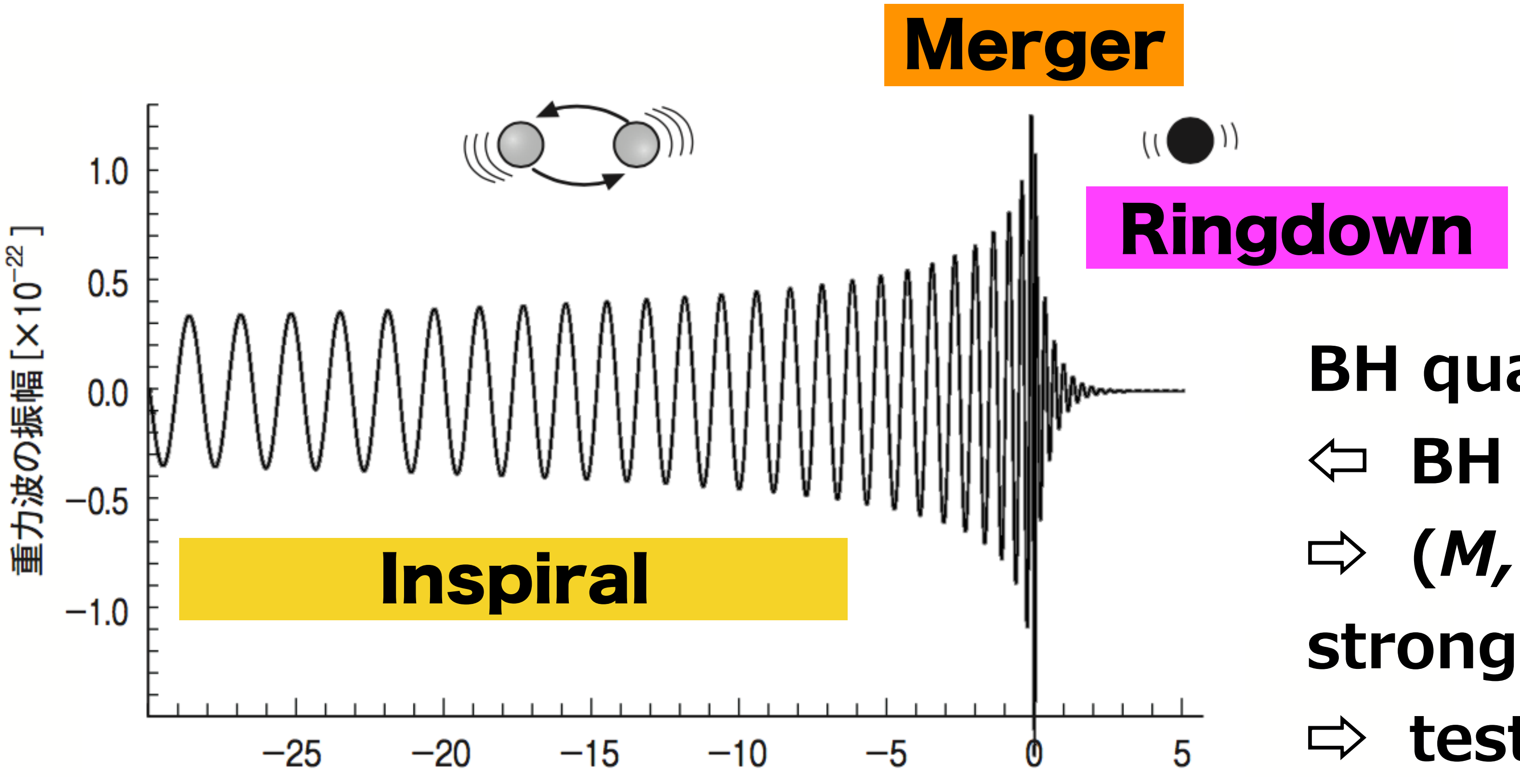
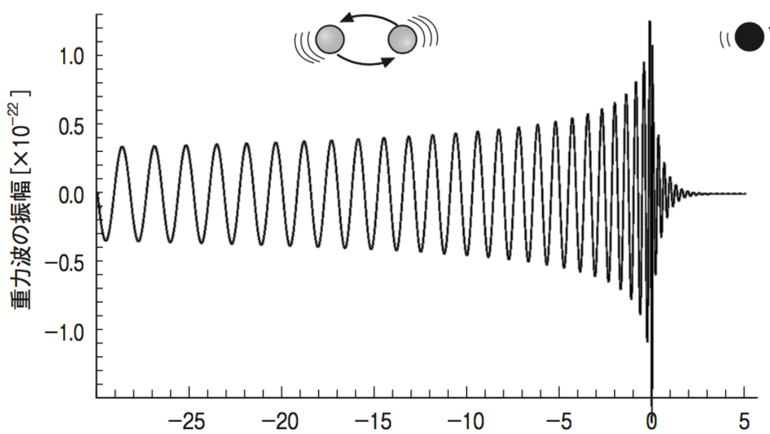


BH quasi-normal modes
⇦ BH perturbation theory
⇨ (*M*, *a*)
strongest gravity we can observe
⇨ test of gravity theories

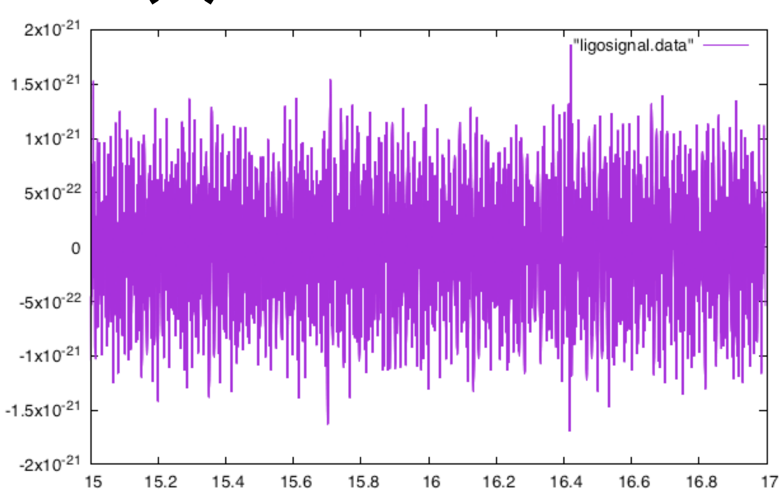
We propose new method, which does not use any templates.

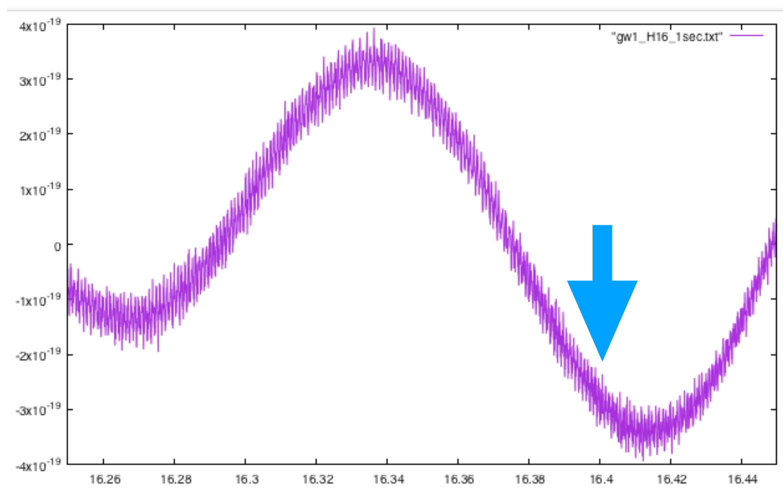
Ideal vs Reality (Theory vs Data Analysis)

GW150914 (S/N=23.7)

