

Ring-down GW search using Auto-Regressive model



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Outline & Summary

The ring-down part of gravitational waves in the final stage of merger of compact objects tells us the nature of strong gravity which can be used for testing the theories of gravity. The ring-down wave, however, fades out in a very short time with a few cycles, and hence it is challenging for gravitational wave data analysis to extract the ringdown frequency and its damping time scale.

We develop a new method, the autoregressive modeling (AR) approach, which extracts waveform by fitting a linear function from bare data. It works well for small number of data points, and does not require any templates. After obtaining the best parameters using mockdata, we applied this method for black-hole merger events of the LIGO/Virgo O1 and O2. We find that for high SNR events, we can extract ring-down waves properly.

This method may work for extracting higher modes of ring-down waves, and implementations are on-going.

Motivation & O1/O2 data

Towards testing gravity theories → Ringdown-part extraction is a key

For 60M BH of a=0.75,
frequency = 300 Hz
damping time scale = 3.7 ms

LVC, PRX9 (2019) 031040
LIGO/Virgo O1/O2 catalogue

Event	m_1/M_\odot	m_2/M_\odot	M/M_\odot	χ_{eff}	M/M_\odot	a_j	$E_{\text{rad}}(M_\odot c^2)$	$\epsilon_{\text{peak}}(\text{ergs}^{-1})$	d_j/Mpc	z	$\Delta\Omega/\text{deg}^2$
GW150914	35.6 $^{+7.7}_{-7.1}$	30.6 $^{+2.0}_{-1.8}$	28.6 $^{+1.7}_{-1.5}$	-0.01 $^{+0.01}_{-0.01}$	63.1 $^{+3.4}_{-3.0}$	0.69 $^{+0.05}_{-0.04}$	3.1 $^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4} \times 10^{56}$	440-150	0.09 $^{+0.03}_{-0.03}$	182
GW151012	23.2 $^{+14.9}_{-8.8}$	13.6 $^{+2.1}_{-2.0}$	15.2 $^{+2.1}_{-2.0}$	0.05 $^{+0.31}_{-0.31}$	35.6 $^{+10.8}_{-10.8}$	0.67 $^{+0.13}_{-0.13}$	1.6 $^{+0.6}_{-0.6}$	$3.2^{+0.8}_{-0.8} \times 10^{56}$	1080-550	0.21 $^{+0.09}_{-0.09}$	1523
GW151226	13.7 $^{+5.8}_{-5.2}$	7.7 $^{+2.2}_{-2.0}$	8.9 $^{+2.3}_{-2.0}$	0.18 $^{+0.20}_{-0.12}$	20.5 $^{+5.4}_{-4.9}$	0.74 $^{+0.05}_{-0.05}$	1.0 $^{+0.1}_{-0.1}$	$3.4^{+0.7}_{-0.7} \times 10^{56}$	450-180	0.09 $^{+0.04}_{-0.04}$	1033
GW170104	30.8 $^{+7.3}_{-6.6}$	20.0 $^{+2.9}_{-2.6}$	21.4 $^{+2.2}_{-2.2}$	-0.04 $^{+0.03}_{-0.03}$	48.9 $^{+5.1}_{-4.6}$	0.66 $^{+0.08}_{-0.08}$	2.2 $^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-0.6} \times 10^{56}$	990-440	0.20 $^{+0.08}_{-0.08}$	921
