Ring-down GW search using Auto-Regressive model

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Outline & Summary

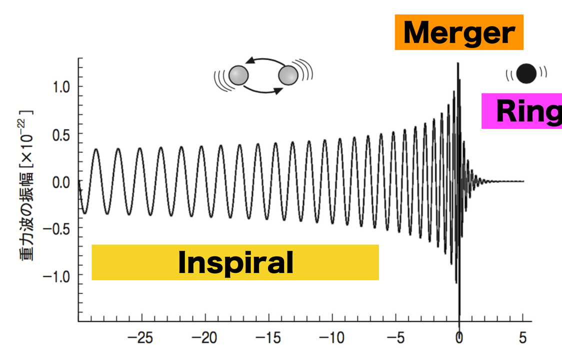
The ring-down part of gravitational waves in the final stage of merger of compact objects tells us the nature of strong gravity which can be used for testing the theories of gravity. The ring-down wave, however, fades out in a very short time with a few cycles, and hence it is challenging for gravitational wave data analysis to extract the ringdown frequency and its damping time scale.

We develop a new method, the autoregressive modeling (AR) approach, which extracts waveform by fitting a linear function from bare data. It works well for small number of data points, and does not require any templates. After obtaining the best parameters using mockdata, we applied this method for black-hole merger events of the LIGO/Virgo O1 and O2. We find that for high SNR events, we can extract ring-down waves properly.

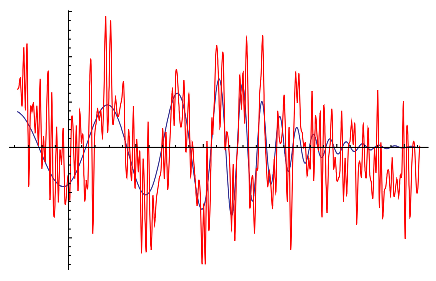
This method may work for extracting higher modes of ring-down waves, and implementations are on-going.

Motivation & O1/O2 data

Towards testing gravity theories ⇒ Ringdown-part extraction is a key



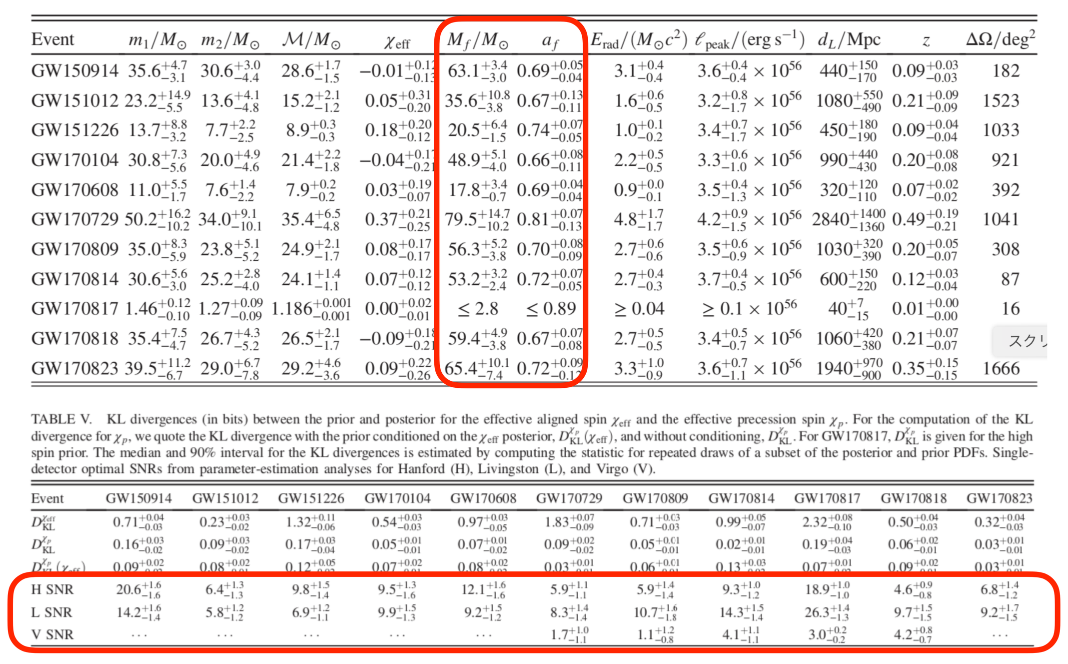
BH quasi-normal modes
⇦ BH perturbation theory
⇨ *(M, a)*
strongest gravity we can observe
⇨ test of gravity theories