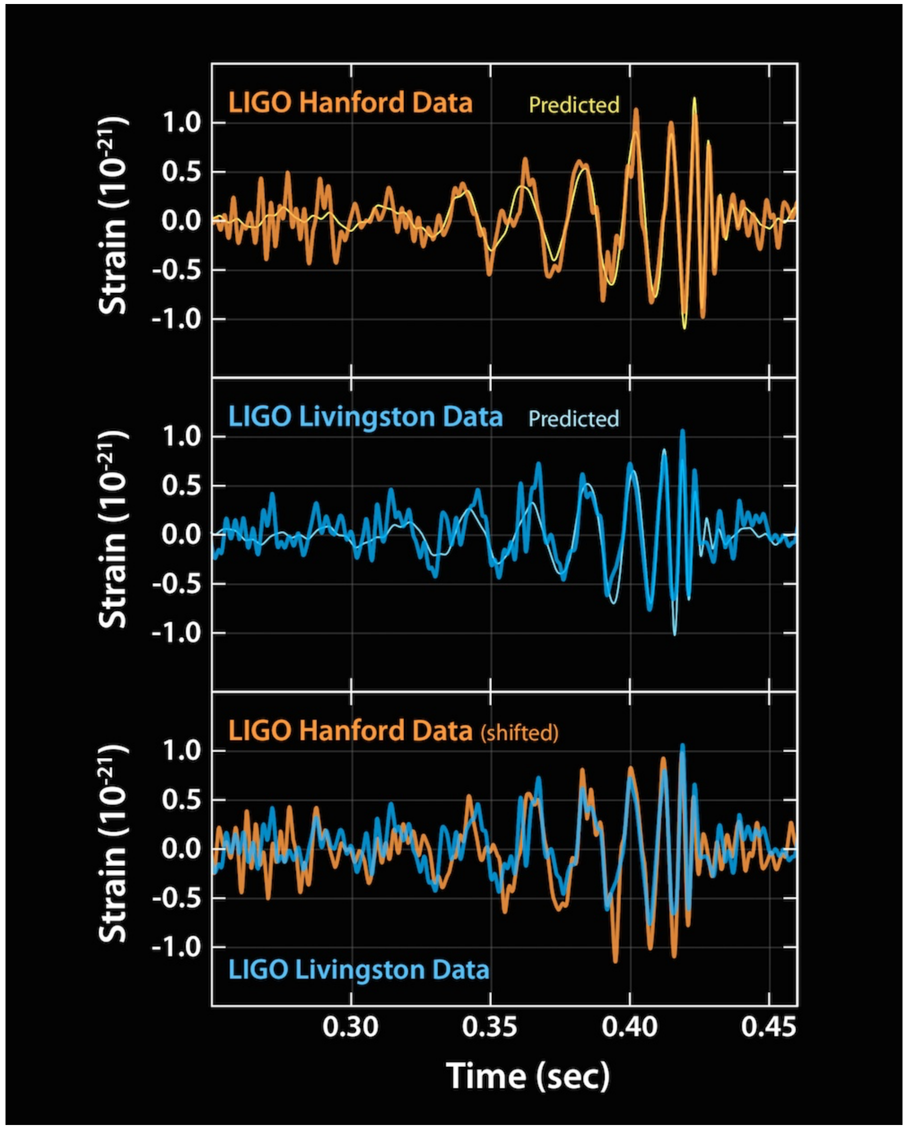


h(t)





16.25 16.45



challenging for data analysis

GW data is with noise

signal quickly decays

(M=60Msun, a=0.75 —> 300Hz, tau = 3 ms)

Mock data example (0) : QNM extraction contest

Mock data challenge for finding ringdown gravitational waves

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modified ringdown signals from GR with LIGO detector's noise

1. Standard Matched-filtering method
2. Improved Matched-filtering method
3. Hilbert-Huang transformation method
4. Auto-Regressive method
5. Neural network method

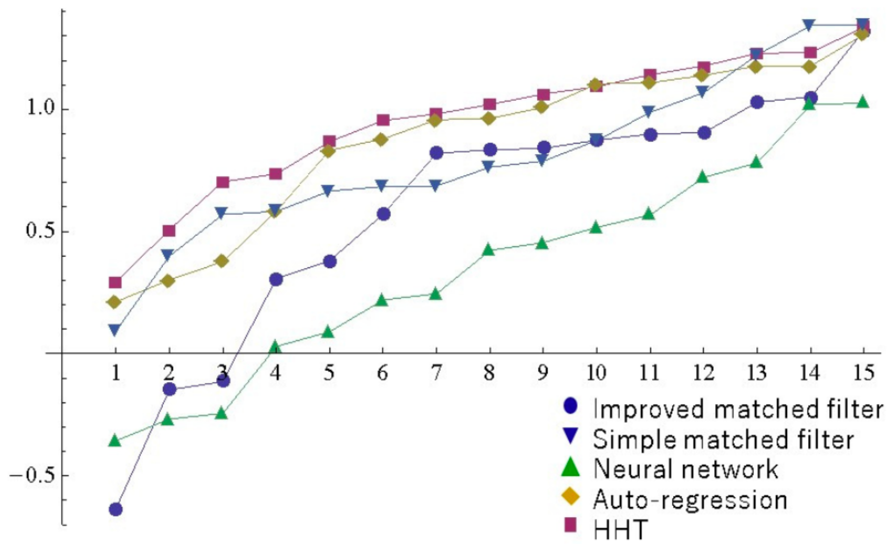


FIG. 1: Real part for Set A

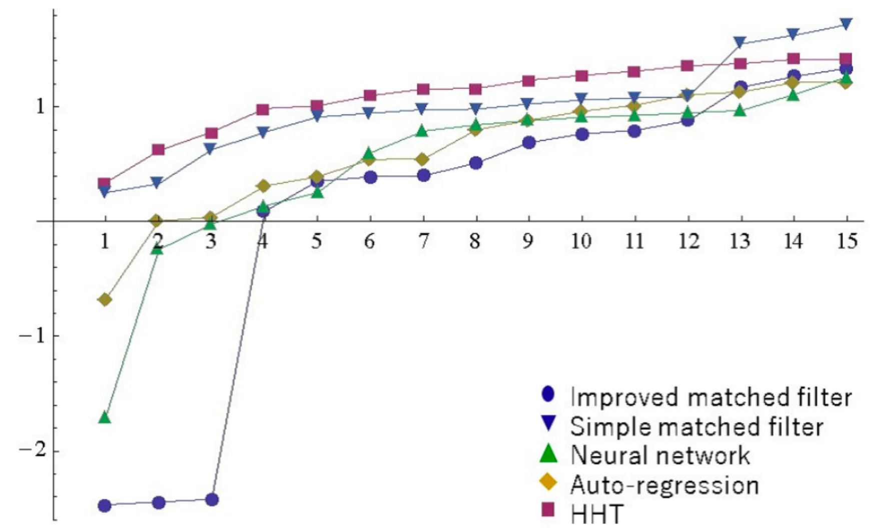


FIG. 3: Imaginary part for Set A

1. Auto-Regressive model (Method, general) I

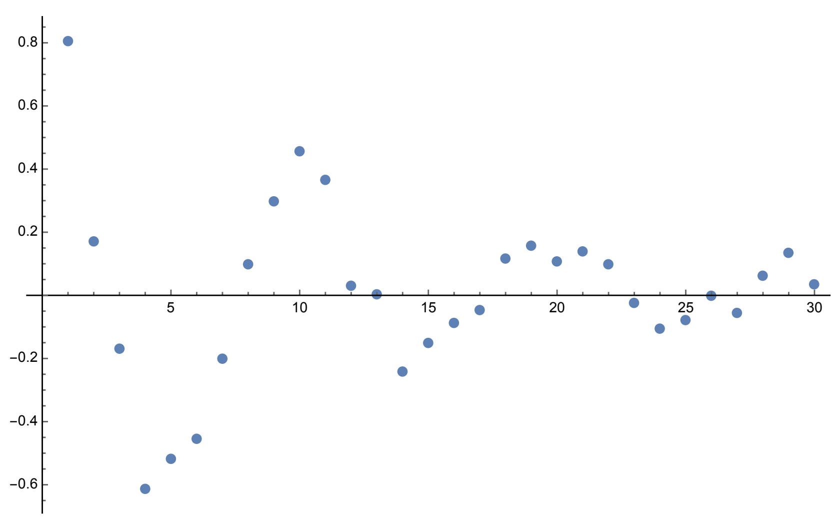
Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon$$
$$= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

e.g. $x_n = Ae^{-rn\Delta t}\cos(\omega n \Delta t)$

$$Z_1 = e^{-(r-j\omega)\Delta t}$$
$$Z_2 = e^{-(r+j\omega)\Delta t}$$

$\rightarrow$ $x_n = \frac{A}{2}(Z_1^n + Z_2^n) = (Z_1 + Z_2)x_{n-1} - Z_1 Z_2 x_{n-2}$



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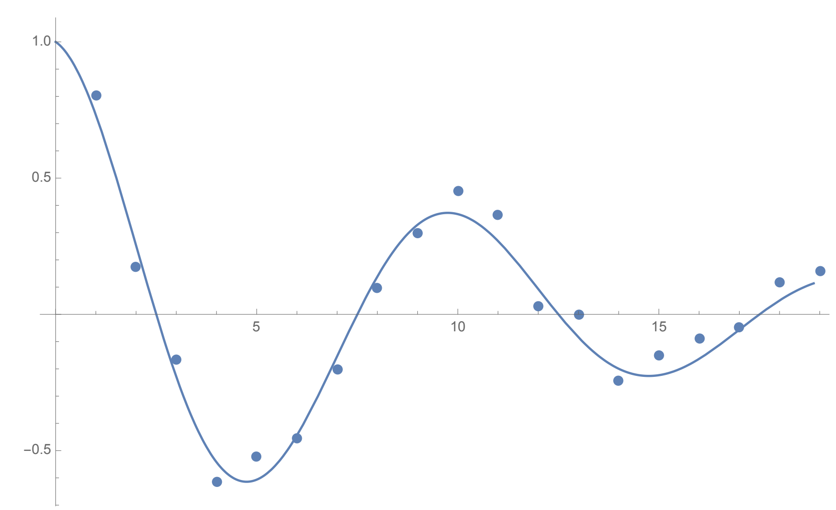