GW170608	11.0 $^{+5.5}_{-5.2}$	7.6 $^{+1.4}_{-1.2}$	7.9 $^{+1.2}_{-1.1}$	0.03 $^{+0.10}_{-0.09}$	17.8 $^{+3.4}_{-3.0}$	0.69 $^{+0.04}_{-0.04}$	0.9 $^{+0.1}_{-0.1}$	$3.5^{+0.4}_{-0.4} \times 10^{56}$	320-120	0.07 $^{+0.02}_{-0.02}$	392
GW170729	50.2 $^{+16.2}_{-15.2}$	34.0 $^{+6.1}_{-5.6}$	35.4 $^{+5.5}_{-5.1}$	0.37 $^{+0.21}_{-0.21}$	79.5 $^{+14.7}_{-13.7}$	0.81 $^{+0.07}_{-0.07}$	4.8 $^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-0.9} \times 10^{56}$	2840-1400	0.49 $^{+0.19}_{-0.19}$	1041
GW170809	35.0 $^{+8.3}_{-8.0}$	23.8 $^{+3.1}_{-2.9}$	24.9 $^{+2.1}_{-2.1}$	0.08 $^{+0.12}_{-0.12}$	56.3 $^{+5.2}_{-4.8}$	0.70 $^{+0.08}_{-0.08}$	2.7 $^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.6} \times 10^{56}$	1030-520	0.20 $^{+0.05}_{-0.05}$	308
GW170814	30.6 $^{+5.6}_{-5.0}$	25.2 $^{+2.8}_{-2.6}$	24.1 $^{+1.4}_{-1.4}$	0.07 $^{+0.12}_{-0.12}$	53.2 $^{+3.2}_{-2.8}$	0.72 $^{+0.07}_{-0.07}$	2.7 $^{+0.4}_{-0.4}$	$3.7^{+0.4}_{-0.4} \times 10^{56}$	600-150	0.12 $^{+0.03}_{-0.03}$	87
GW170817	1.46 $^{+0.12}_{-0.10}$	1.27 $^{+0.09}_{-0.09}$	1.186 $^{+0.001}_{-0.001}$	0.00 $^{+0.02}_{-0.02}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1 \times 10^{56}$	40 $^{+7}_{-7}$	0.01 $^{+0.00}_{-0.00}$	16
GW170818	35.4 $^{+1.7}_{-1.7}$	26.7 $^{+2.3}_{-2.3}$	26.5 $^{+2.1}_{-2.1}$	-0.09 $^{+0.11}_{-0.11}$	59.4 $^{+4.9}_{-4.5}$	0.67 $^{+0.07}_{-0.07}$	2.7 $^{+0.3}_{-0.3}$	$3.4^{+0.5}_{-0.5} \times 10^{56}$	1060 $^{+420}_{-280}$	0.21 $^{+0.07}_{-0.07}$	227
GW170823	39.5 $^{+2.7}_{-2.7}$	29.0 $^{+2.7}_{-2.7}$	29.2 $^{+2.6}_{-2.6}$	0.09 $^{+0.23}_{-0.23}$	65.4 $^{+10.1}_{-9.7}$	0.72 $^{+0.09}_{-0.09}$	3.3 $^{+1.0}_{-1.0}$	$3.6^{+0.7}_{-0.7} \times 10^{56}$	1940 $^{+920}_{-500}$	0.35 $^{+0.15}_{-0.15}$	1666

TABLE V. KL divergences (in bits) between the prior and posterior for the effective aligned spin χ_{eff} and the effective precession spin a_j . For the computation of the KL divergence for χ_{eff} , we quote the KL divergence with the prior conditioned on the χ_{eff} posterior, $D_{\text{KL}}(p_{\text{post}}||p_{\text{prior}})$, and without conditioning, $D_{\text{KL}}^{\text{uncond}}$. For GW170817, $D_{\text{KL}}^{\text{uncond}}$ is given for the high spin prior. The median and 90% interval for the KL divergences is estimated by computing the statistics for repeated draws of a subset of the posterior and prior PDFs. Single-detector optimal SNRs from parameter-estimation analyses for Hanford (H), Livingston (L), and Virgo (V).