For *60M* BH of *a*=0.75,
frequency = 300 Hz
damping time scale = 3.7 ms

LVC, PRX9 (2019) 031040

LIGO/Virgo O1/O2 catalogue

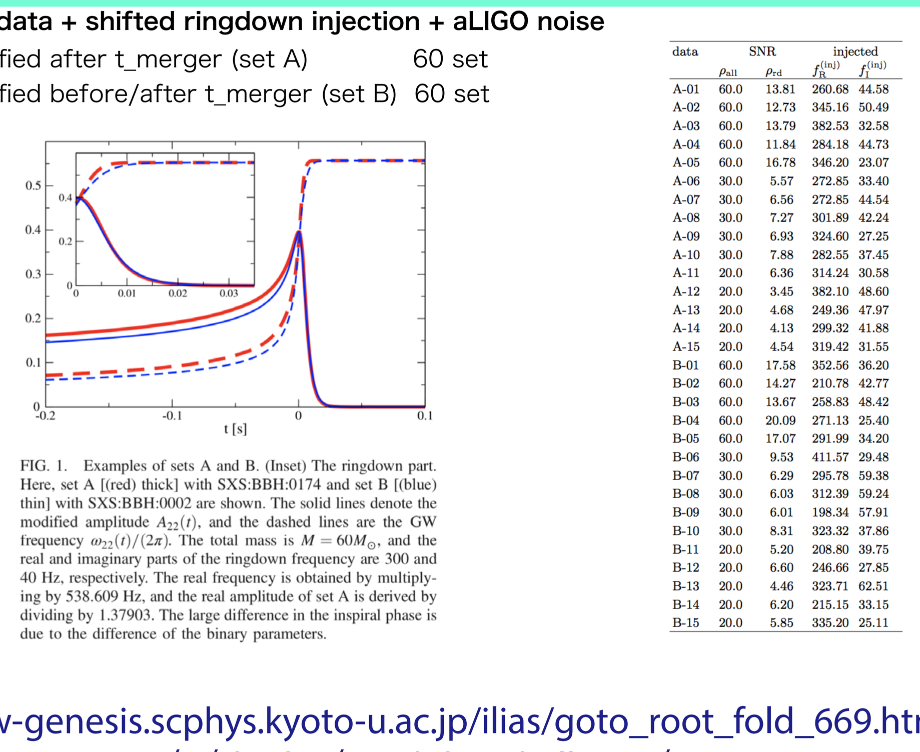


Mockdata Comparison

Phys. Rev. D 99, 124032 (2019) [arXiv:1811.06443]