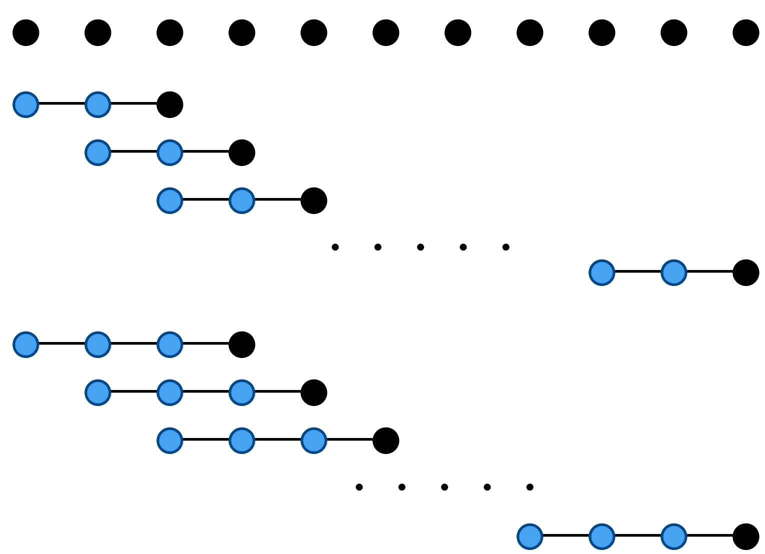
can be applied also to noisy data by adjusting *M*

1. Auto-Regressive model (Method, general) II

Fitting data with linear func.

$$\begin{aligned} x_n &= a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \\ &= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon \end{aligned}$$

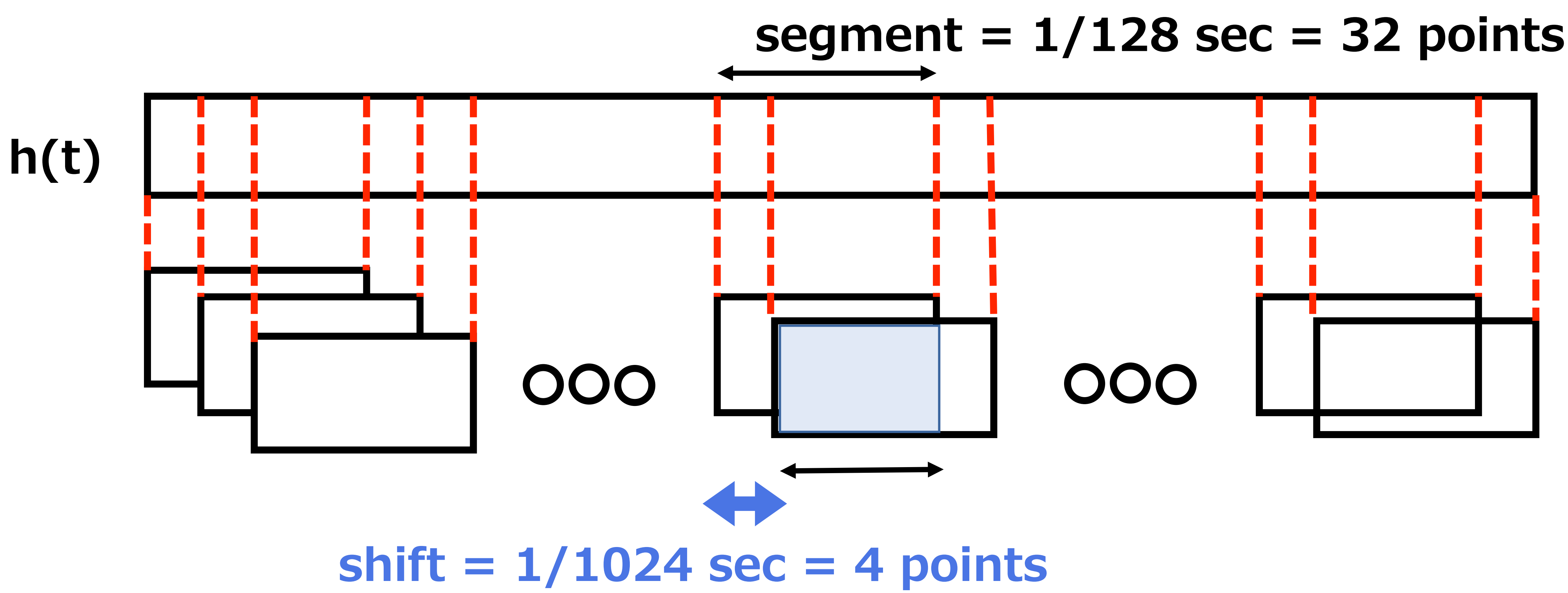
- find a_j (Burg method)
- find M (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

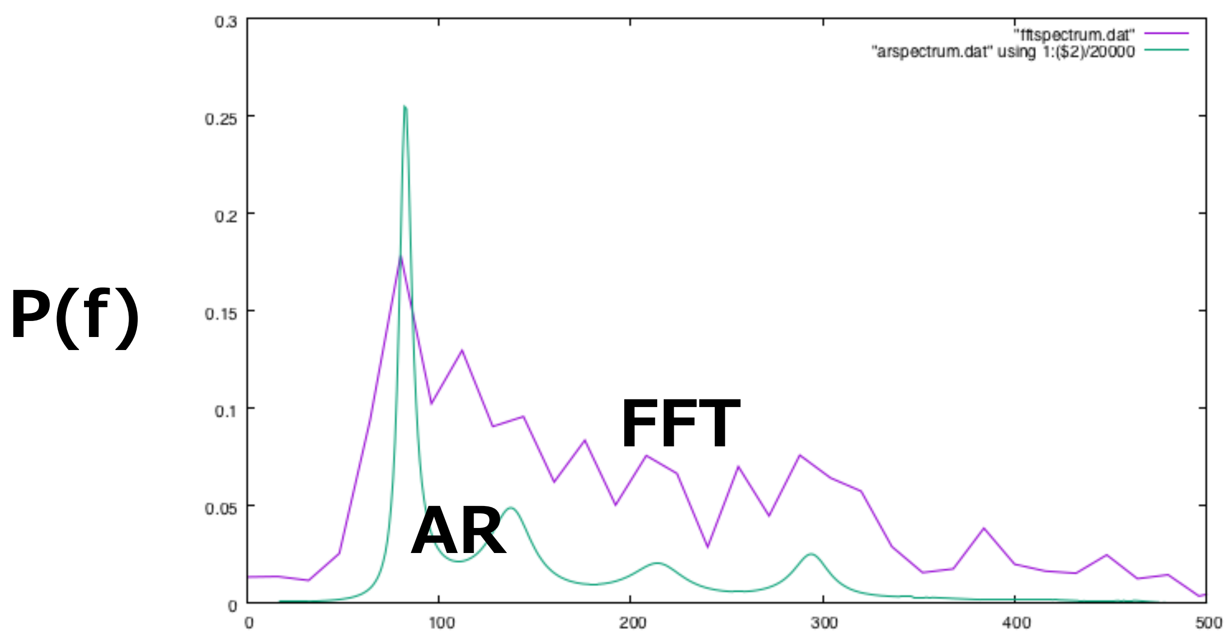


power spectrum

$$p(f) = \frac{\sigma^2}{\left| 1 - \sum_{j=1}^{M} a_j e^{-I 2\pi j f \Delta t} \right|^2}$$

Auto-Regressive model vs Short FFT





freq. [mock data, SNR=40, inspiral part]

The order M can be fixed at 2～8.

Even for short segment, AR model shows precise power-spectrum.

1. Auto-Regressive model (Method, general) III

Fitting data with linear func.

$$\begin{aligned} x_n &= a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \\ &= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon \end{aligned}$$

- find a_j (Burg method)
- find M (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

power spectrum

$$p(f) = \frac{\sigma^2}{\left| 1 - \sum_{j=1}^{M} a_j e^{-I 2\pi j f \Delta t} \right|^2}$$

characteristic eq.