Event	GW150914	GW151012	GW151226	GW170104	GW170608	GW170729	GW170809	GW170814	GW170817	GW170818	GW170823
$D_{\text{KL}}^{\text{uncond}}(\chi_{\text{eff}})$	0.71 $^{+0.20}_{-0.20}$	0.23 $^{+0.02}_{-0.02}$	1.32 $^{+0.26}_{-0.26}$	0.54 $^{+0.20}_{-0.20}$	0.97 $^{+0.20}_{-0.20}$	1.83 $^{+0.20}_{-0.20}$	0.79 $^{+0.20}_{-0.20}$	0.99 $^{+0.20}_{-0.20}$	2.32 $^{+0.20}_{-0.20}$	0.50 $^{+0.20}_{-0.20}$	0.32 $^{+0.20}_{-0.20}$
$D_{\text{KL}}^{\text{uncond}}(a_j)$	0.16 $^{+0.02}_{-0.02}$	0.09 $^{+0.02}_{-0.02}$	0.17 $^{+0.02}_{-0.02}$	0.05 $^{+0.02}_{-0.02}$	0.05 $^{+0.02}_{-0.02}$	0.05 $^{+0.02}_{-0.02}$	0.05 $^{+0.02}_{-0.02}$	0.05 $^{+0.02}_{-0.02}$	0.19 $^{+0.02}_{-0.02}$	0.06 $^{+0.02}_{-0.02}$	0.03 $^{+0.02}_{-0.02}$
$D_{\text{KL}}^{\text{uncond}}(\chi_{\text{eff}}, a_j)$	0.29 $^{+0.05}_{-0.05}$	0.05 $^{+0.02}_{-0.02}$	0.13 $^{+0.02}_{-0.02}$	0.07 $^{+0.02}_{-0.02}$	0.06 $^{+0.02}_{-0.02}$	0.06 $^{+0.02}_{-0.02}$	0.06 $^{+0.02}_{-0.02}$	0.06 $^{+0.02}_{-0.02}$	0.19 $^{+0.02}_{-0.02}$	0.06 $^{+0.02}_{-0.02}$	0.03 $^{+0.02}_{-0.02}$
H SNR	20.6 $^{+1.2}_{-1.2}$	6.4 $^{+1.1}_{-1.1}$	9.8 $^{+1.2}_{-1.2}$	12.1 $^{+1.2}_{-1.2}$	5.9 $^{+1.2}_{-1.2}$	18.9 $^{+1.2}_{-1.2}$	10.7 $^{+1.2}_{-1.2}$	14.3 $^{+1.2}_{-1.2}$	26.3 $^{+1.2}_{-1.2}$	9.7 $^{+1.2}_{-1.2}$	9.2 $^{+1.2}_{-1.2}$
L SNR	14.2 $^{+1.1}_{-1.1}$	5.8 $^{+1.1}_{-1.1}$	6.9 $^{+1.1}_{-1.1}$	9.2 $^{+1.1}_{-1.1}$	8.3 $^{+1.1}_{-1.1}$	10.7 $^{+1.1}_{-1.1}$	10.7 $^{+1.1}_{-1.1}$	14.3 $^{+1.1}_{-1.1}$	26.3 $^{+1.1}_{-1.1}$	9.7 $^{+1.1}_{-1.1}$	9.2 $^{+1.1}_{-1.1}$
V SNR				1.7 $^{+1.1}_{-1.1}$	1.7 $^{+1.1}_{-1.1}$	4.1 $^{+1.1}_{-1.1}$	3.0 $^{+1.1}_{-1.1}$	4.2 $^{+1.1}_{-1.1}$			

Mockdata Comparison

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Mockdata preparation

SXS data + shifted ringdown injection + aLIGO noise
modified after t_merger (set A) 60 set
modified before/after t_merger (set B) 60 set