Mockdata preparation

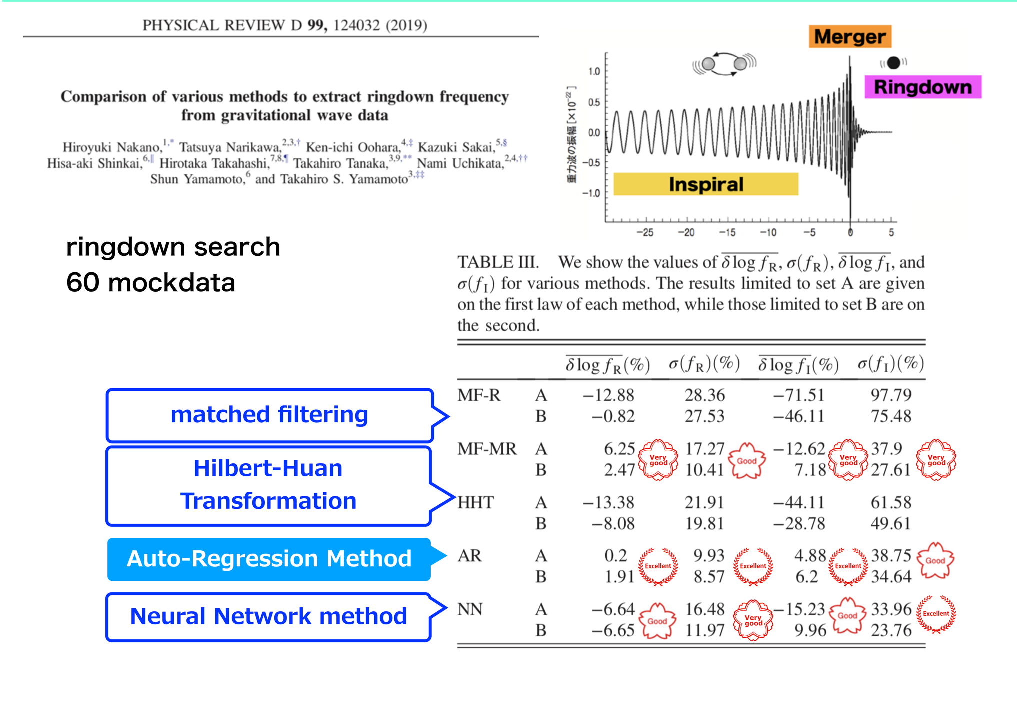
SXS data + shifted ringdown injection + aLIGO noise
modified after t_merger (set A) 60 set
modified before/after t_merger (set B) 60 set



https://gw-genesis.scphys.kyoto-u.ac.jp/ilias/goto_root_fold_669.html
http://www.oit.ac.jp/is/shinkai/mockdatachallenge/

Nakano+, PRD99 (2019) 124032

mockdata-challenge comparison



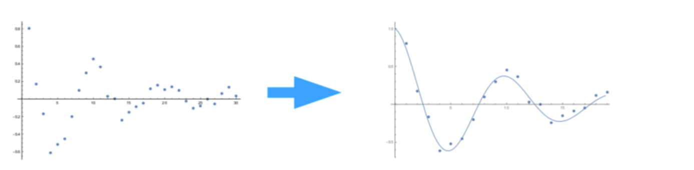
Method

Auto-Regressive model (idea)

Fitting data with linear func.

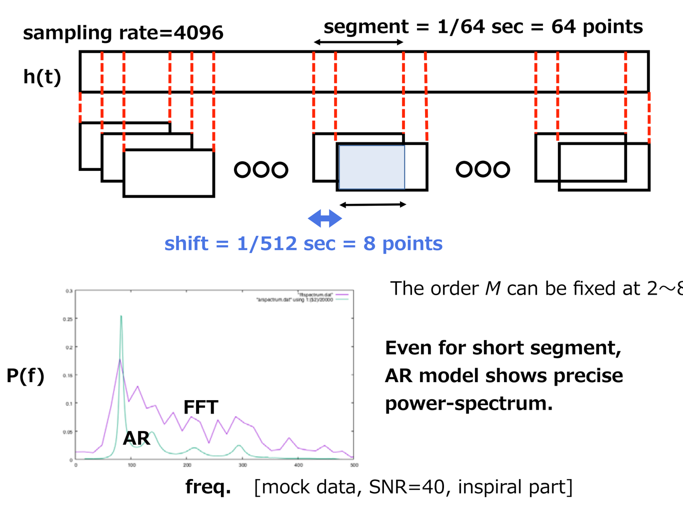
$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon = \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

e.g. $x_n = Ae^{-\gamma n\Delta t}\cos(\omega n\Delta t)$, $Z_1 = e^{-(\gamma - j\omega)\Delta t}$, $Z_2 = e^{-(\gamma + j\omega)\Delta t}$ → $x_n = \frac{A}{2}(Z_1^n + Z_2^n) = (Z_1 + Z_2)x_{n-1} - Z_1 Z_2 x_{n-2}$



can be applied also to noisy data by adjusting *M*

Auto-Regressive model vs Short FFT



The order *M* can be fixed at 2~8.

Even for short segment, AR model shows precise power-spectrum.