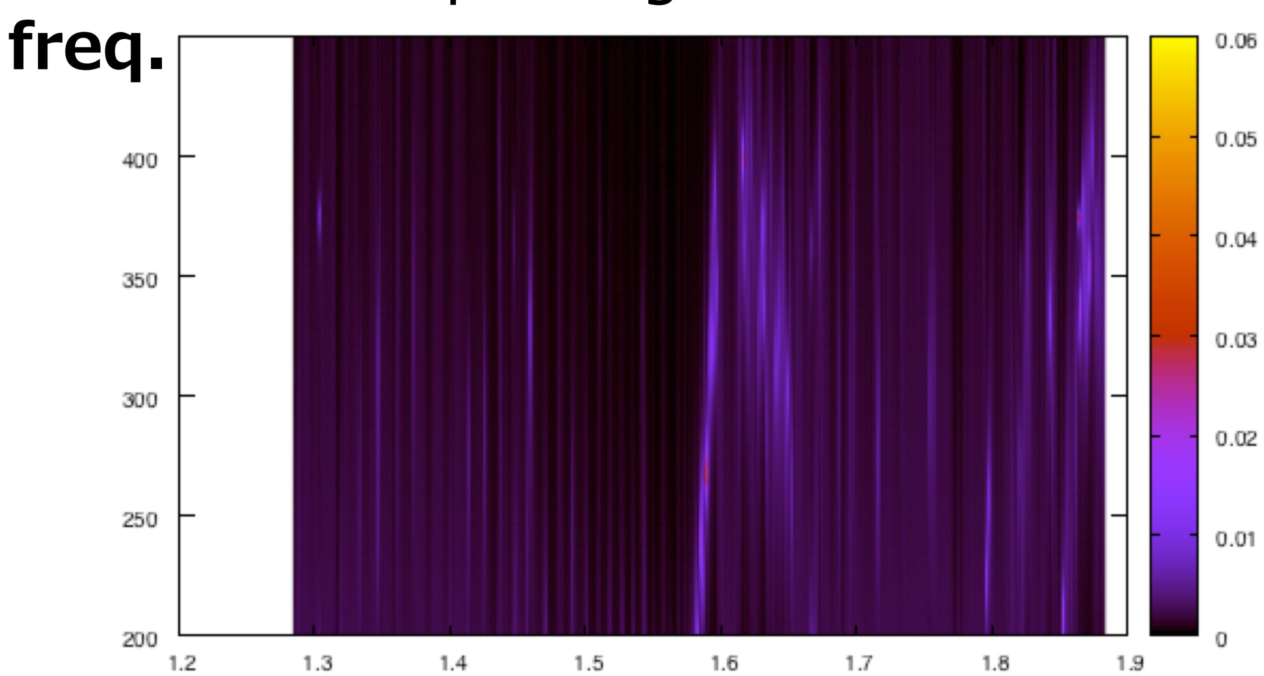
$$f(z) = 1 - \sum_{j=1}^{M} a_j z^j = 0$$

$|z_k|$ says amplitude,
$\arg(z_k)$ says frequency.

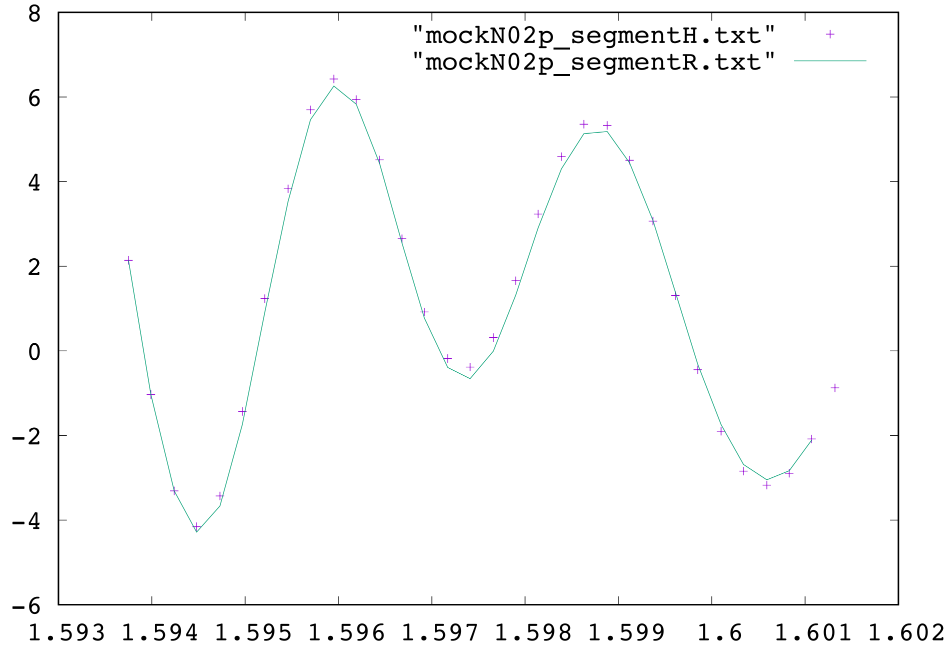
Mock data example (1) fitting well

Mock Data (Nakano02 p)

spectrogram



h(t) x original data, — fitted



Fitting data with linear func.

$$
\begin{aligned}
x_n &= a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \\
&= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon
\end{aligned}
$$

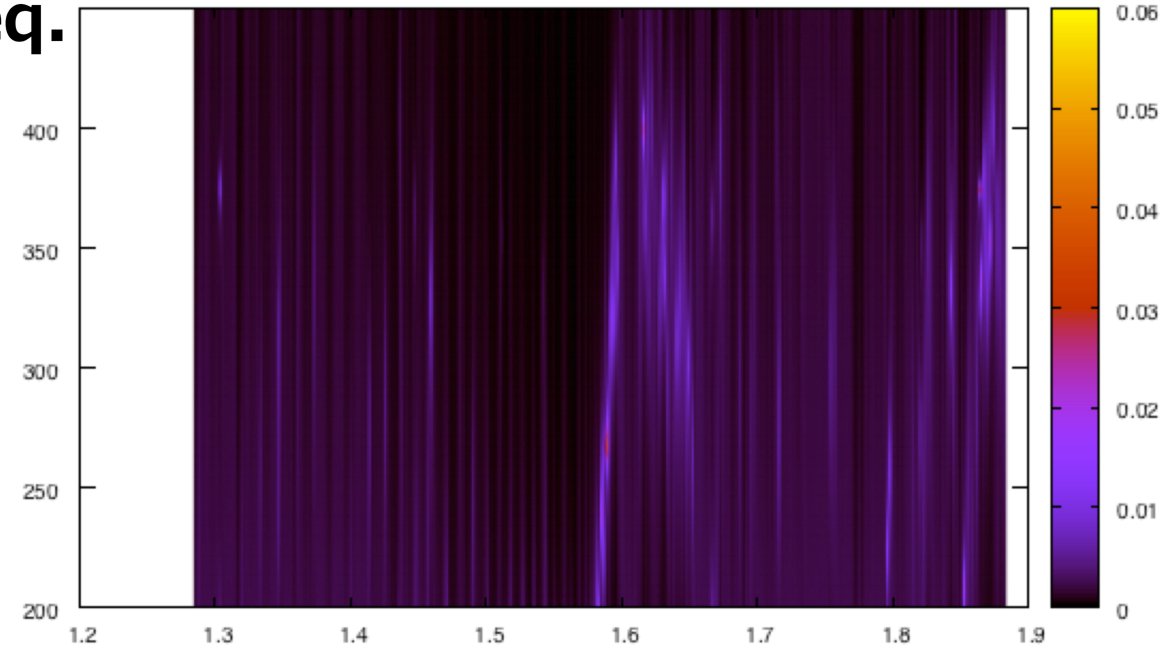
```
a.1   = -2.235e+00
a.2   =  1.869e+00
a.3   = -5.545e-01
```

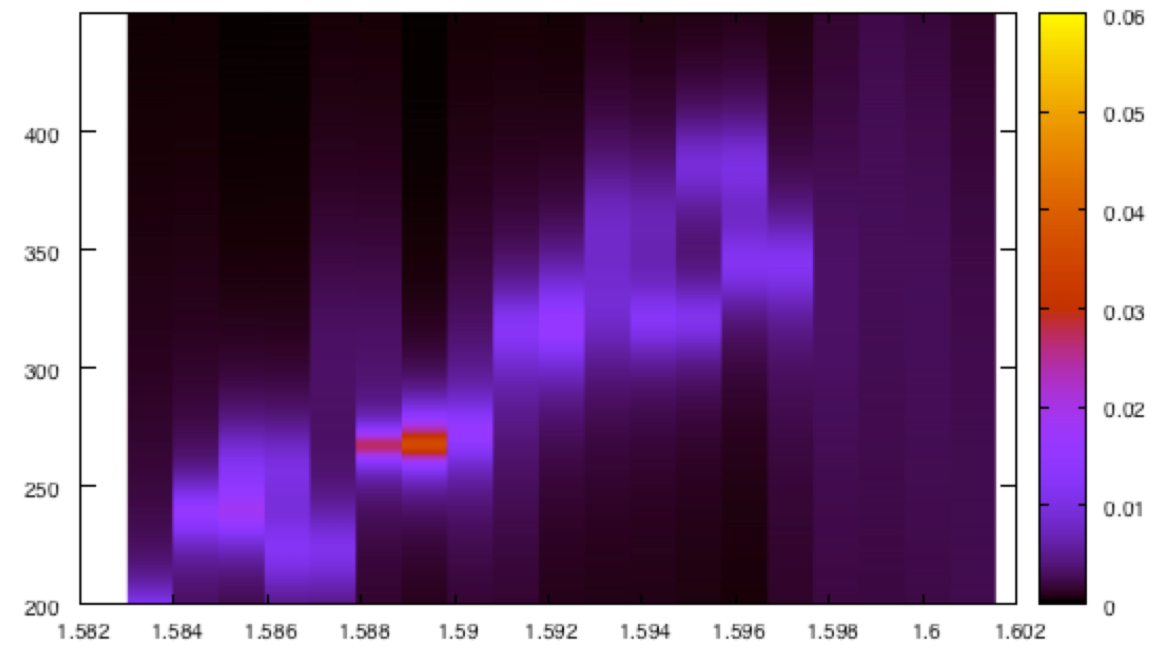
Mock data example (2) spectrogram

Mock Data (Nakano02 p)

