Introduction to Sparse Modeling

Toward GW data analysis without templates

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- ★ The standard way for detecting GW signals from noisy data is the matched filtering technique. GR researchers prepare gravitational waveforms for that purposes.
- ★ But **how about the theory is differ from GR?**
  How about unknown wave sources?
- ★ **Remember the Pulsar was first detected as unknown radio sources 50 years ago.**
- ★ **Remember GRB was first detected as suspicious bomb.**
- ★ **Auto-regressive method/Sparse modeling**

Hisaaki Shinkai, ShinGakuJutsu BootCamp@Sendai, 2017/12/8-9 http://sparse-modeling.jp

Sparse Modeling

English　お問い合わせ　サイトマップ

文部科学省科学研究費補助金「新学術領域研究」平成25年度〜29年度

スパースモデリングの深化と高次元データ駆動科学の創成

Initiative for High-Dimensional Data-Driven Science through Deepening of Sparse Modeling

- ホーム
- 領域代表のあいさつ
- 領域概要
- 総括班
- 計画研究 >
- 公募研究 >
- 活動記録 >
- 技術支援
- 研究業績 >
- ニュースレター

News & Topics

2019.4.10　天文班(A02-3)・計測モデリング班(B01-1)が共に参加した国際プロジェクトが，ブラックホールシャドウの撮像に関するプレスリリースを世界６か所で同時に行いました．国内では国立天文台や統計数理研究所などが共同で発表を行いました．

2018.1.4　2018年3月31日(土)に東京大学小柴ホールにて開催される公開シンポジウム「データ駆動科学の深化と展開」のウェブページを開設しました．

2017.12.22　【速報】2018年3月31日(土)に，東京大学小柴ホールにて公開シンポジウム「データ駆動科学の深化と展開」が開催されます．

2017.11.20　2017年12月17日(日)-20(水)に，東京大学武田ホールにて「疎性モデリング」最終成果報告会が開催されます．

2017.7.21　中田公募班(A02)・非線形班(C01-1)・地球科学班(A02-1)・スパースモデリング班(B01-2)による領域内共同研究の成果に関するプレスリリースが海洋研究開発機構より発表されました．

文部科学省

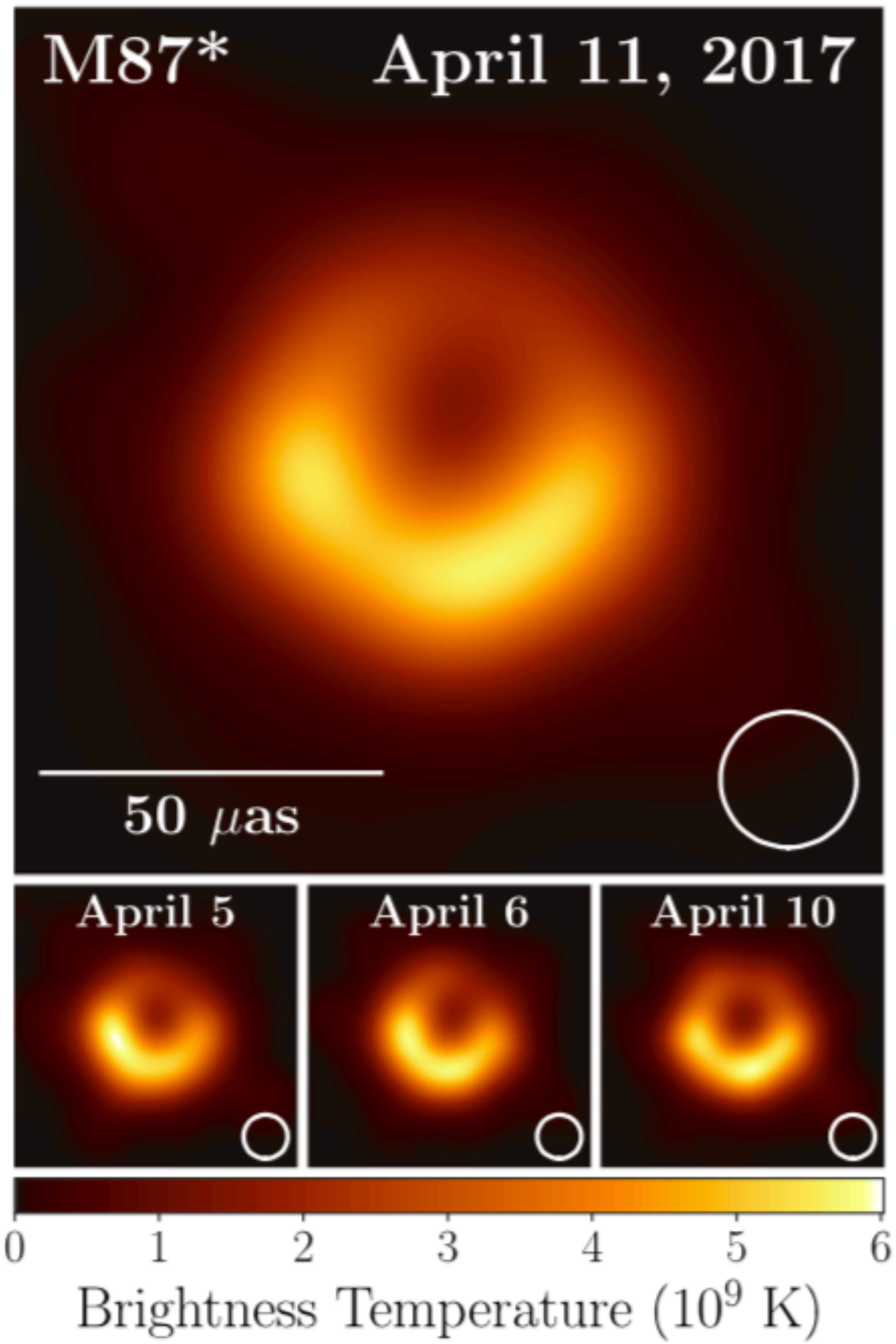
JSPS日本学術振興会

スクリ

First Image of a black hole : center of M87

地球から5500万光年

https://alma-telescope.jp/news/press/eht-201904



M=(6.5±0.7)x 10^9 Msun

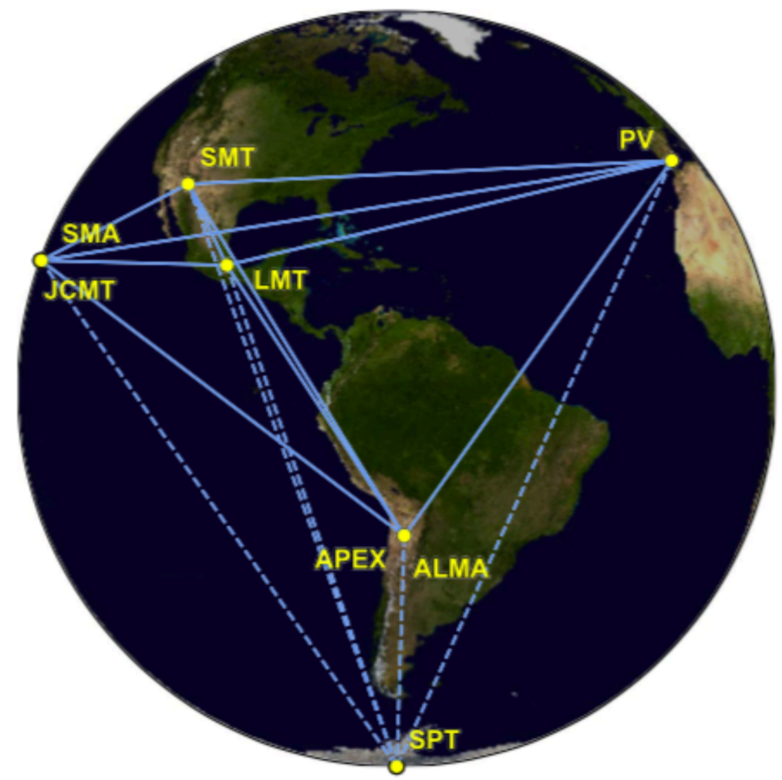


Figure 1. Eight stations of the EHT 2017 campaign over six geographic locations as viewed from the equatorial plane. Solid baselines represent mutual visibility on M87* (+12° declination). The dashed baselines were used for the calibration source 3C279 (see Papers III and IV).

eye sight
=3x10^6

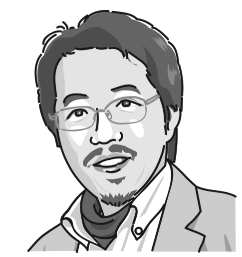
The Astrophysical Journal Letters, 875:L1 (17pp), 2019 April 10
https://doi.org/10.3847/2041-8213/ab0ec7

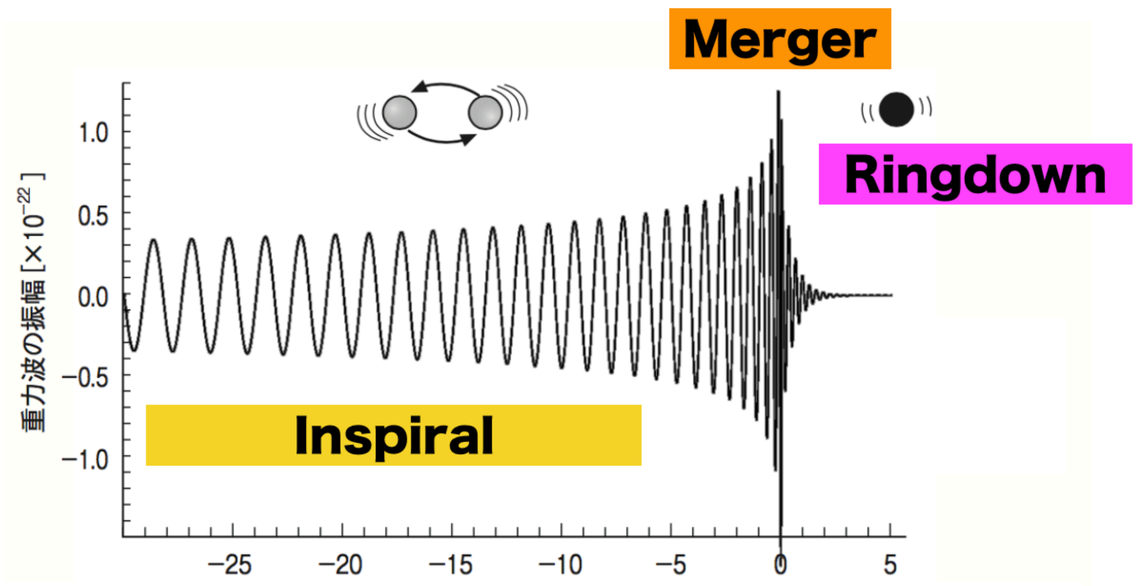
First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole

The Event Horizon Telescope Collaboration
(See the end matter for the full list of authors.)
Received 2019 March 1; revised 2019 March 12; accepted 2019 March 12; published 2019 April 10

Ring-down GW search using Auto-Regressive model

Hisaaki Shinkai (OIT)
真貝寿明（大阪工大）





AR model

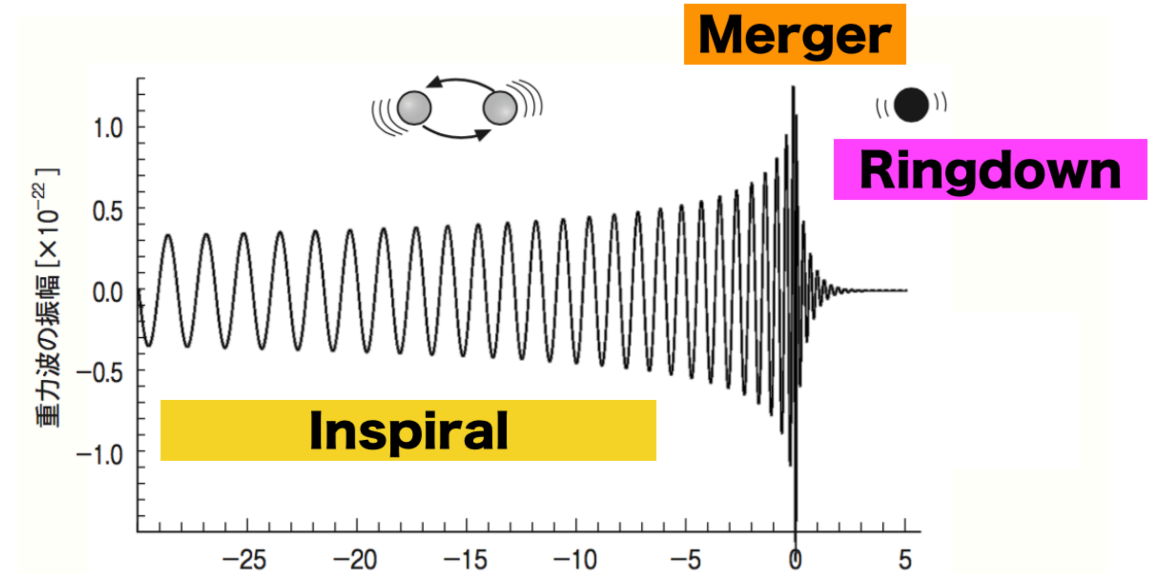
$$
\begin{aligned}
x_n &= a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \\
&= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon
\end{aligned}
$$

mockdata-challenge comparison

PHYSICAL REVIEW D **99**, 124032 (2019)

Comparison of various methods to extract ringdown frequency from gravitational wave data

Hiroyuki Nakano,[1,*] Tatsuya Narikawa,[2,3,†] Ken-ichi Oohara,[4,‡] Kazuki Sakai,[5,§] Hisa-aki Shinkai,[6,∥] Hirotaka Takahashi,[7,8,¶] Takahiro Tanaka,[3,9,**] Nami Uchikata,[2,4,††] Shun Yamamoto,[6] and Takahiro S. Yamamoto[3,‡‡]



ringdown search

60 mockdata

TABLE III. We show the values of $\overline{\delta \log f_R}$, $\sigma(f_R)$, $\overline{\delta \log f_I}$, and $\sigma(f_I)$ for various methods. The results limited to set A are given on the first law of each method, while those limited to set B are on the second.