Auto-Regressive model (Method, general)

Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon = \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

- find a_j (Burg method)
- find *M* (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

power spectrum

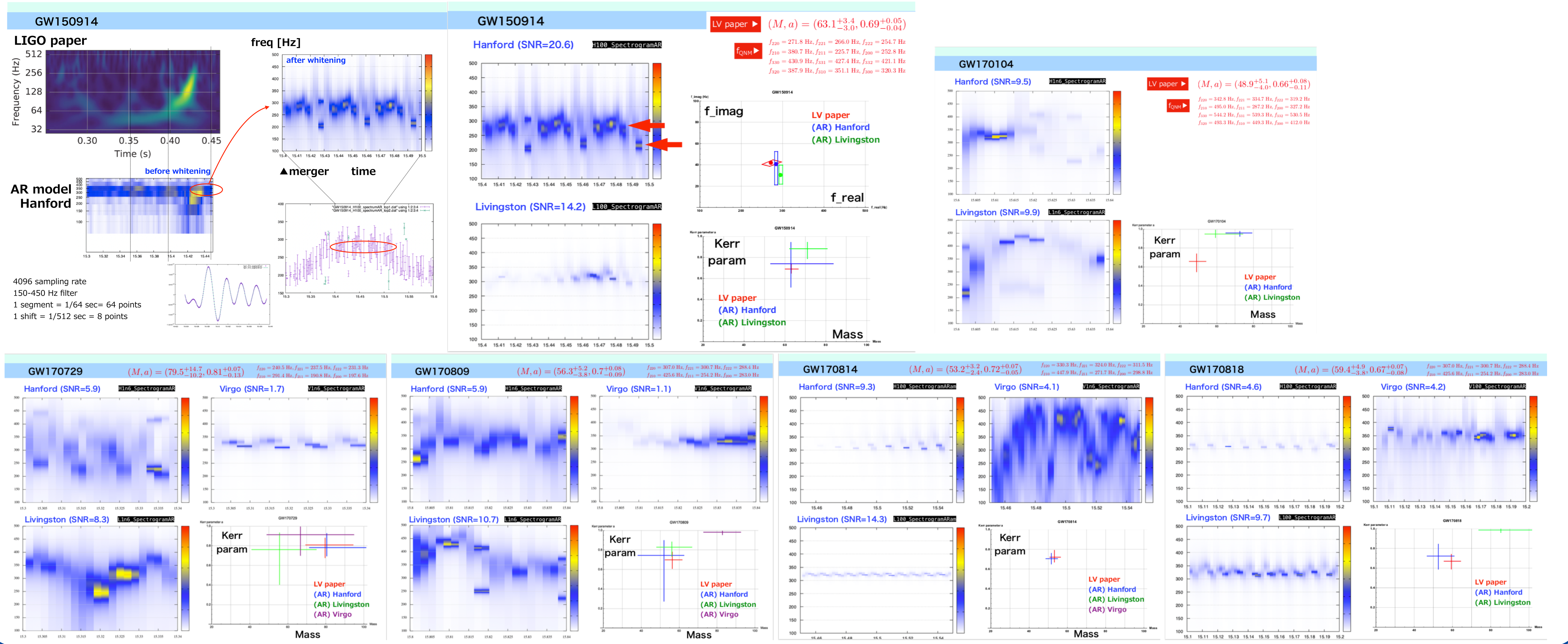
$$p(f) = \frac{\sigma^2}{\left|1 - \sum_{j=1}^{M} a_j e^{-i2\pi j f \Delta t}\right|^2}$$

characteristic eq.

$$f(z) = 1 - \sum_{j=1}^{M} a_j z^j = 0$$

$|z_k|$ says amplitude, $\arg(z_k)$ says frequency.

Results



ACKNOWLEDGMENTS

This work was supported by JSPS KAKENHI Grant No. JP17H06358 (and also JP17H06357) [A01: Testing gravity theories using gravitational waves, as a part of the innovative research area, "Gravitational wave physics and astronomy: Genesis"], by JSPS KAKENHI Grant No. JP18K03630 [Non-linear dynamics in the modified gravity theories as a test of string theory] and by No. 19H01901 [New directions in gravitational-wave data analysis: both in computing algorithms and hardwares including its outreach activities].