spectrogram

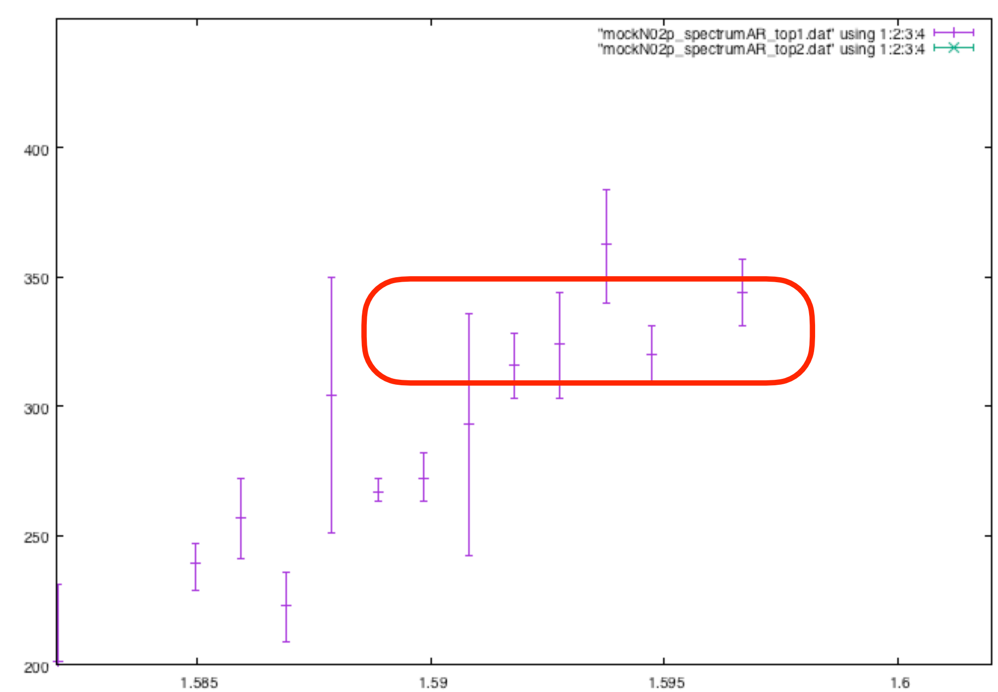
freq.





power spectrum

$$p(f) = \frac{\sigma^2}{\left| 1 - \sum_{j=1}^{M} a_j e^{-I 2\pi j f \Delta t} \right|^2}$$



Mock data example (3) characteristic eq.

Mock Data (Nakano02 p)

Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon$$
$$= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

```
a.1  = -2.235e+00
a.2  =  1.869e+00
a.3  = -5.545e-01
```

characteristic eq.

$$f(z) = 1 - \sum_{j=1}^{M} a_j z^j = 0$$

$$x_{n-1} = z x_n$$

$$z = \exp[-2\pi i f \Delta t]$$

$|z_k|$ says amplitude,
$\arg(z_k)$ says frequency.

	x.r	x.i	f_R[Hz]	|x|	f_I[Hz]
1	0.962	0.566	346.800	8.025e-01	71.721
2	0.962	-0.566	-346.800	8.025e-01	71.721
3	1.447	0.000	0.000	4.775e-01	240.931

Mock data example (4) identify ring-down freq.

Mock Data (Nakano02 p)

characterestic eq.

$$f(z) = 1 - \sum_{j=1}^{M} a_j z^j = 0$$

$$x_{n-1} = z x_n$$

$$z = \exp[-2\pi i f \Delta t]$$

$|z_k|$ says amplitude,
$\arg(z_k)$ says frequency.

	x.r	x.i	f_R[Hz]	|x|	f_I[Hz]
1	0.962	0.566	346.800	8.025e-01	71.721
2	0.962	-0.566	-346.800	8.025e-01	71.721
3	1.447	0.000	0.000	4.775e-01	240.931

t	f_R (z_plane)	f_I (z_plane)	f_Rh(spectr)	f_Rmax(spectr)	f_Rh(spectr)
0.159375E+01	0.363837E+03	0.280414E+02	0.340000E+03	0.363000E+03	0.384000E+03
0.159668E+01	0.344258E+03	0.166608E+02	0.331000E+03	0.344000E+03	0.357000E+03
0.159766E+01	0.346800E+03	0.717212E+02	0.240000E+03	0.329000E+03	0.382000E+03
0.161230E+01	0.357677E+03	0.122067E+03	0.213000E+03	0.338000E+03	0.431000E+03
0.161328E+01	0.361098E+03	0.948919E+02	0.261000E+03	0.350000E+03	0.422000E+03
0.161523E+01	0.379918E+03	0.772796E+02	0.304000E+03	0.373000E+03	0.432000E+03

average & variance zfr = 0.359E+03 0.118E+02 fr(sp) = 0.350E+03 0.148E+02
average & variance zfi = 0.684E+02 0.365E+02