		$\overline{\delta \log f_R}$(%)	$\sigma(f_R)$(%)	$\overline{\delta \log f_I}$(%)	$\sigma(f_I)$(%)
MF-R (matched filtering)	A	−12.88	28.36	−71.51	97.79
	B	−0.82	27.53	−46.11	75.48
MF-MR	A	6.25 (Very good)	17.27 (Good)	−12.62 (Very good)	37.9 (Very good)
	B	2.47	10.41	7.18	27.61
HHT (Hilbert-Huan Transformation)	A	−13.38	21.91	−44.11	61.58
	B	−8.08	19.81	−28.78	49.61
AR (Auto-Regression Method)	A	0.2 (Excellent)	9.93 (Excellent)	4.88 (Excellent)	38.75 (Good)
	B	1.91	8.57	6.2	34.64
NN (Neural Network method)	A	−6.64 (Good)	16.48 (Very good)	−15.23 (Good)	33.96 (Excellent)
	B	−6.65	11.97	9.96	23.76

Auto-Regressive model (idea)

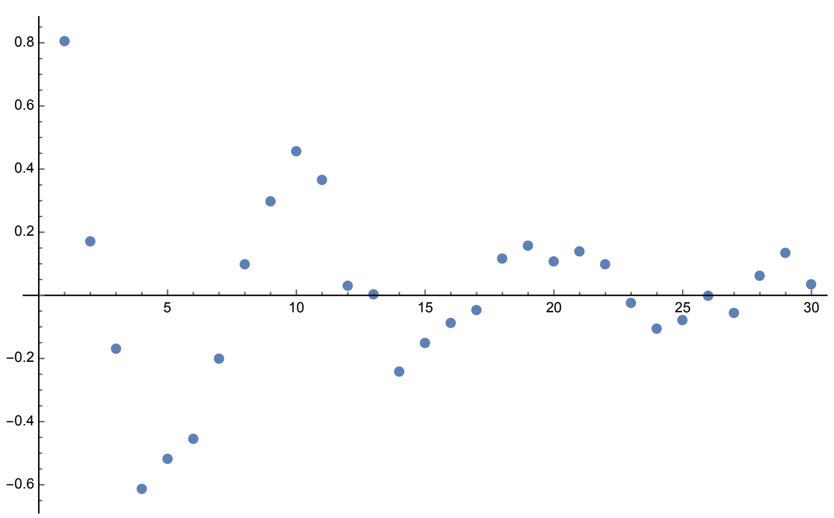
Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon$$
$$= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

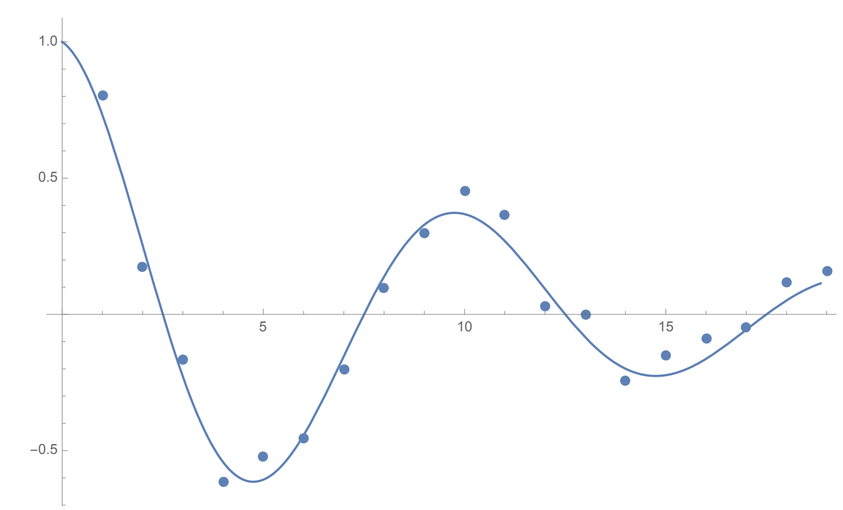
e.g. $x_n = A e^{-rn\Delta t} \cos(\omega n \Delta t)$

$$Z_1 = e^{-(r-j\omega)\Delta t}$$
$$Z_2 = e^{-(r+j\omega)\Delta t}$$

$$\rightarrow \quad x_n = \frac{A}{2}(Z_1^n + Z_2^n) = (Z_1 + Z_2) x_{n-1} - Z_1 Z_2 x_{n-2}$$



→



can be applied also to noisy data by adjusting *M*

Auto-Regressive model (Method, general)

Fitting data with linear func.

$$\begin{aligned} x_n &= a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon \\ &= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon \end{aligned}$$

- find a_j (Burg method)
- find M (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

power spectrum

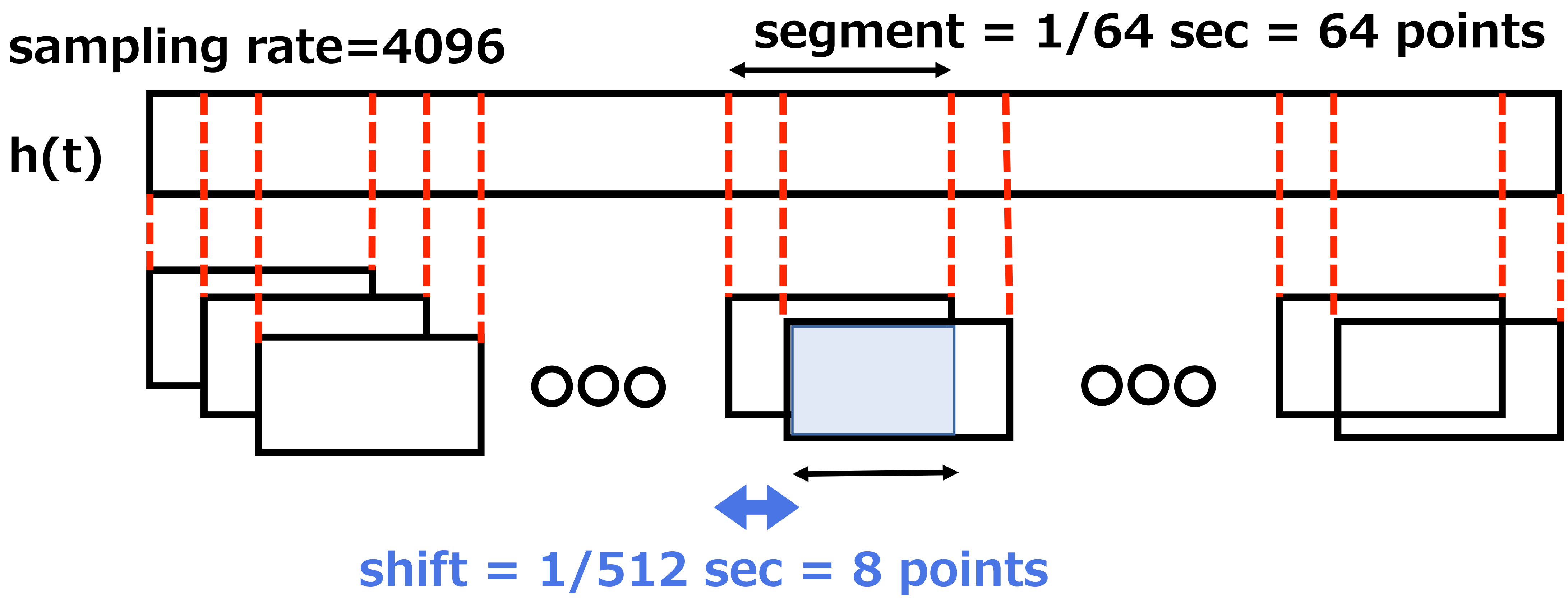
$$p(f) = \frac{\sigma^2}{\left| 1 - \sum_{j=1}^{M} a_j e^{-I 2\pi j f \Delta t} \right|^2}$$

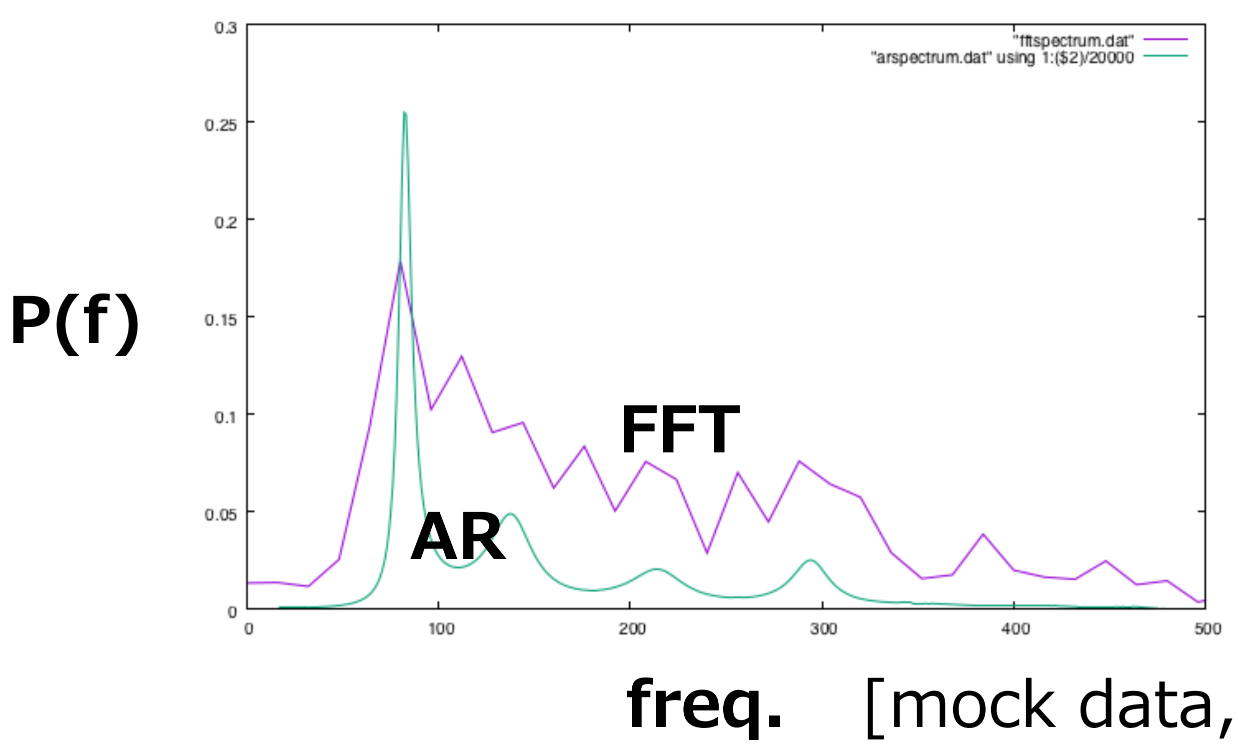
characteristic eq.

$$f(z) = 1 - \sum_{j=1}^{M} a_j z^j = 0$$

$|z_k|$ says amplitude,
$\arg(z_k)$ says frequency.

Auto-Regressive model vs Short FFT





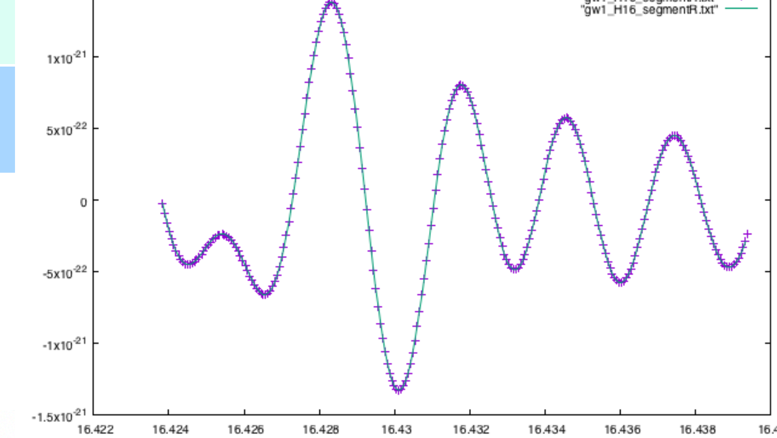
[mock data, SNR=40, inspiral part]

The order *M* can be fixed at 2〜8.

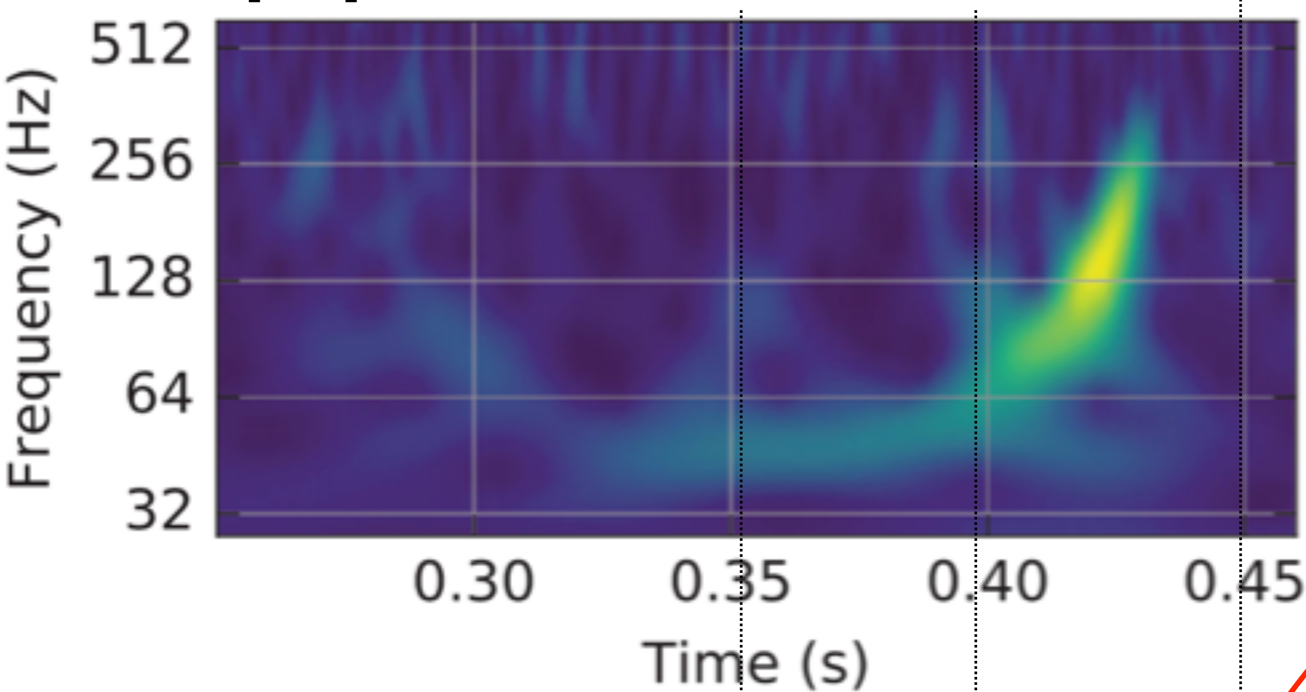
Even for short segment, AR model shows precise power-spectrum.