FIG. 1. Examples of sets A and B. (Inset) The ringdown part. Here, set A (red) is the SXS:BBH0174 and set B (blue) is the modified amplitude $A_{\text{mod}}(t)$, and the dashed lines are the GW frequency $\omega_{\text{ring}}(t)/2\pi$. The total mass is $M = 60M_\odot$, and the real and imaginary parts of the ringdown frequency are 300 and 40 Hz, respectively. The real frequency is obtained by multiplying by 538.609 Hz, and the real amplitude of set A is derived by dividing by 1.37903. The large difference in the inspiral phase is due to the difference of the binary parameters.

https://gw-genesis.scphys.kyoto-u.ac.jp/ilias/goto_root_fold_669.html
<http://www.oit.ac.jp/is/shinkai/mockdatachallenge/>

Method

Auto-Regressive model (idea)

Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \dots + a_M x_{n-M} + \epsilon$$

$$= \sum_{j=1}^M a_j x_{n-j} + \epsilon$$

e.g. $x_n = A e^{-\gamma n \Delta t} \cos(\omega n \Delta t)$

$$Z_1 = e^{-(\gamma - j\omega)\Delta t} \rightarrow x_n = \frac{A}{2} (Z_1^n + Z_2^n) = (Z_1 + Z_2)x_{n-1} - Z_1 Z_2 x_{n-2}$$

can be applied also to noisy data by adjusting M

Auto-Regressive model vs Short FFT

sampling rate=4096 segment = 1/64 sec = 64 points

shift = 1/512 sec = 8 points

The order M can be fixed at 2~8.

Even for short segment, AR model shows precise power-spectrum.

Auto-Regressive model (Method, general)

Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \dots + a_M x_{n-M} + \epsilon$$

$$= \sum_{j=1}^M a_j x_{n-j} + \epsilon$$

- find a_j (Burg method)
- find M (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

power spectrum

$$p(f) = \frac{\sigma^2}{|1 - \sum_{j=1}^M a_j e^{-j2\pi f \Delta t}|^2}$$

characteristic eq.

$$f(z) = 1 - \sum_{j=1}^M a_j z^j = 0$$

$|z_k|$ says amplitude, $\arg(z_k)$ says frequency.

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mockdata-challenge comparison

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Comparison of various methods to extract ringdown frequency from gravitational wave data

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ringdown search for 60 mockdata

TABLE III. We show the values of $\delta \log f_R$, $\sigma(f_R)$, $\delta \log f_I$, and $\sigma(f_I)$ for various methods. The results limited to set A are given on the first row of each method, while those limited to set B are on the second.

Method	Set	$\delta \log f_R$ (%)	$\sigma(f_R)$ (%)	$\delta \log f_I$ (%)	$\sigma(f_I)$ (%)
matched filtering	A	-12.88	28.36	-71.51	97.79
	B	-0.82	27.53	-46.11	75.48
Hilbert-Huan Transformation	A	6.25	17.27	-12.62	37.9
	B	2.47	10.41	7.18	27.61
Auto-Regression Method	A	-13.38	21.91	-44.11	61.58
	B	-8.08	19.81	-28.78	49.61
Neural Network method	A	0.2	9.93	4.88	38.75
	B	1.91	8.57	6.2	34.64
NN	A	-6.64	16.48	-15.23	33.96
	B	-6.65	11.97	9.96	23.76

Results

GW150914

LIGO paper

AR model Hanford

4096 sampling rate
150-450 Hz filter
1 segment = 1/64 sec = 64 points
1 shift = 1/512 sec = 8 points

GW170729

Hanford (SNR=5.9)

Virgo (SNR=1.7)

Livingston (SNR=8.3)

Kerr param

Mass

GW170809

Hanford (SNR=5.9)

Virgo (SNR=1.1)

Livingston (SNR=10.7)

Kerr param

Mass

GW170814

Hanford (SNR=9.3)

Virgo (SNR=4.1)

Livingston (SNR=14.3)

Kerr param

Mass

GW170818

Hanford (SNR=4.6)

Virgo (SNR=4.2)

Livingston (SNR=9.7)

Kerr param

Mass

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