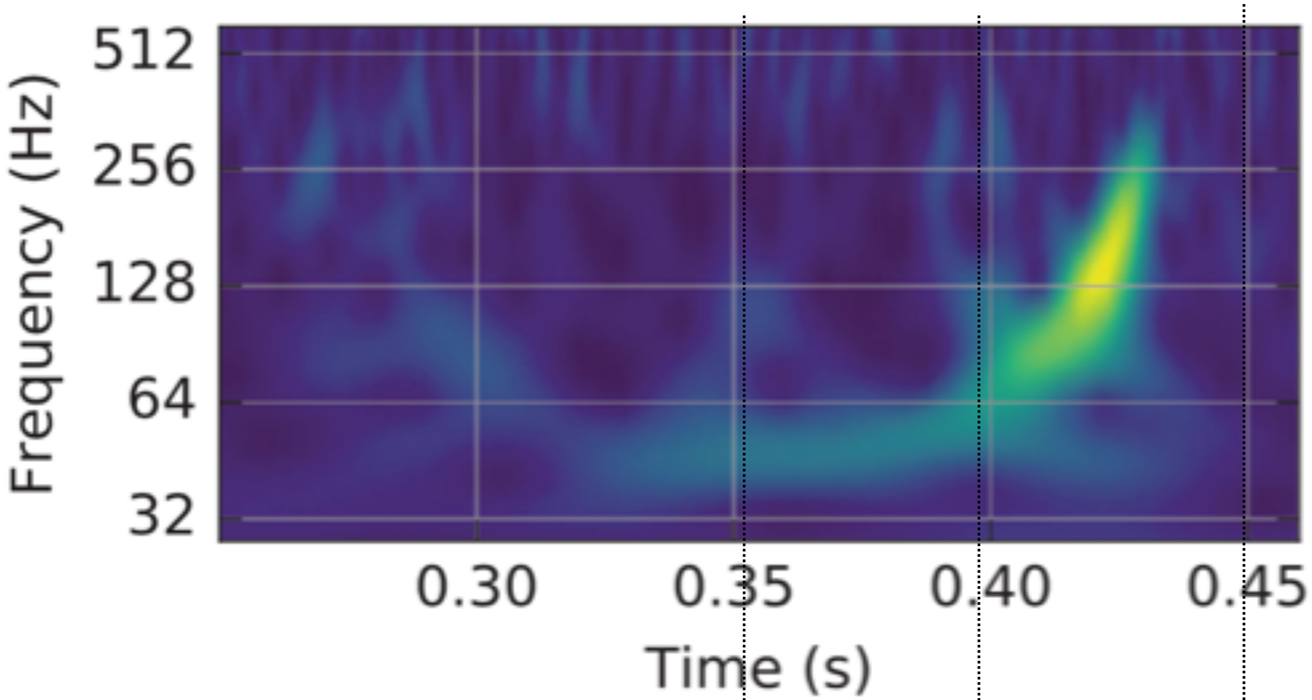
Application to the LIGO/Virgo data

List of Detected GW events

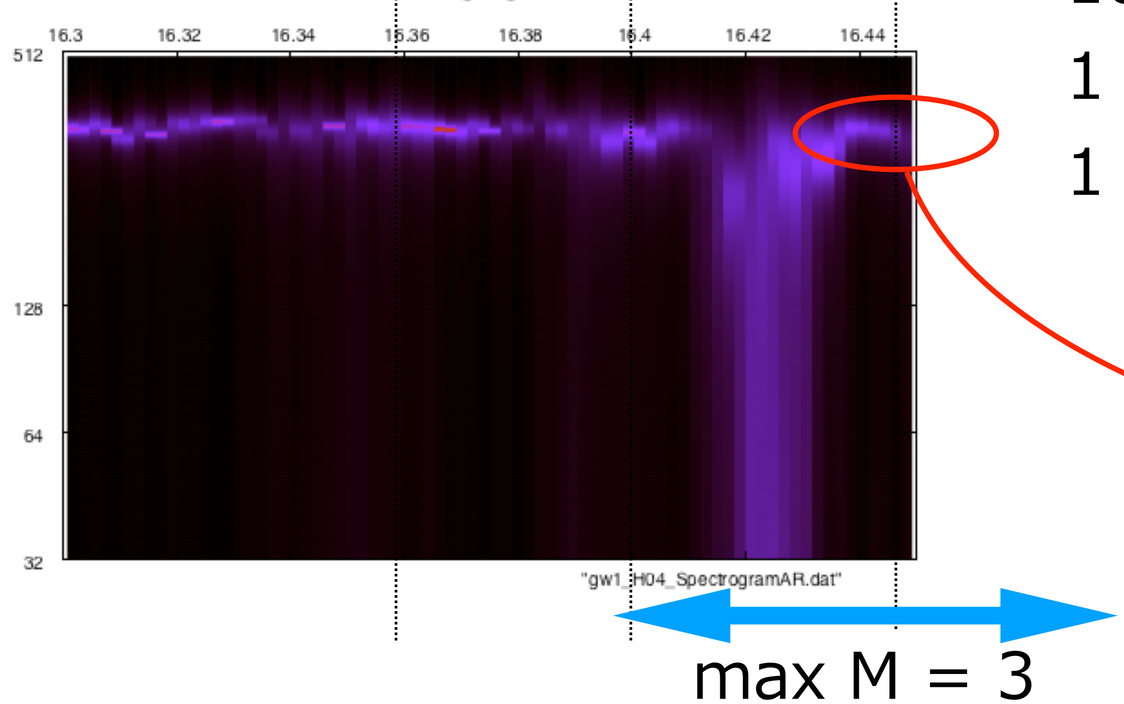
	ref.	M1+M2=Mf, Mdiff/Mtotal	spin a_final	Mpc z	SNR	deg^2
GW150914	PRL116, 061102 (2016/2/11)	36.2+29.1=62.3+3.0 **4.59%**	0.68	410Mpc 0.09	**23.7**	600
LVT151012	(2016/2/11)	23+13=35+1.5 **2.78%**	0.66	1000Mpc 0.20	9.7	
GW151226	PRL116, 241103 (2016/6/15)	14.2+7.5=20.8+0.9 **4.15%**	0.74	440Mpc 0.09	13.0	850
GW170104	PRL118, 221101 (2017/6/1)	31.2+19.4=48.7+1.9 **3.75%**	0.64	880Mpc 0.18	**13**	1300
GW170608	ApJ 851, L35 (2017/12/18)	12+7=18.0+1.0 **5.2%**	0.69	340Mpc 0.07	13	520
GW170814	PRL119,141101 (2017/10/6)	30.5+25.3=53.2+2.6 **4.66%**	0.70	540Mpc 0.11	**18**	60
GW170817	PRL119, 161101 (2017/10/16)	1.36〜1.60 + 1.17〜1.36 = 2.74 + ?	?	40Mpc	32.4	28