Ringdown Search by Auto-Regressive Approach (O1/O2 public data, GWTC1 catalogue)

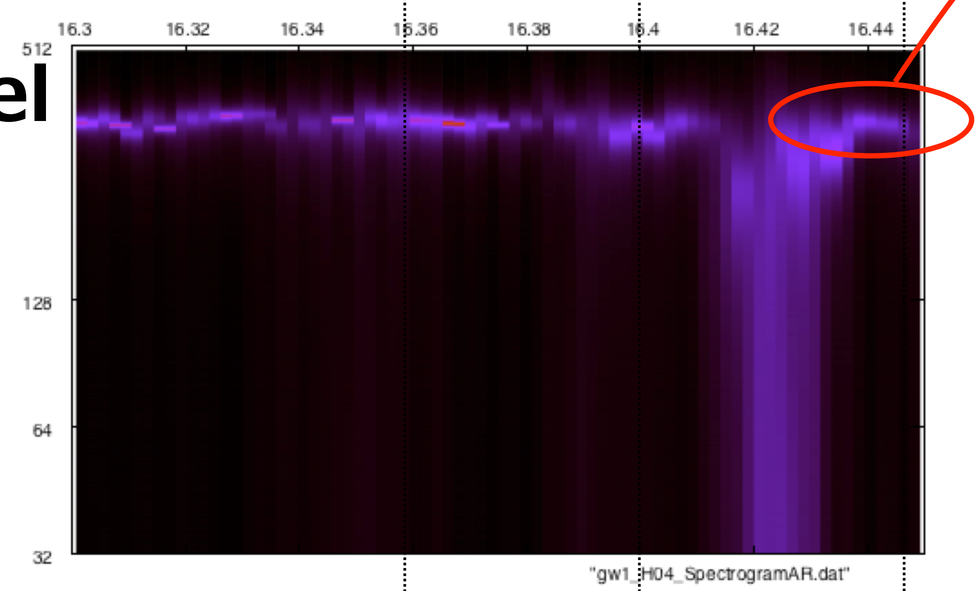
GW150914



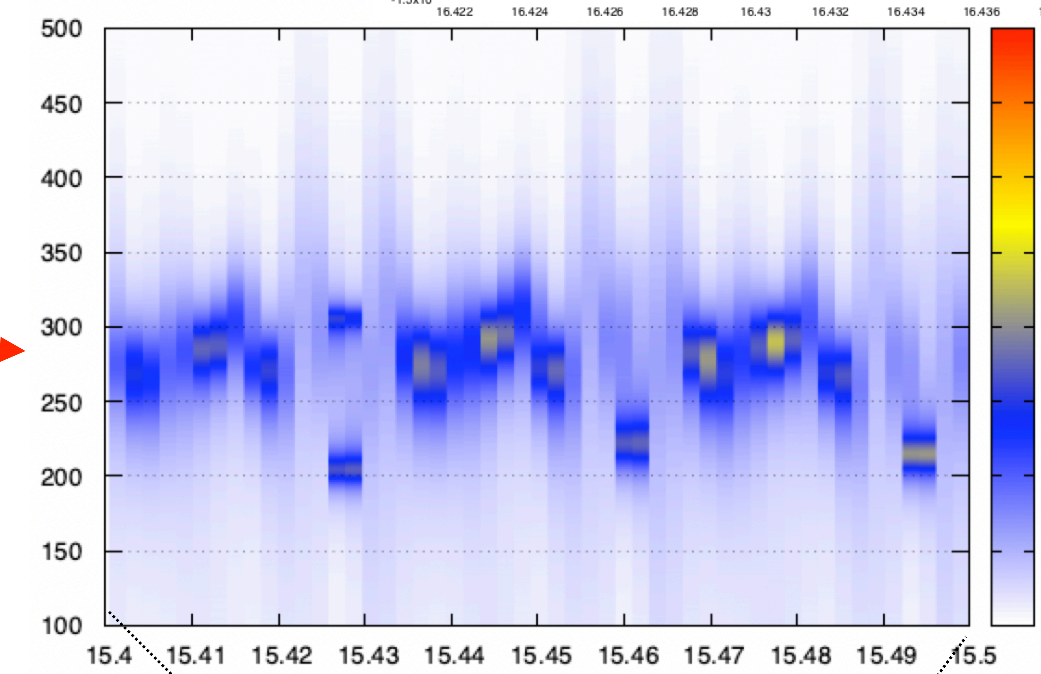
LIGO paper



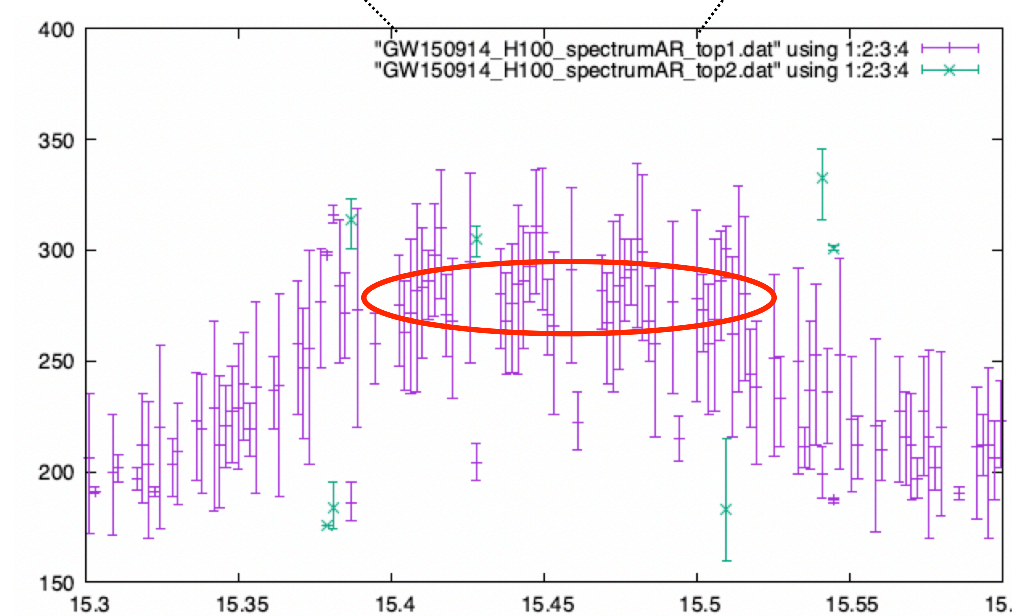
AR model Hanford



freq [Hz]