Ringdown wave of GW150914

LIGO paper



AR model Hanford

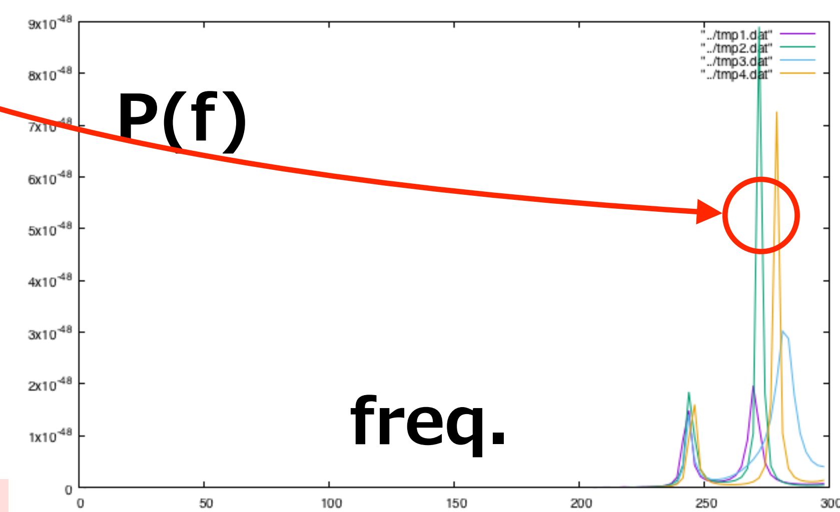


4096 sampling rate

100-400 Hz filter

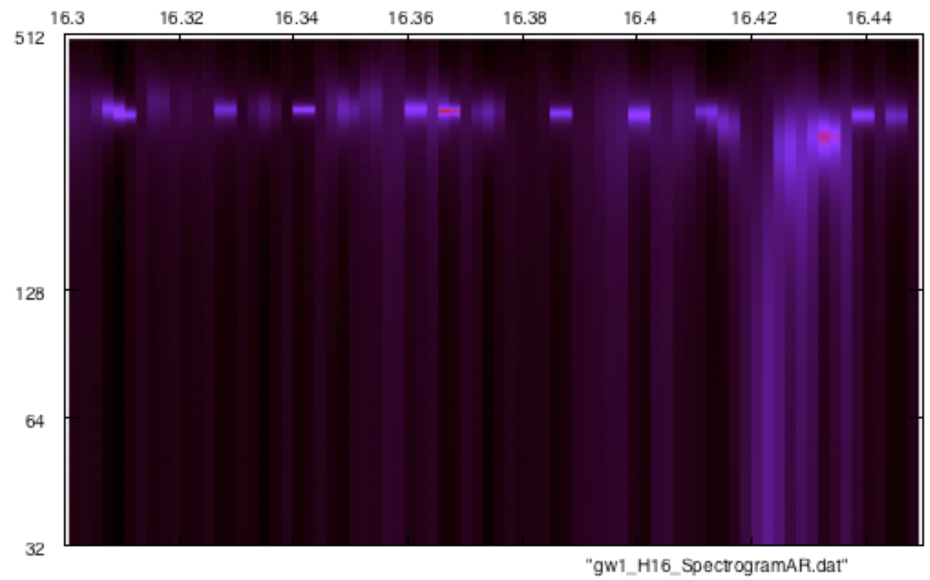
1 segment = 1/64 sec= 64 points

1 shift = 1/512 sec = 8 points



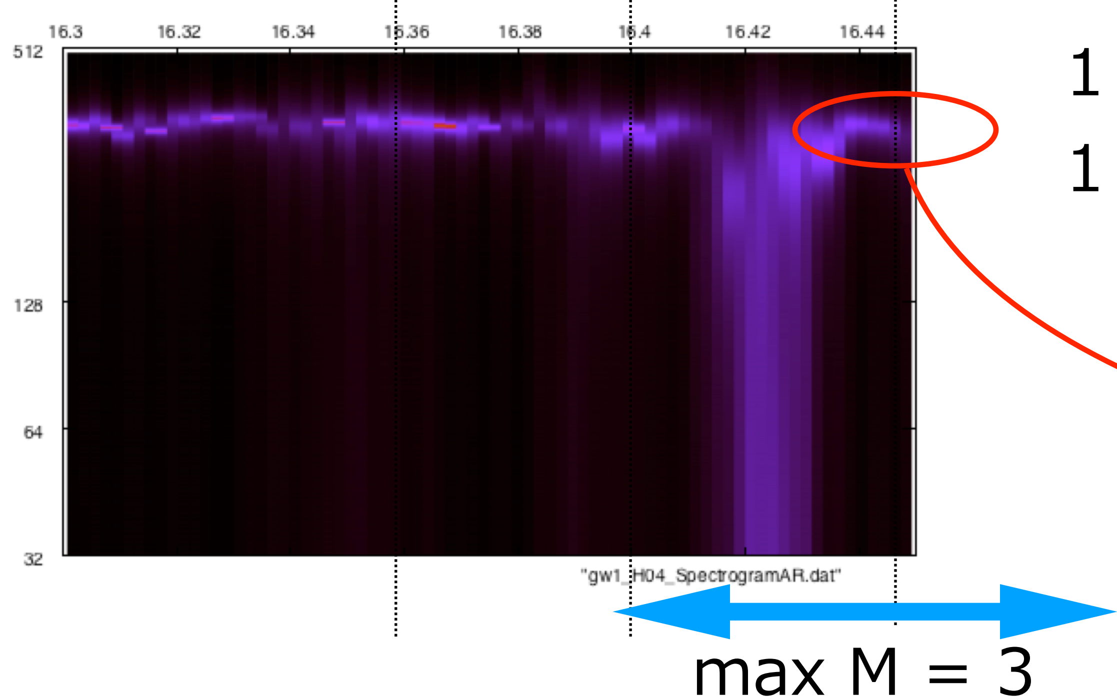
Ringdown wave of GW150914

AR model Hanford



16384 sampling rate

AR model Hanford

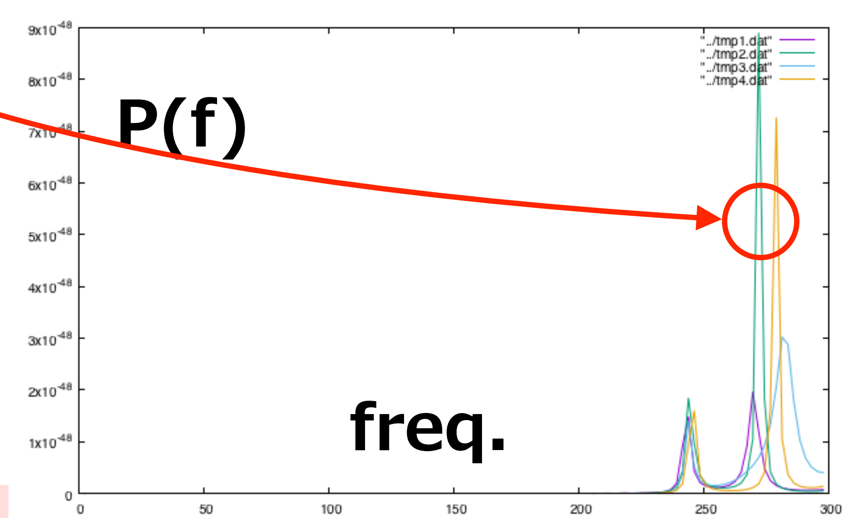


4096 sampling rate

100-400 Hz filter

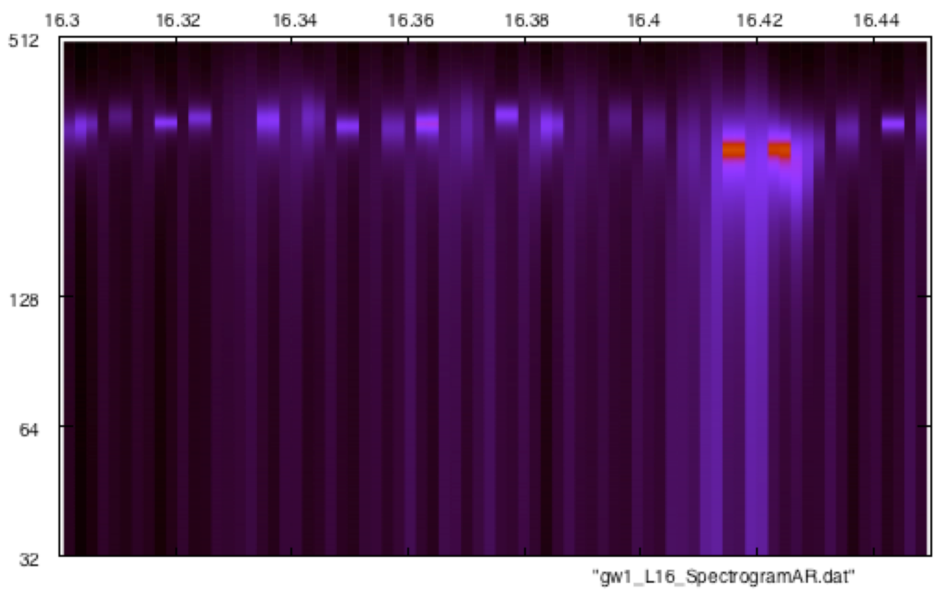
1 segment = 1/64 sec= 64 points

1 shift = 1/512 sec = 8 points



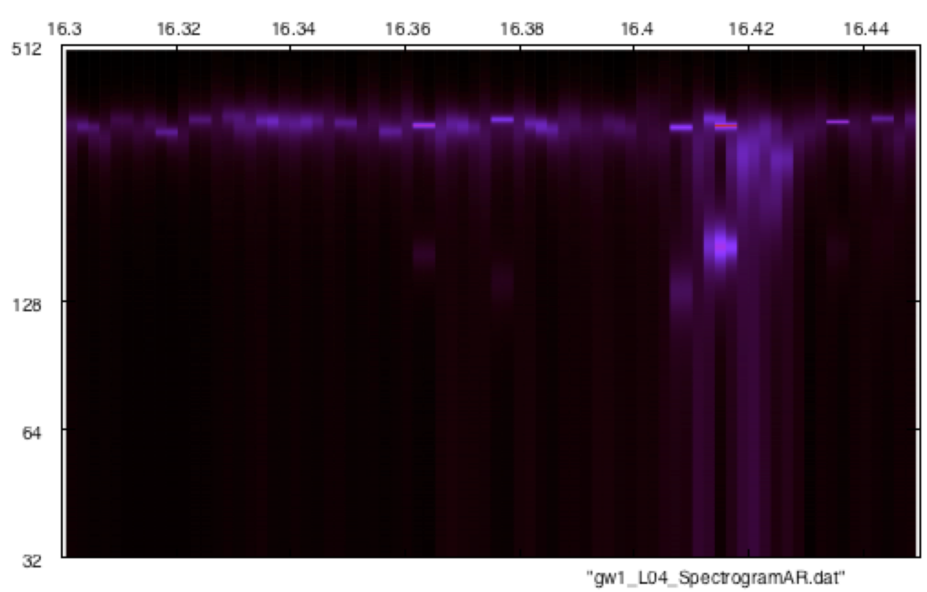
Ringdown wave of GW150914

AR model
Livingston



16384 sampling rate

AR model
Livingston



4096 sampling rate
100-400 Hz filter
1 segment = 1/64 sec= 64 points
1 shift = 1/512 sec = 8 points

max M = 3

Ringdown wave of GW150914

t f real f imag

```
4486      1  0.164258E+02  0.313508E+03  0.432791E+02  0.259000E+03  0.305000E+03  0.337000E+03
4488xx    2  0.164297E+02  0.300353E+03  0.578188E+02  0.218000E+03  0.287000E+03  0.330000E+03
4489      1  0.164316E+02  0.317507E+03  0.382182E+02  0.274000E+03  0.311000E+03  0.339000E+03
4498      1  0.164492E+02  0.314336E+03  0.538556E+02  0.261000E+03  0.309000E+03  0.349000E+03
4501      1  0.164551E+02  0.317640E+03  0.349751E+02  0.282000E+03  0.314000E+03  0.340000E+03
4505      1  0.164629E+02  0.316355E+03  0.429281E+02  0.277000E+03  0.314000E+03  0.346000E+03
4508      1  0.164688E+02  0.311752E+03  0.297619E+02  0.285000E+03  0.310000E+03  0.332000E+03
data points =    7
average & variance zfr =   0.313E+03   0.556E+01  fr(sp) =    0.307E+03   0.871E+01
average & variance zfi =   0.430E+02   0.926E+01
```

We see QNM at 300Hz, 0.003s after the merger.

$$f_R = f_1 + f_2(1-a)^{f_3}$$

$$Q \equiv \frac{f_R}{2f_I} = q_1 + q_2(1-a)^{q_3}$$

$$f_{\rm qnm}[{\rm Hz}] = \frac{c^3}{2\pi GM} f_R \sim 32314.1 \left(\frac{M_\odot}{M}\right) f_R.$$

$$a = 1 - \left(\frac{Q-q_1}{q_2}\right)^{1/q_3}$$

$$M[M_\odot] = 32314.1 \times \frac{f_1 + f_2(1-a)^{f_3}}{f_{\rm qnm}[{\rm Hz}]}$$

Berti, Cardoso & Will PRD 73, 064030 (2006).