▲merger time



$f_{220} = 249.4$ Hz, $f_{221} = 244.0$ Hz, $f_{222} = 233.7$ Hz

$f_{210} = 349.3$ Hz, $f_{211} = 207.1$ Hz, $f_{200} = 231.9$ Hz

$f_{330} = 395.3$ Hz, $f_{331} = 392.1$ Hz, $f_{332} = 386.3$ Hz

$f_{320} = 355.9$ Hz, $f_{310} = 322.1$ Hz, $f_{300} = 293.9$ Hz

4096 sampling rate

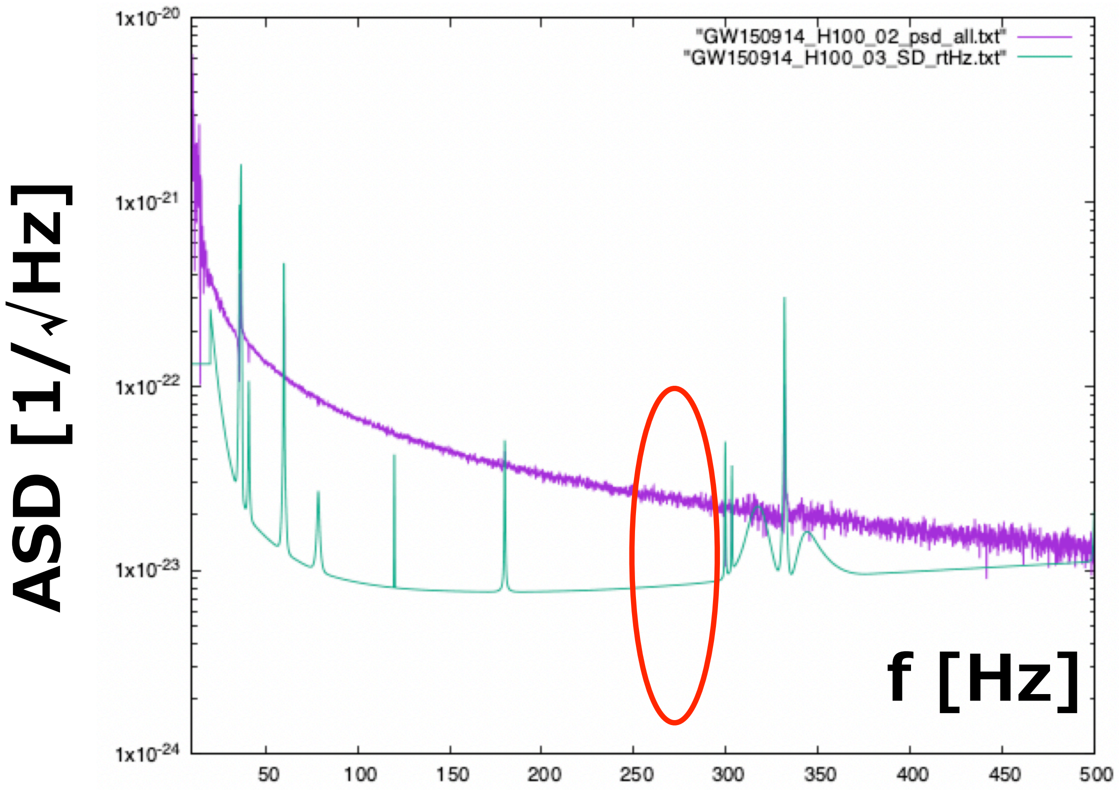
150-450 Hz filter

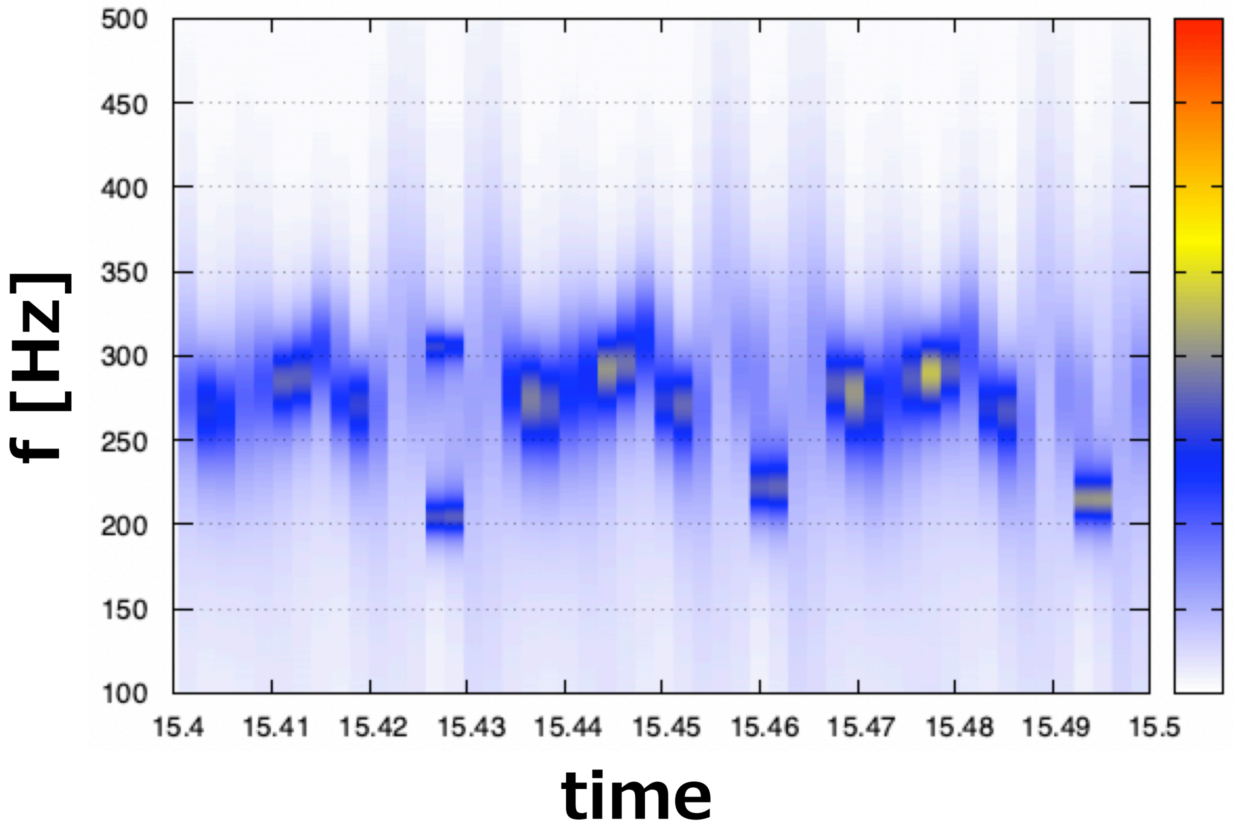
1 segment = 1/64 sec= 64 points

1 shift = 1/512 sec = 8 points

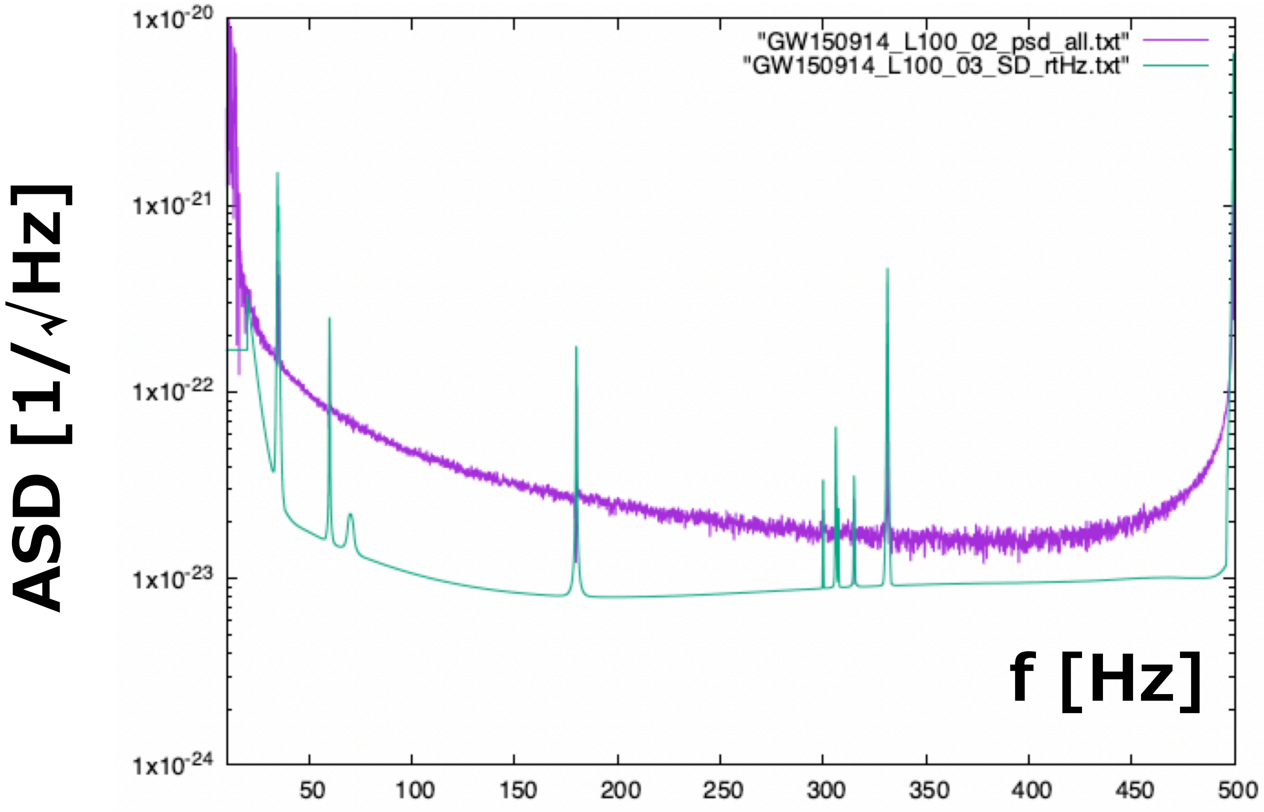
GW150914

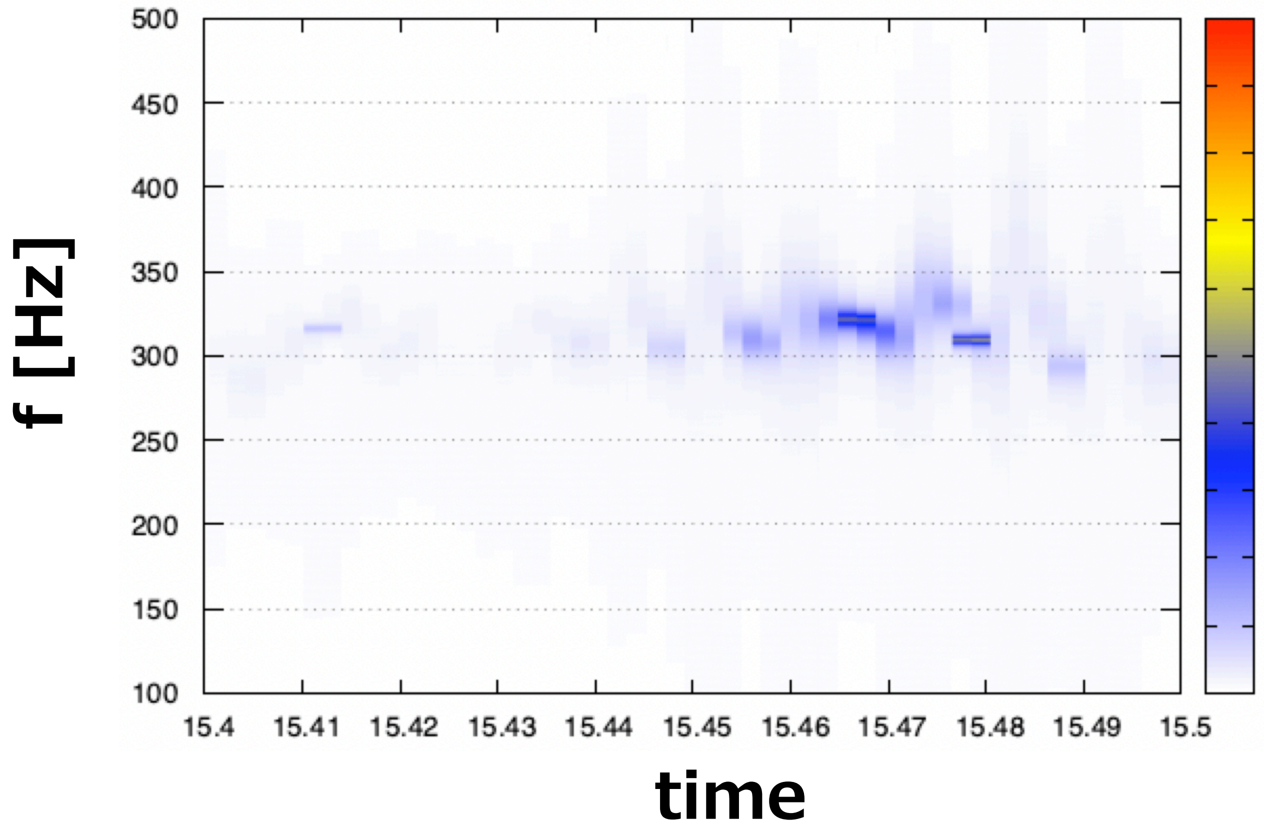
Hanford (SNR=20.6)





Livingston (SNR=14.2)





Ringdown Search by Auto-Regressive Approach (O1/O2 public data, GWTC1 catalogue)

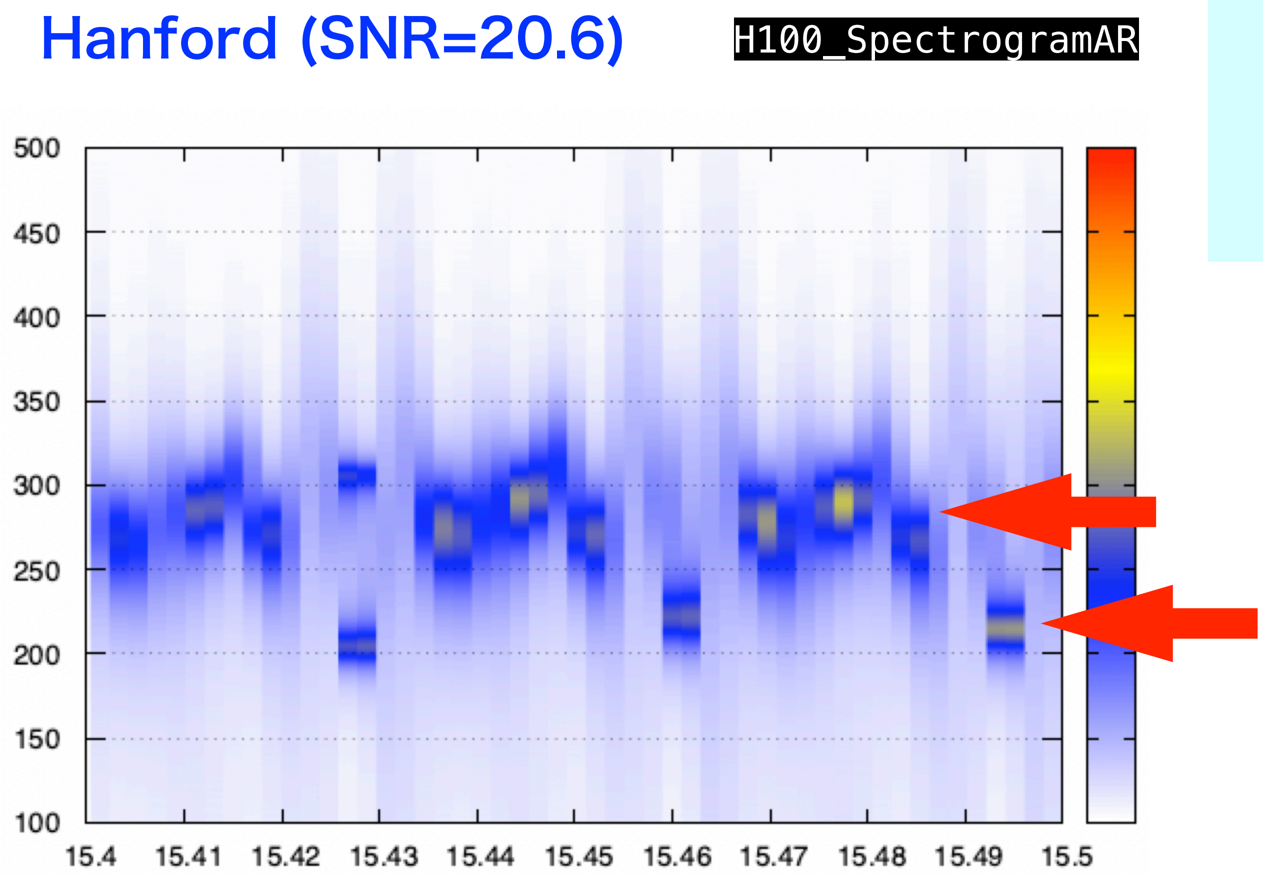
GW150914

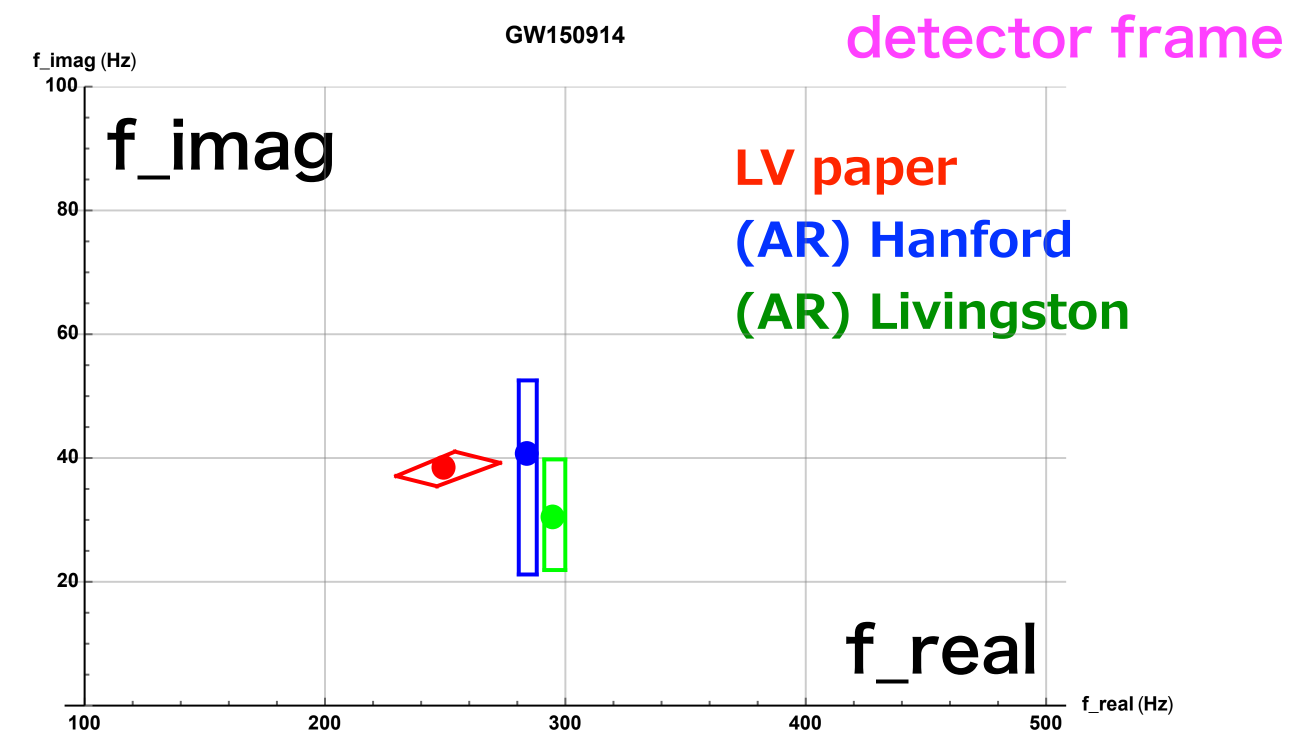
LV paper ▶ $(M, a, z) = (63.1^{+3.4}_{-3.0}, 0.69^{+0.05}_{-0.04}, 0.09^{+0.03}_{-0.03})$

f_{QNM} ▶

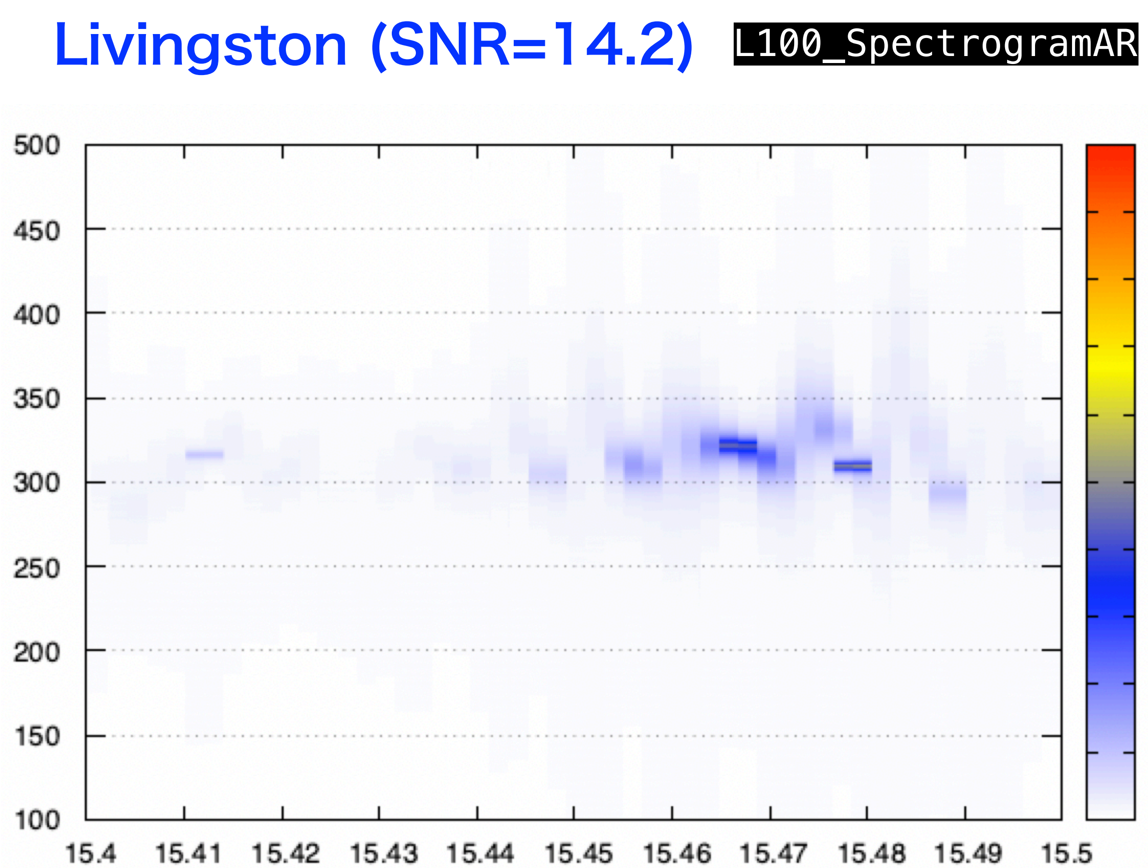
$f_{220} = 249.4$ Hz, $f_{221} = 244.0$ Hz, $f_{222} = 233.7$ Hz
$f_{210} = 349.3$ Hz, $f_{211} = 207.1$ Hz, $f_{200} = 231.9$ Hz
$f_{330} = 395.3$ Hz, $f_{331} = 392.1$ Hz, $f_{332} = 386.3$ Hz
$f_{320} = 355.9$ Hz, $f_{310} = 322.1$ Hz, $f_{300} = 293.9$ Hz