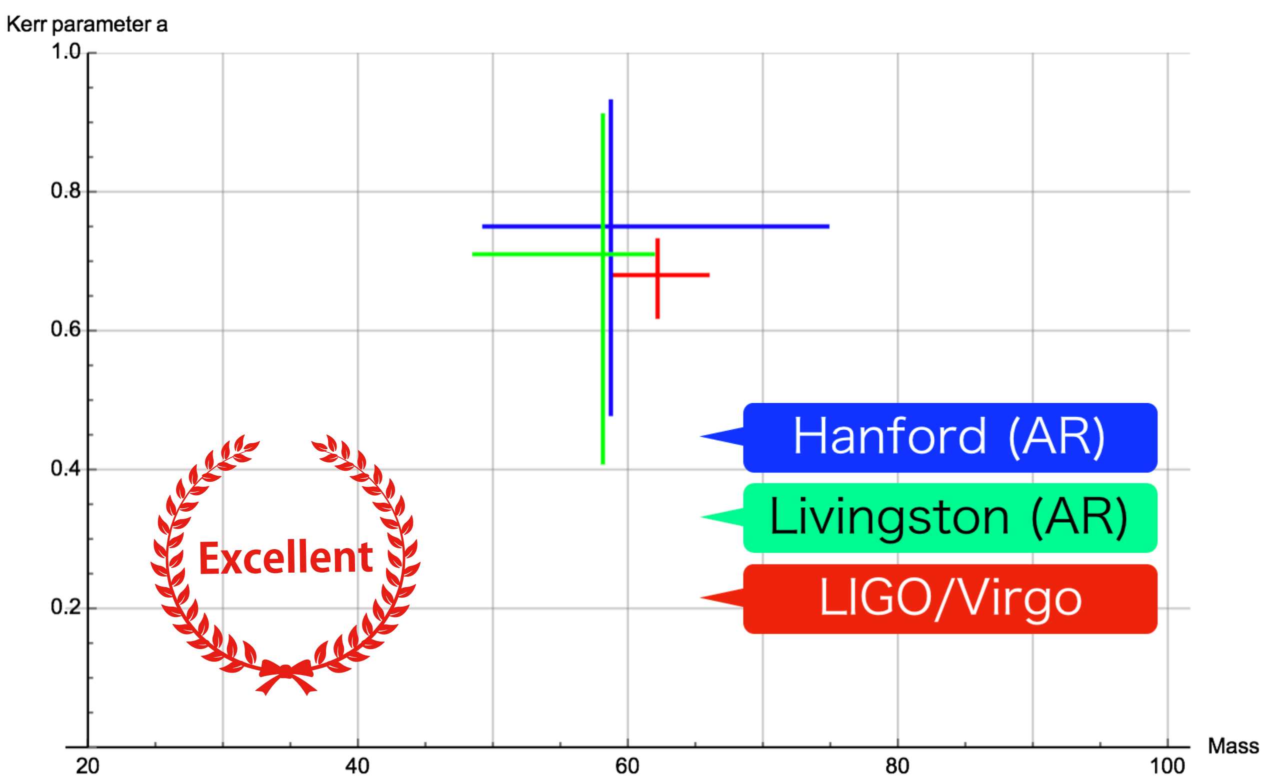
Table 1. Results of frequency and damping rate of ring-down gravitational wave of GW150914.

data	$f_{\rm real}$[Hz]	$f_{\rm imag}$[Hz]	mass $(M/M_\odot)$	Kerr parameter a/M
Hanford	$305.94^{+18.68}_{-27.82}$	$43.55^{+13.00}_{-17.99}$	$58.74^{+16.03}_{-9.37}$	$0.75^{+0.18}_{-0.27}$
Livingston	$300.02^{+17.49}_{-27.21}$	$44.94^{+12.88}_{-18.30}$	$58.15^{+16.49}_{-9.53}$	$0.71^{+0.20}_{-0.30}$

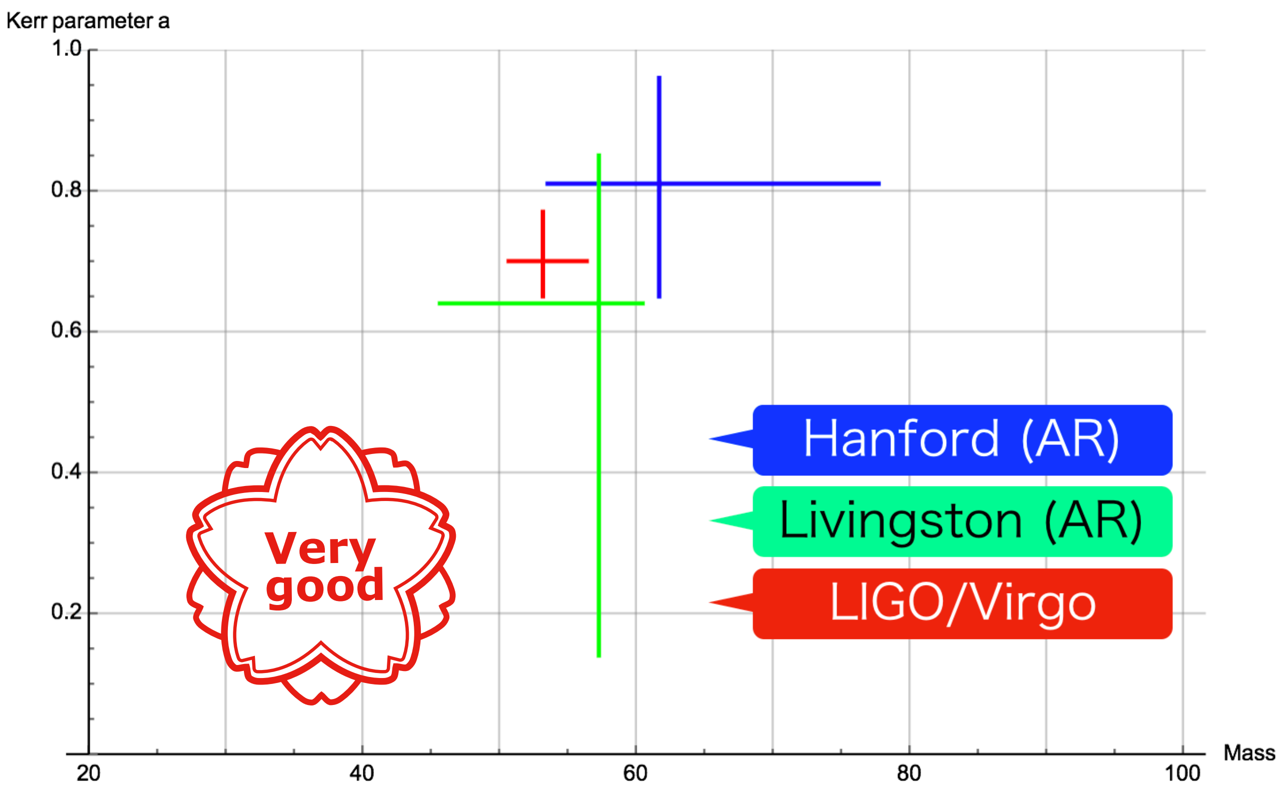
LIGO paper says $62.2^{+3.7}_{-3.4}\ M_\odot$ $a/M = 0.68^{+0.05}_{-0.06}$

Ringdown wave of GW150914, GW170814, GW170104

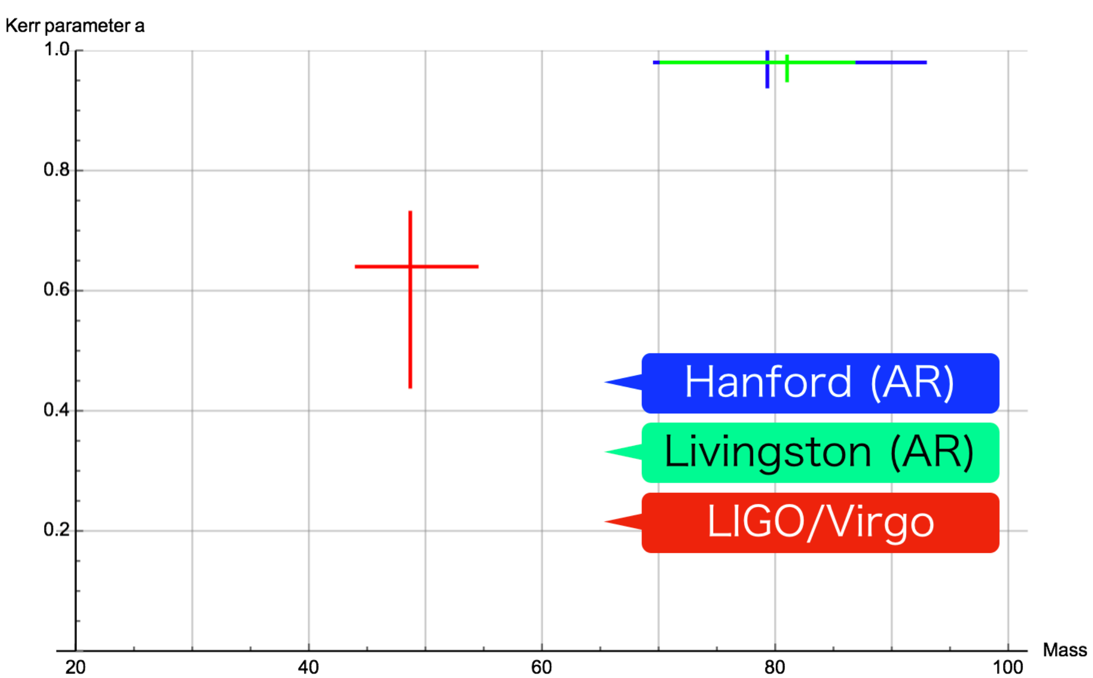
GW150914 (S/N = 23.7)



GW170814 (S/N = 18)



GW170104 (S/N = 13)



Summary & Outlook

New method for extracting GW signal from *x(t)*

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon$$
$$= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

It works for short segment data ($\sim$ 30 pts), accurate freq.
It does not require any templates for finding signals.

Applied to 3 real events, for extracting ring-down part.

Obtained similar *(M, a)* with LIGO/Virgo-papers for **GW150914, GW170814**
(S/N=23.7) (S/N=18)

But not for **GW170104**. (S/N=13)

By combining with other methods, we expect to test the theories of gravity using the ring-down part of BH merger.

We hope template-independent wave extraction method will contribute to find gravitational wave from unknown sources.