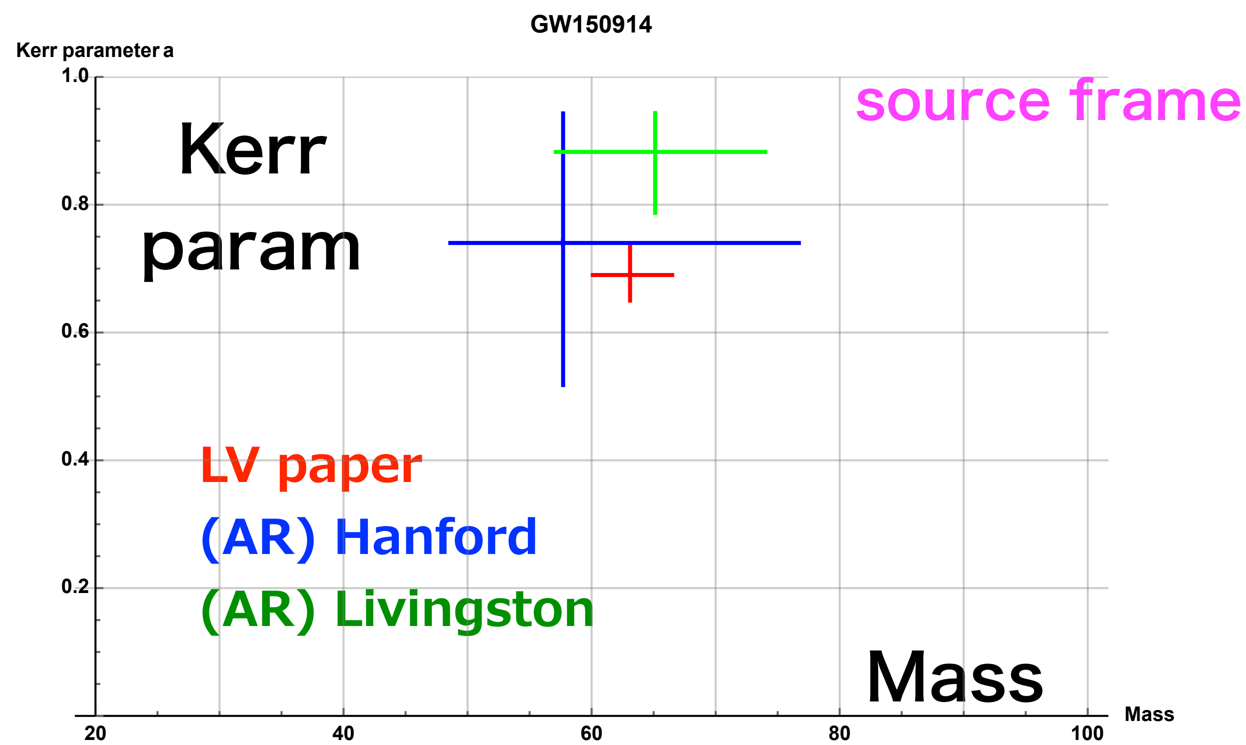
Hanford (SNR=20.6)





Livingston (SNR=14.2)

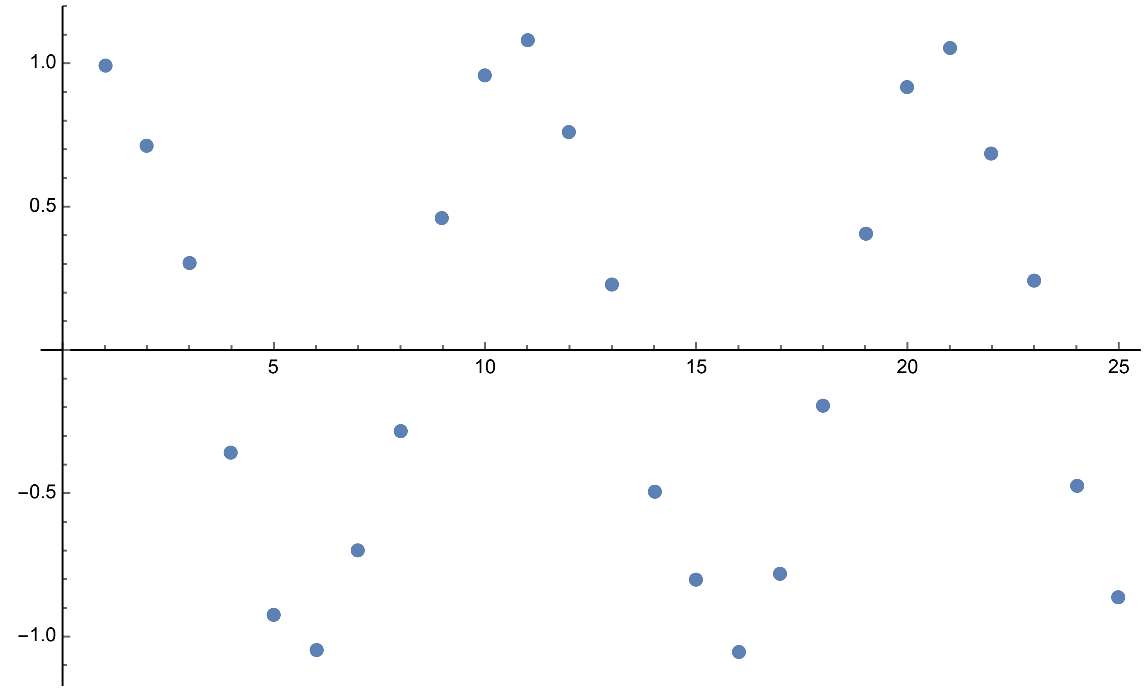




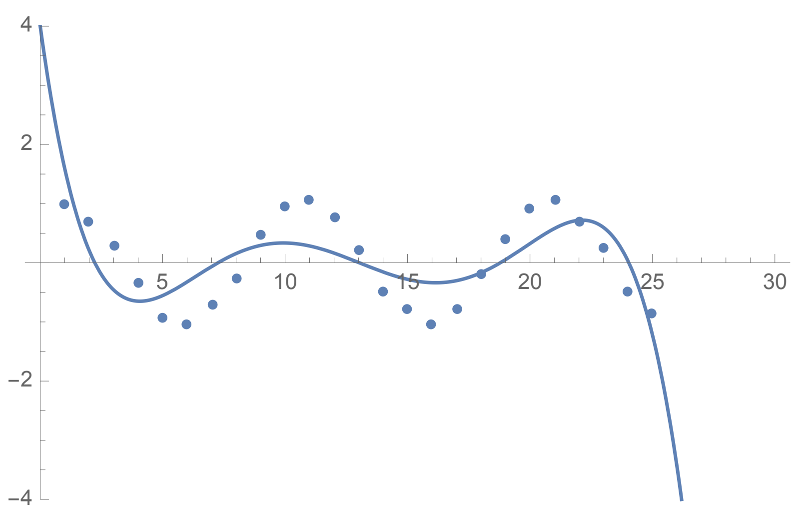
Sparse modeling (疎性モデリング)　introduction

```
Sin[i Pi/5 + Pi/3]
 + RandomReal[{-0.1, 0.1}]
```

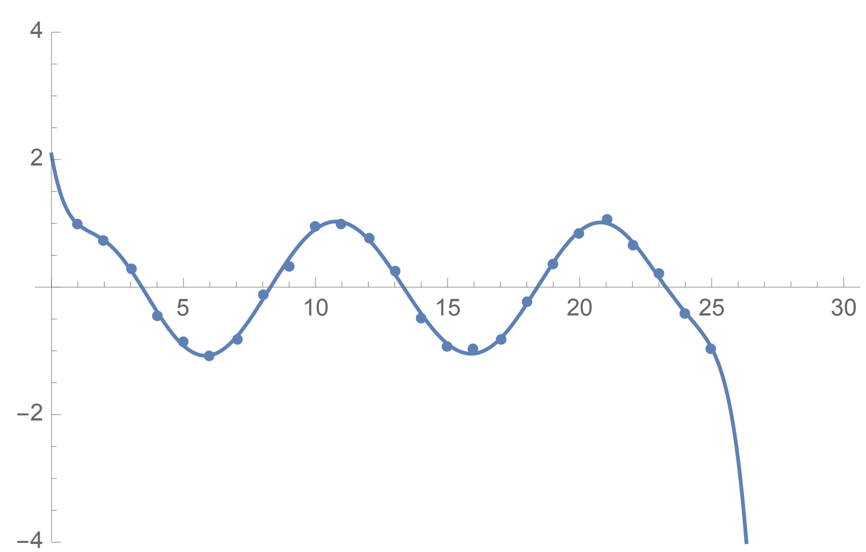
25 pt data



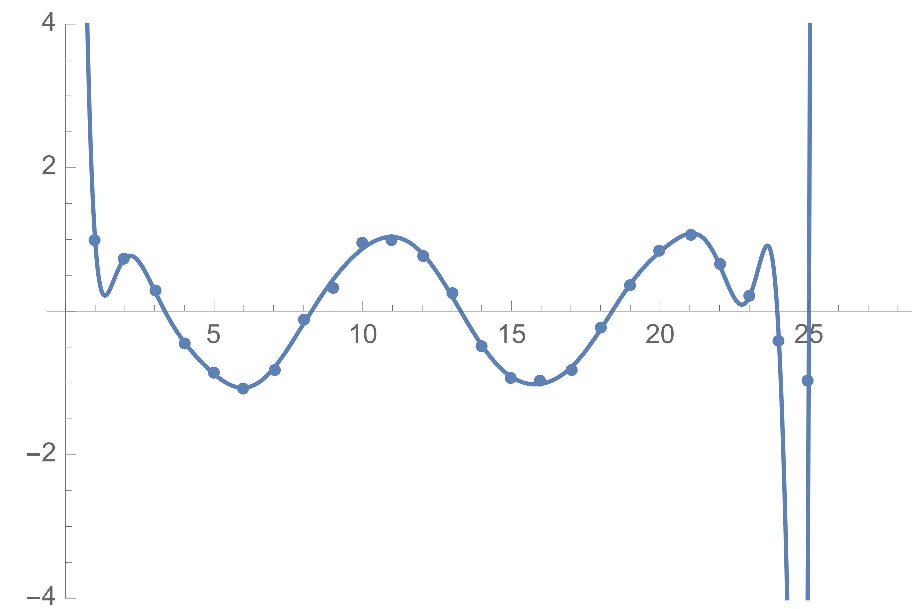
fitting up to x^5



fitting up to x^10



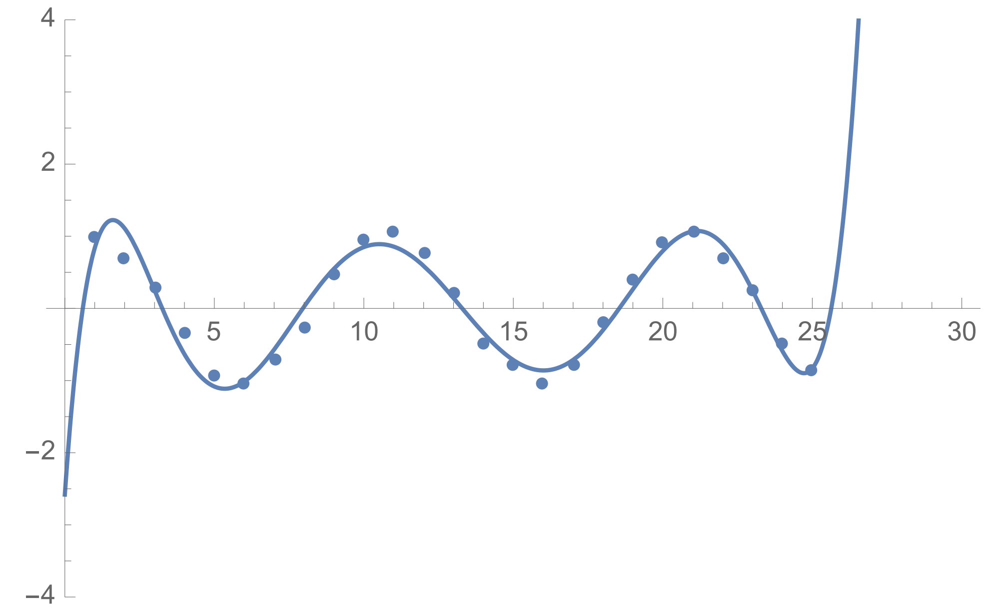
fitting up to x^25



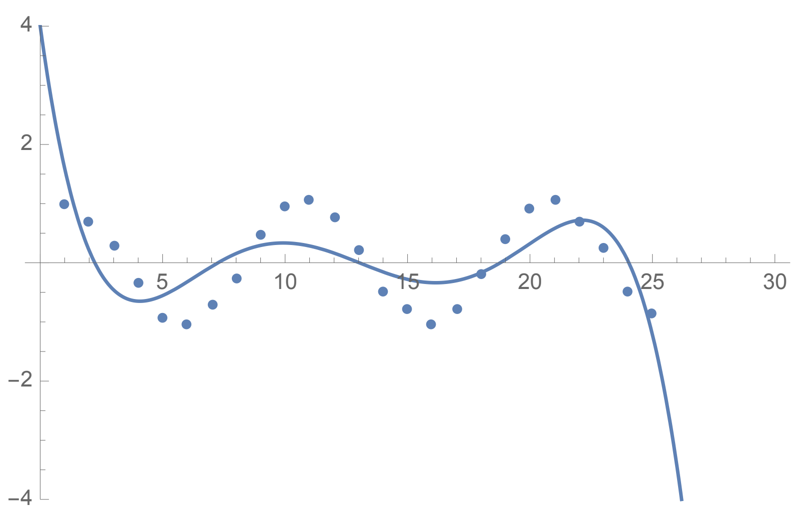
$$53.7899 - 151.583\,x + 178.296\,x^2 - 115.918\,x^3 + 46.9704\,x^4 - 12.6065\,x^5 + 2.30156\,x^6 - 0.28538\,x^7 + 0.0230585\,x^8 - 0.00104673\,x^9 + 8.43045\times10^{-6}\,x^{10} + 1.54191\times10^{-6}\,x^{11} - 4.25305\times10^{-8}\,x^{12} - 2.23891\times10^{-9}\,x^{13} + 7.47783\times10^{-11}\,x^{14} + 4.10585\times10^{-12}\,x^{15} - 9.09328\times10^{-14}\,x^{16} - 8.04044\times10^{-15}\,x^{17} + 4.79003\times10^{-17}\,x^{18} + 1.46216\times10^{-17}\,x^{19} + 1.02793\times10^{-19}\,x^{20} - 2.5521\times10^{-20}\,x^{21} - 2.74335\times10^{-22}\,x^{22} + 5.38566\times10^{-23}\,x^{23} - 1.39172\times10^{-24}\,x^{24} + 1.17278\times10^{-26}\,x^{25}$$

fitting up to x^7

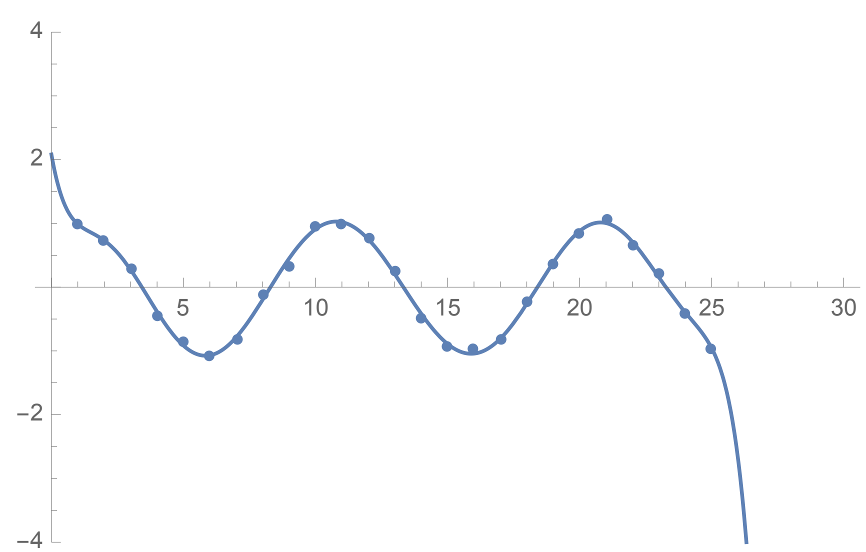
$$-2.58003 + 5.95476\,x - 3.14124\,x^2 + 0.668408\,x^3 - 0.0698719\,x^4 + 0.0038123\,x^5 - 0.000104163\,x^6 + 1.12509\times10^{-6}\,x^7$$



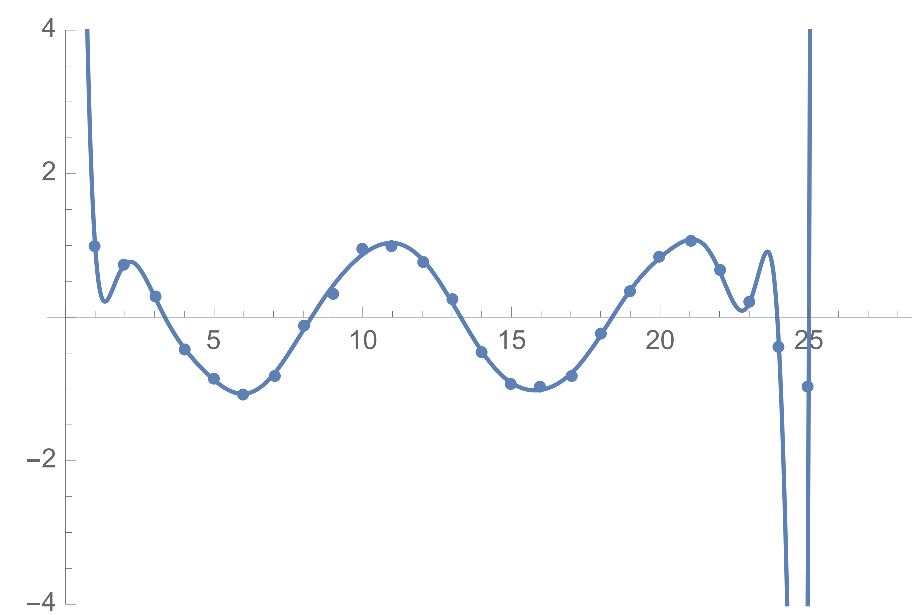
fitting up to x^5



fitting up to x^10



fitting up to x^25



$$53.7899 - 151.583\,x + 178.296\,x^2 - 115.918\,x^3 + 46.9704\,x^4 - 12.6065\,x^5 + 2.30156\,x^6 - 0.28538\,x^7 + 0.0230585\,x^8 - 0.00104673\,x^9 + 8.43045\times10^{-6}\,x^{10} + 1.54191\times10^{-6}\,x^{11} - 4.25305\times10^{-8}\,x^{12} - 2.23891\times10^{-9}\,x^{13} + 7.47783\times10^{-11}\,x^{14} + 4.10585\times10^{-12}\,x^{15} - 9.09328\times10^{-14}\,x^{16} - 8.04044\times10^{-15}\,x^{17} + 4.79003\times10^{-17}\,x^{18} + 1.46216\times10^{-17}\,x^{19} + 1.02793\times10^{-19}\,x^{20} - 2.5521\times10^{-20}\,x^{21} - 2.74335\times10^{-22}\,x^{22} + 5.38566\times10^{-23}\,x^{23} - 1.39172\times10^{-24}\,x^{24} + 1.17278\times10^{-26}\,x^{25}$$

fitting the data with noise
=> we do not need a fitting function which passes all the data
(overfitting, 過適合)

=> rather we should find a fitting as it has more zero-components

signal　redundant dictionary　data

$$\begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_m \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & \ldots & A_{1n} \\ A_{21} & A_{22} & \ldots & A_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1} & A_{m2} & \ldots & A_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{pmatrix} \qquad (m < n), \quad \mathrm{rank}(A) = \min(m, n)$$

with many zero components
=> minimize

$$||\boldsymbol{x}||_0 \equiv |\mathrm{supp}(\boldsymbol{x})| \equiv |\{i = \{1, 2, \cdots, n\} : x_i \neq 0\}|$$

LASSO

Linear regression problem.

▼ error

$$\boldsymbol{y_i} = \boldsymbol{\beta_1 x_i^{(1)}} + \boldsymbol{\beta_2 x_i^{(2)}} + \cdots + \boldsymbol{\beta_p x_i^{(p)}} + \boldsymbol{\varepsilon_i} \tag{1}$$

Find β, which minimize

$$\min_\beta \quad \sum_{i=1}^{N} \left(y_i - \sum_{j=1}^{p} \beta_j x_i^{(j)} \right)^2 \tag{2}$$

and also with sparse β. ◀ **sparse = as much zero-components**

LASSO (least absolute shrinkage and selection operator)

Tibshirani (1996)

$$\min_\beta \quad \sum_{i=1}^{N} \left(y_i - \sum_{j=1}^{p} \beta_j x_i^{(j)} \right)^2 \qquad \text{subject to} \quad \sum_{i=1}^{N} |\beta_j| \leq t$$

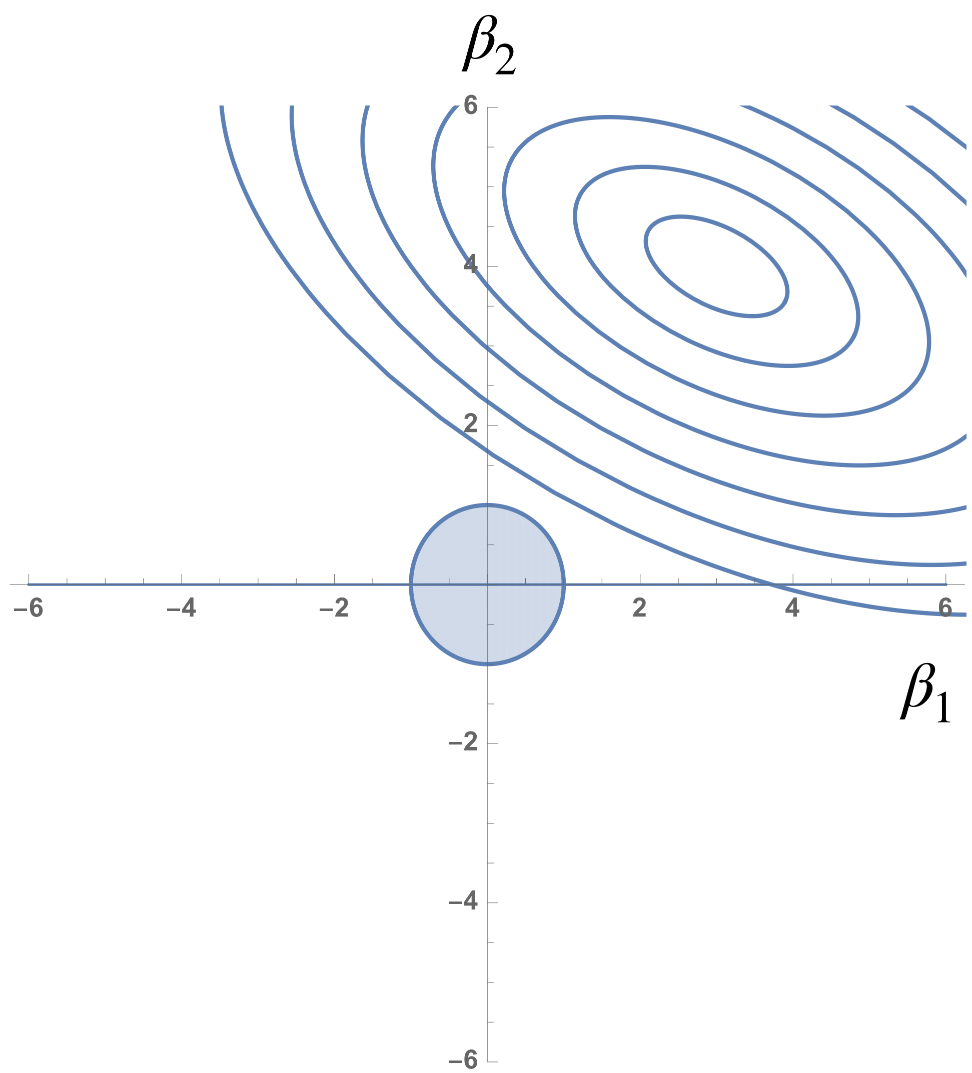
which is equivalent with finding β which minimize the following equation:

$$\min_\beta \quad \left[\sum_{i=1}^{N} \left(y_i - \sum_{j=1}^{p} \beta_j x_i^{(j)} \right)^2 + \lambda \sum_{i=1}^{N} |\beta_j| \right] = \quad \min_\beta \quad \left[\left\| \boldsymbol{y} - \sum_{j=1}^{p} \beta_j \boldsymbol{x}^{(j)} \right\|_2^2 + \lambda \, ||\boldsymbol{\beta}||_1 \right]$$

▲ **L1-norm** ▲ **L1-norm**

Why L1-norm constraint makes sparse solution?

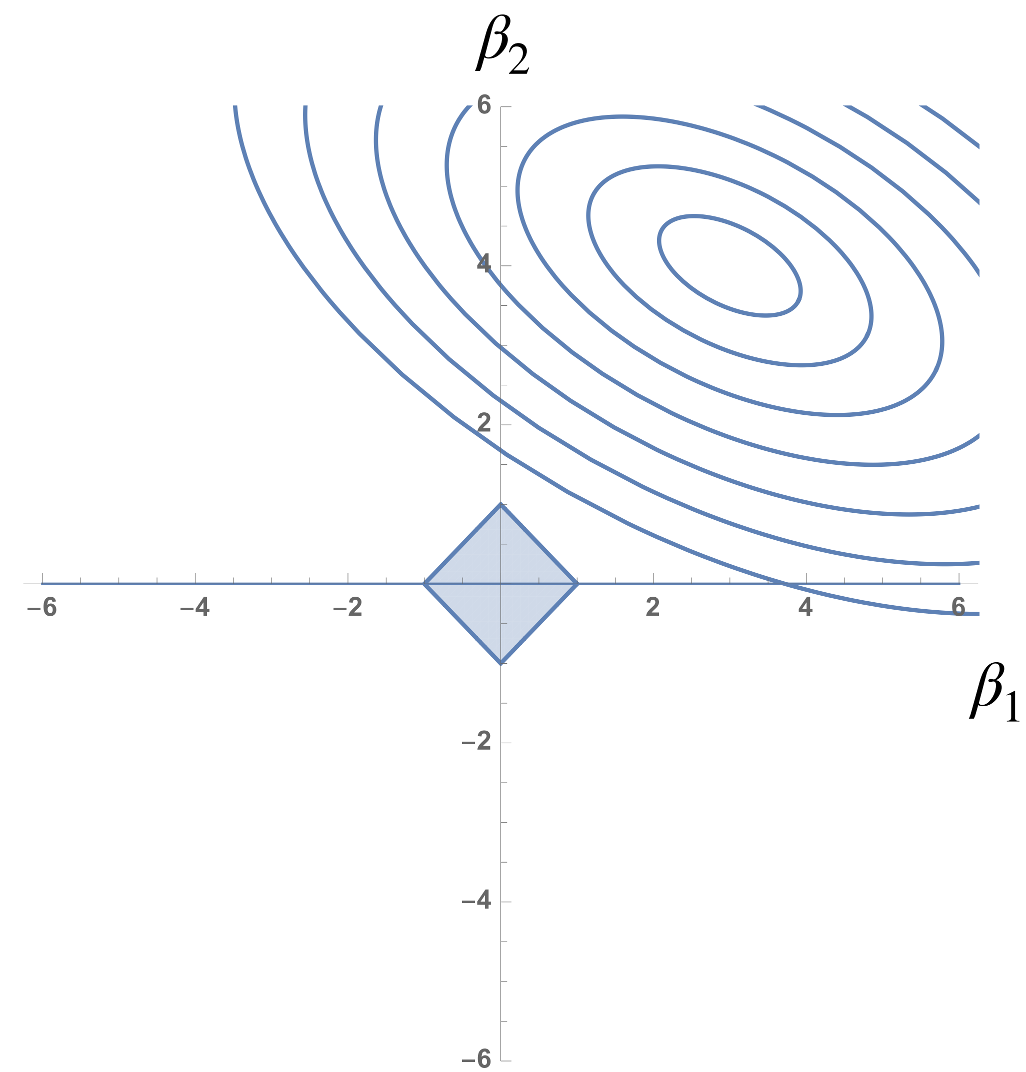
Rigde Regression



$$\min_\beta \quad \left[\left\| \boldsymbol{y} - \sum_{j=1}^{p} \beta_j \boldsymbol{x}^{(j)} \right\|_2^2 + \lambda \, ||\boldsymbol{\beta}||_2 \right]$$

▲ L2-norm

LASSO



$$\min_\beta \quad \left[\left\| \boldsymbol{y} - \sum_{j=1}^{p} \beta_j \boldsymbol{x}^{(j)} \right\|_2^2 + \lambda \, ||\boldsymbol{\beta}||_1 \right]$$

▲ L1-norm

LASSO variations

- elestic net = LASSO + Ridge Regression

$$\min_\beta \quad \left[\left\| \boldsymbol{y} - \sum_{j=1}^{p} \beta_j \boldsymbol{x}^{(j)} \right\|_2^2 + \lambda \left((1-\alpha) \, ||\boldsymbol{\beta}||_1 + \alpha \, ||\boldsymbol{\beta}||_2 \right) \right] \tag{1}$$

- fused LASSO (for time series data)

$$\min_\beta \quad \left[\sum_{i=1}^{N} (y_i - \beta_i)^2 + \lambda \sum_{i=1}^{N-1} |\beta_{i+1} - \beta_i| \right] \tag{2}$$

- Classification problem: Set the logistic regression for probability function

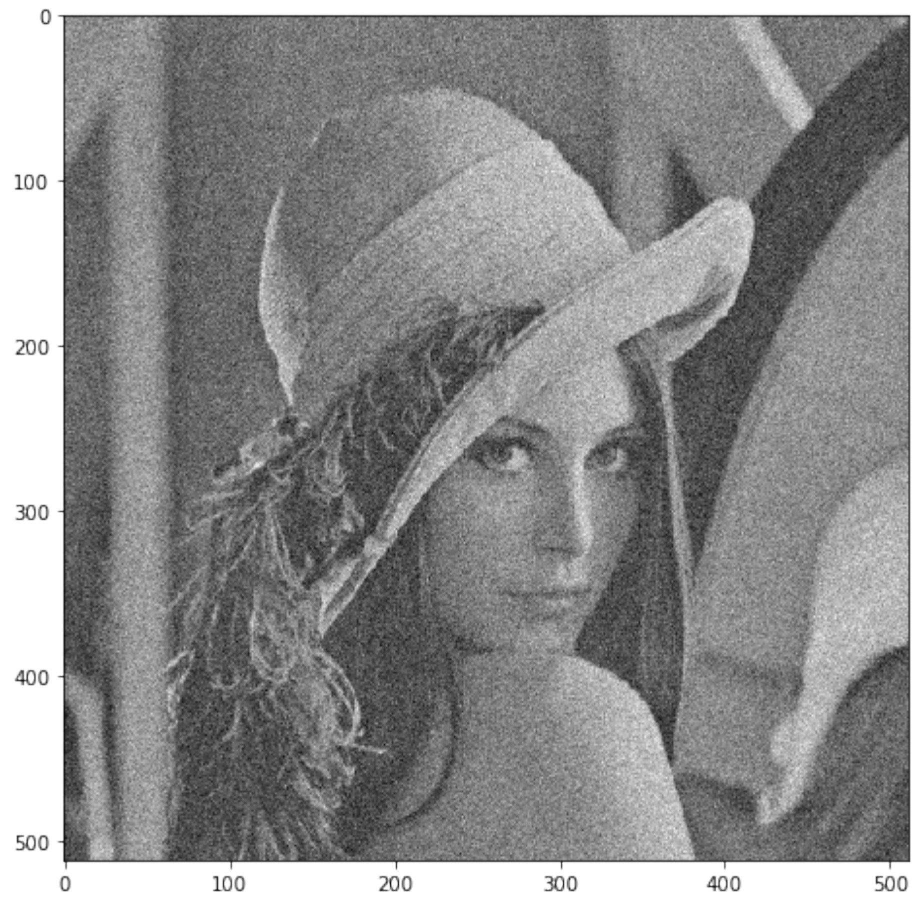
$$p(y \mid \boldsymbol{x}; \boldsymbol{\beta}, \beta_0) = \frac{\exp\left[y(\beta_0 + \sum_j \beta_j \boldsymbol{x}^{(j)}) \right]}{1 + \exp\left[\beta_0 + \sum_j \beta_j \boldsymbol{x}^{(j)} \right]} \tag{3}$$

and consider

$$\min_\beta \quad \sum_{i=1}^{N} \left[-\log p_i(y_i \mid \boldsymbol{x}_i; \boldsymbol{\beta}, \beta_0) + \lambda ||\boldsymbol{\beta}||_1 \right] \tag{4}$$

Exercise of Noise Removing

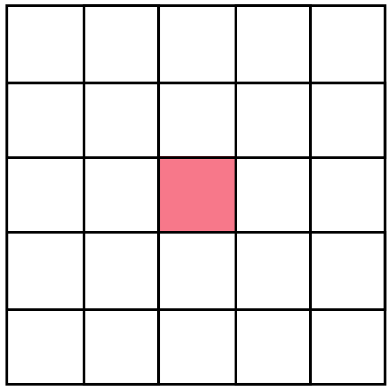
original

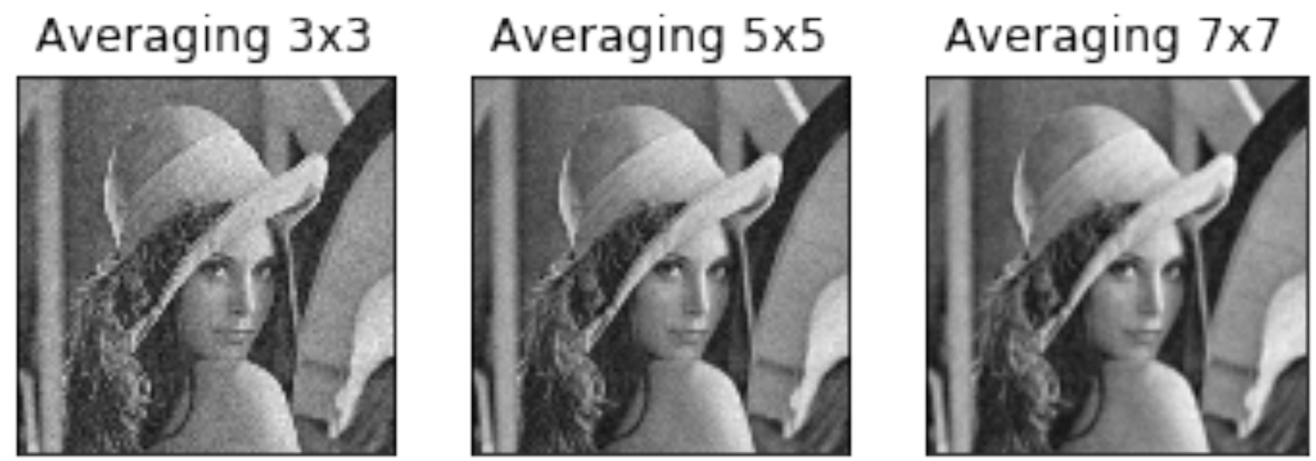


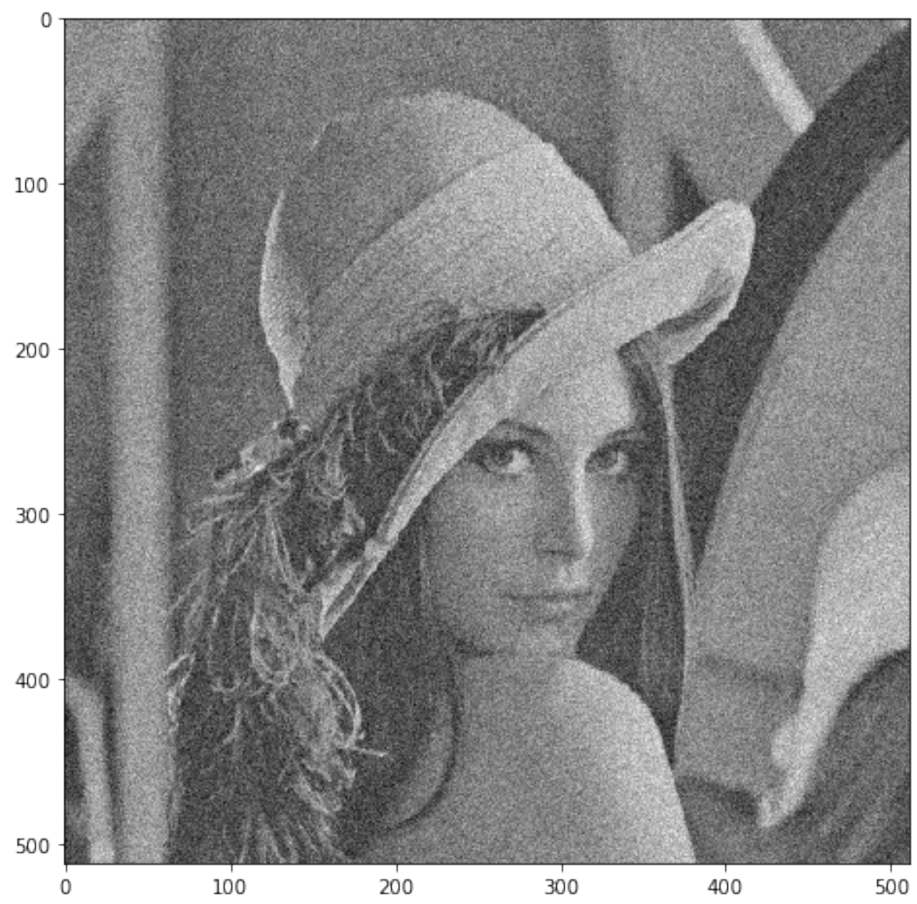
noised

Exercise of Noise Removing

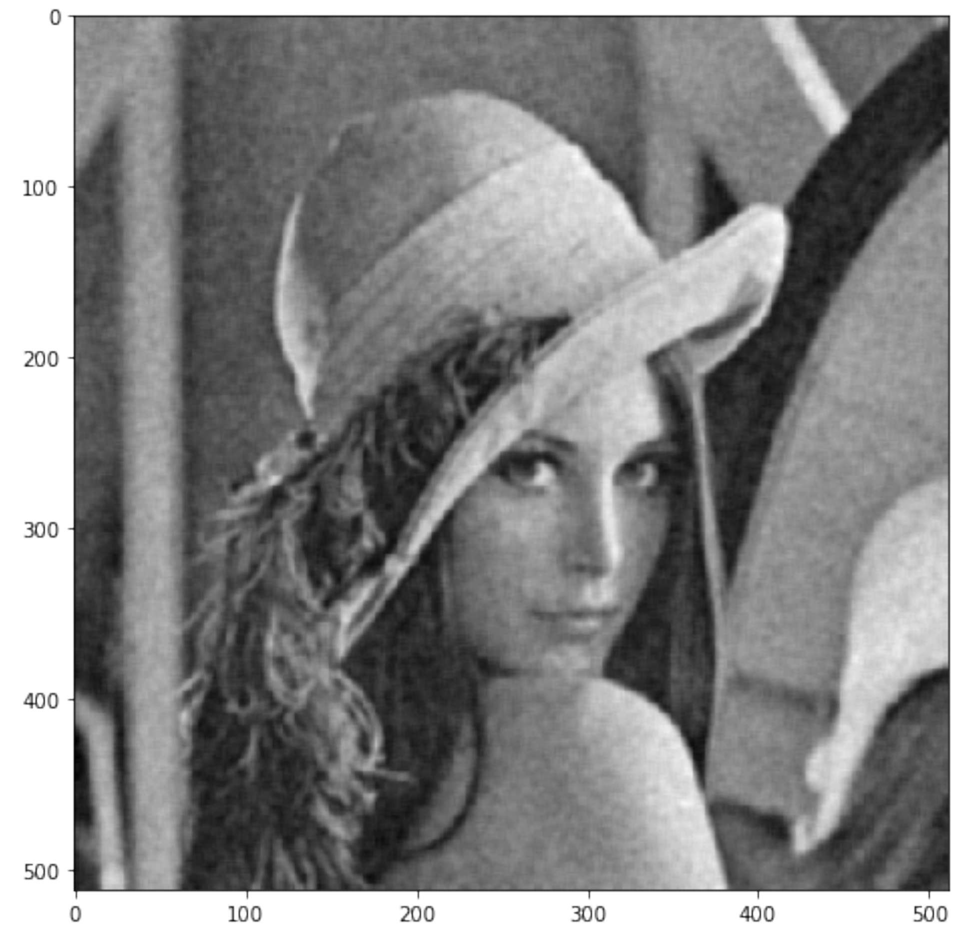
Mean Blur







noised

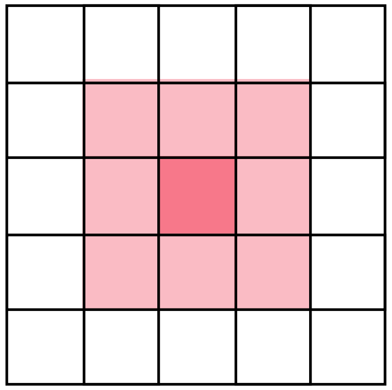


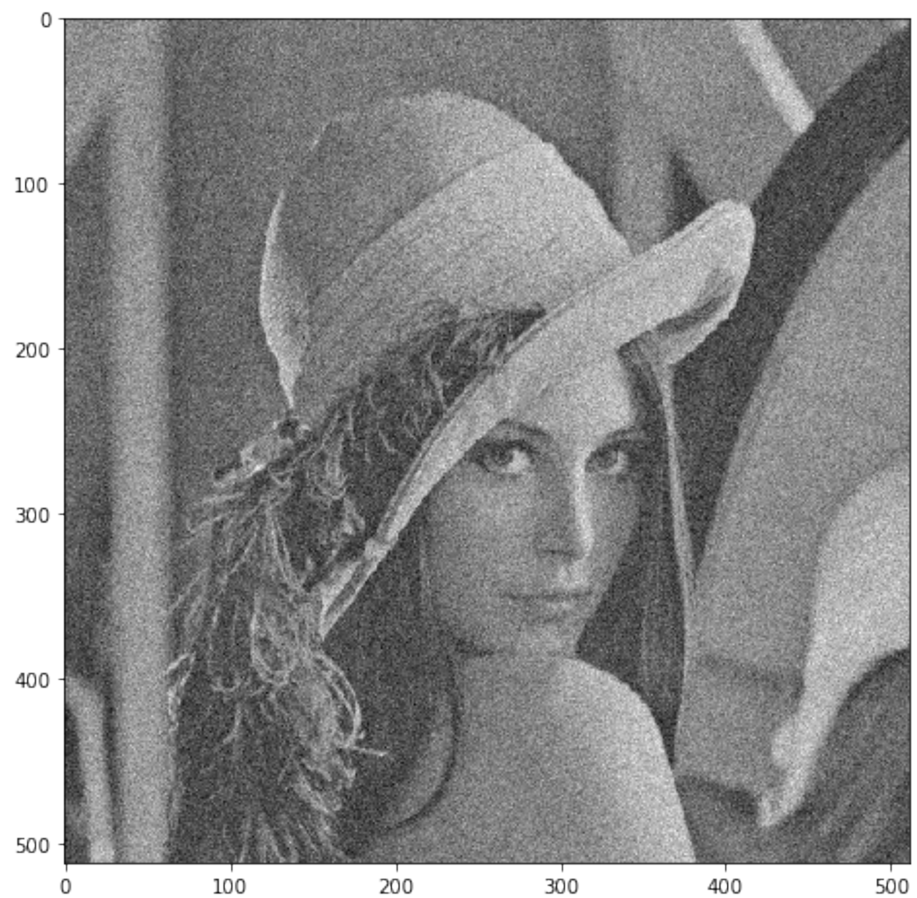
*** remove small scale noises (Low-Pass filter)**

*** Edge is smoothed**

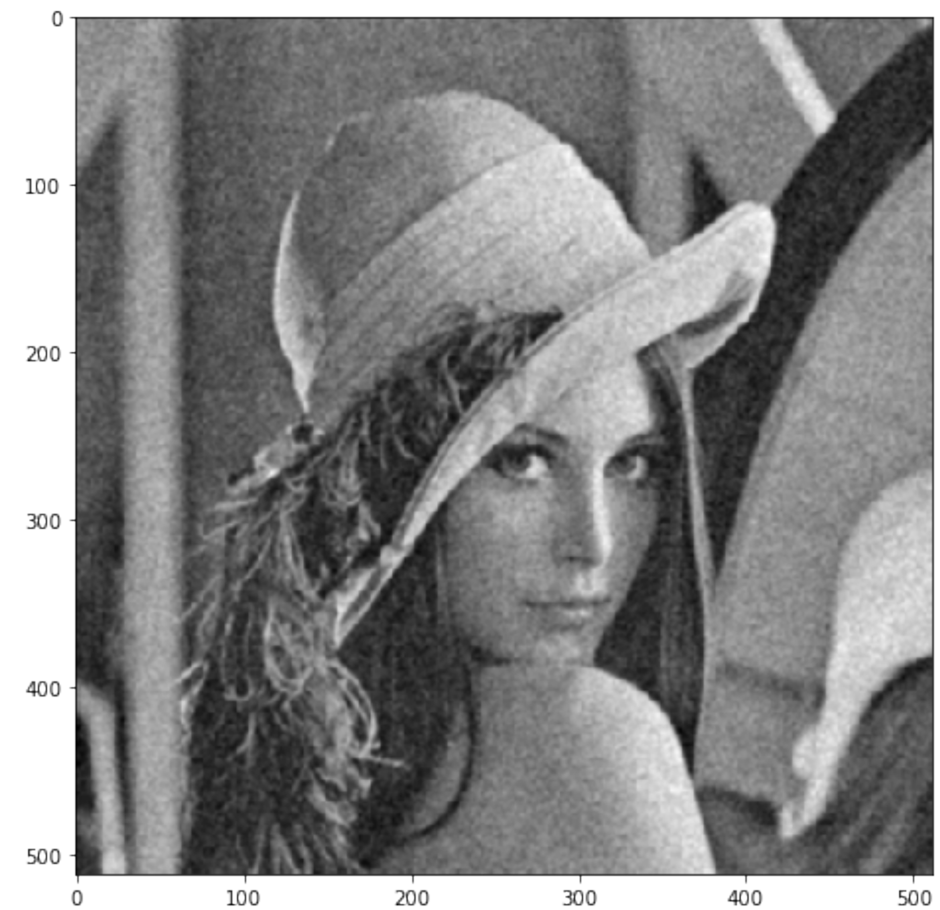
http://labs.eecs.tottori-u.ac.jp/sd/Member/oyamada/OpenCV/html/py_tutorials/py_imgproc/py_filtering/py_filtering.html

Gaussian Blur





noised



* **Center cell is emphasized**
* **Good for removing white noises**
* **Edge is smoothed**

LASSO

From noisy data I_{input} to reconstructed data I_{output}. Try to minimize

$$F = \frac{1}{2}|I_{\text{output}} - I_{\text{input}}|^2 + \lambda |I_{\text{output}}|. \tag{1}$$

In actual, let the output (each cell component) x_i and the input b_i. Set $\boxed{x_i = a_i e_i}$, where $a_i > 0$ and $e_i = +1$ or -1.

$$\begin{aligned} F &= \frac{1}{2}\sum_i (x_i - b_i)^2 + \lambda \sum_i |x_i| \\ &= \frac{1}{2}\sum_i (a_i e_i - b_i)^2 + \lambda \sum_i |e_i| a_i \end{aligned}$$

If we want to minimize this equation, $e_i = \text{sgn}\,(b_i)$, therefore

$$\begin{aligned} F &= \frac{1}{2}\sum_i (a_i \text{sgn}\,(b_i) - b_i)^2 + \lambda \sum_i |e_i| a_i \\ &= \frac{1}{2}\sum_i (a_i - |b_i|)^2 + \lambda \sum_i |e_i| a_i \\ &= \frac{1}{2}\sum_i (a_i - |b_i| + \lambda)^2 + 2\lambda |b_i| - \lambda^2 \end{aligned}$$

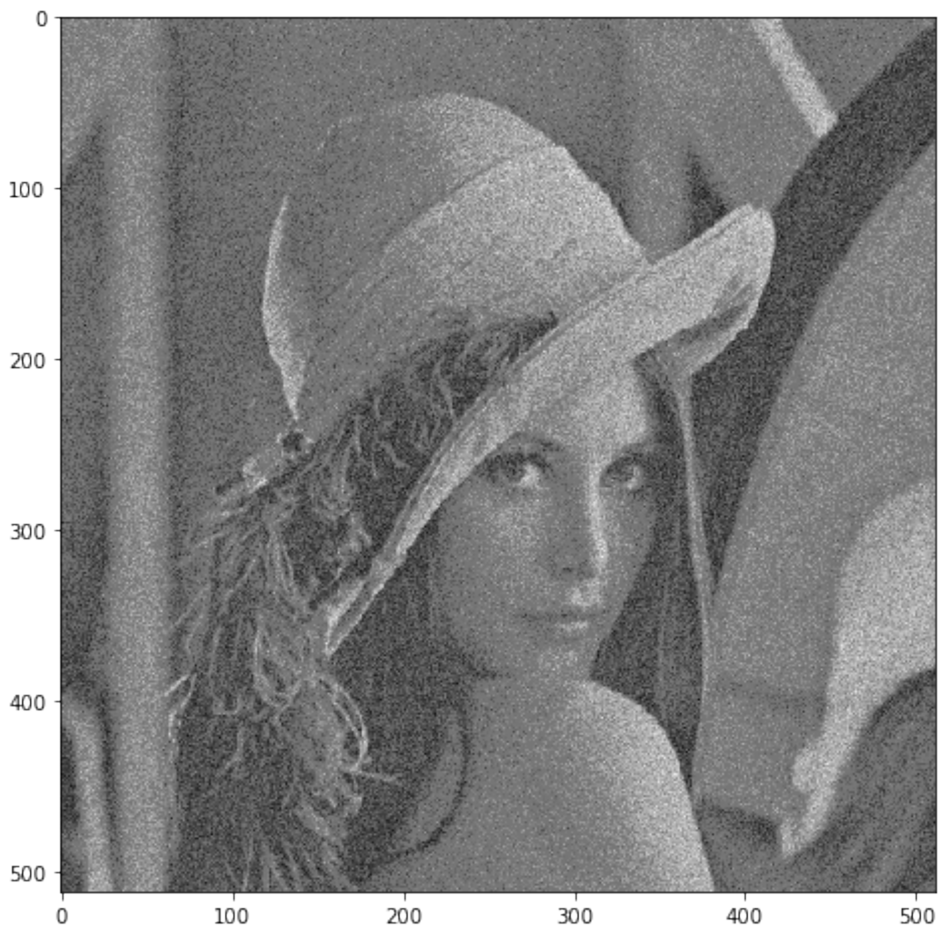
so that we obtain $a_i = \max\,(|b_i| - \lambda, 0)$.

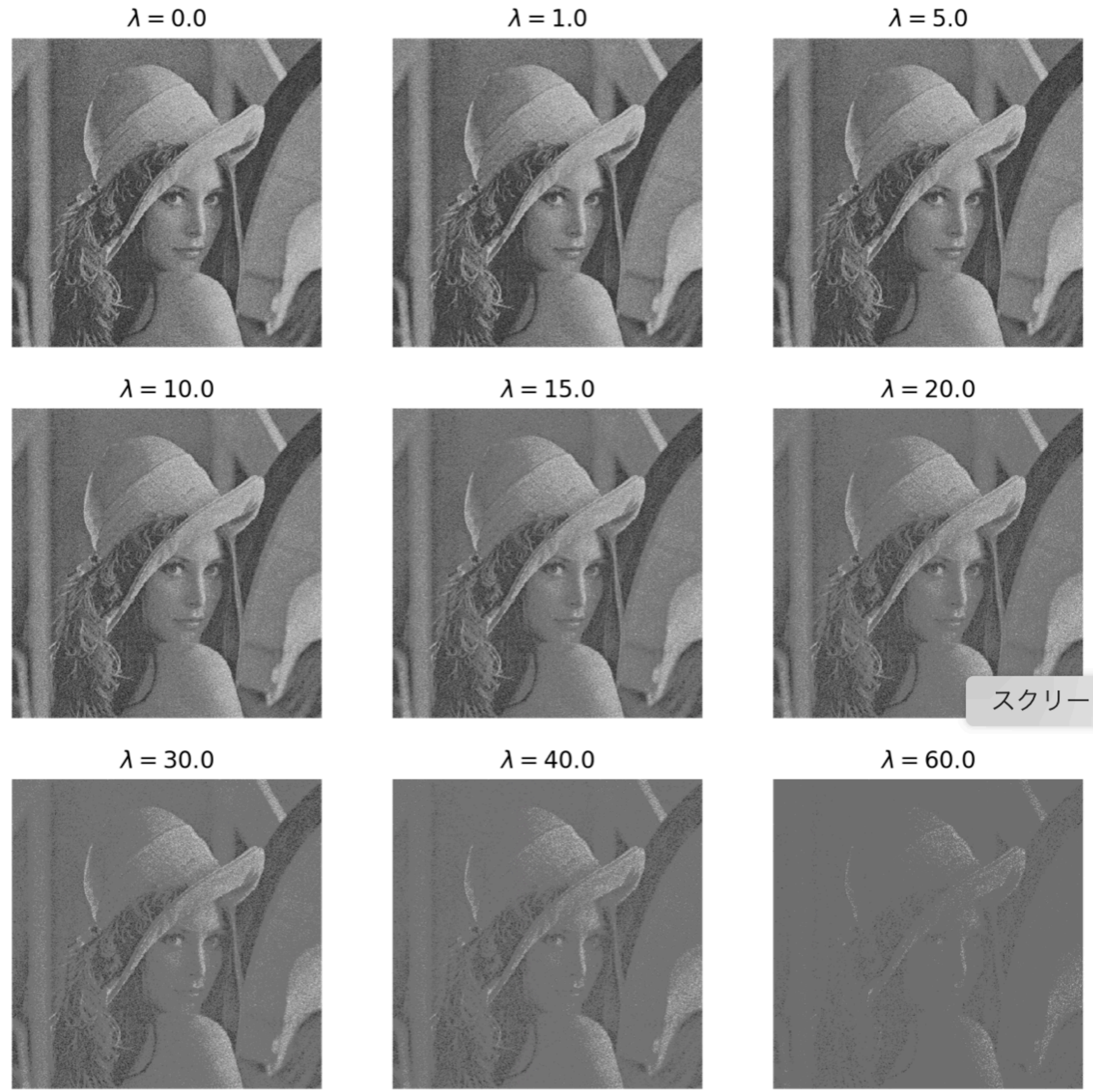
https://lp-tech.net/articles/CY2Kn

LASSO

Original

Noisy





Total Variation (Bregman Iteration)

From noisy data $I_{\rm input}$ to reconstructed data $u \equiv I_{\rm output}$. Try to minimize

$$F = |\nabla_x u| + |\nabla_y u| + \frac{\lambda}{2}|u - I_{\rm input}|^2. \quad (1)$$

This makes differences of each nearest cells are sparse, which makes monotonic colors.

To weakly enforce the constraints in this formulation as to minimize

$$\text{minimize}_{u,d_x,d_y}|d_x| + |d_y| + \frac{\lambda}{2}|u - I_{\rm input}|^2 + \frac{\mu}{2}\left(|\nabla_x u - d_x|^2 + |\nabla_y u - d_y|^2\right) \quad (2)$$

where $d_x \equiv \nabla_x u, d_y \equiv \nabla_y u$ (independent values to $\nabla_x u$ and $\nabla_y u$). **Bregman Iteration** is to split this converging process in k-steps as

$$u^{k+1} = \text{minimize}_{u,d_x,d_y}|d_x| + |d_y| + \frac{\lambda}{2}|u - I_{\rm input}|^2 + \frac{\mu}{2}\left(|\nabla_x u - d_x - b_x^k|^2 + |\nabla_y u - d_y - b_y^k|^2\right) \quad (3)$$

https://lp-tech.net/articles/tkPFr/view

Total Variation (Bregman Iteration)

Original

Noisy

iter=10, lambda=0.1

iter=10, lambda=1.0

iter=10, lambda=10.0

iter=50, lambda=0.1

iter=50, lambda=1.0

iter=50, lambda=10.0

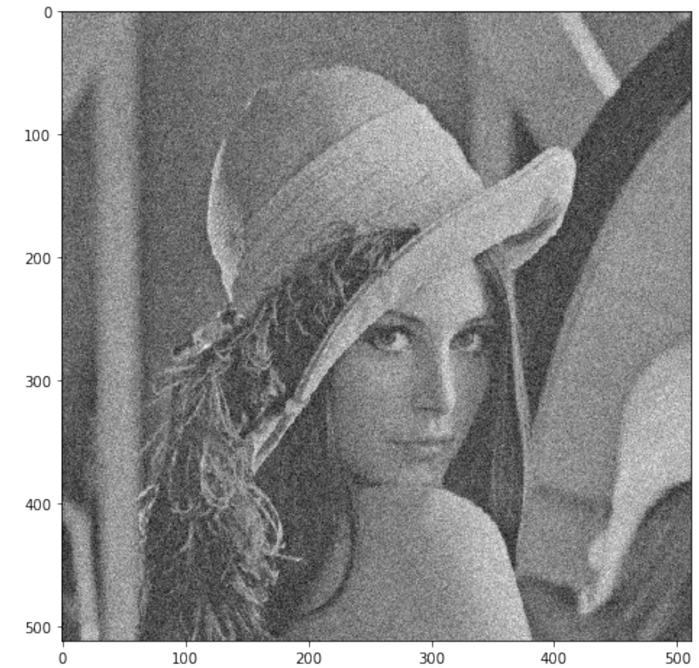
iter=100, lambda=0.1

iter=100, lambda=1.0

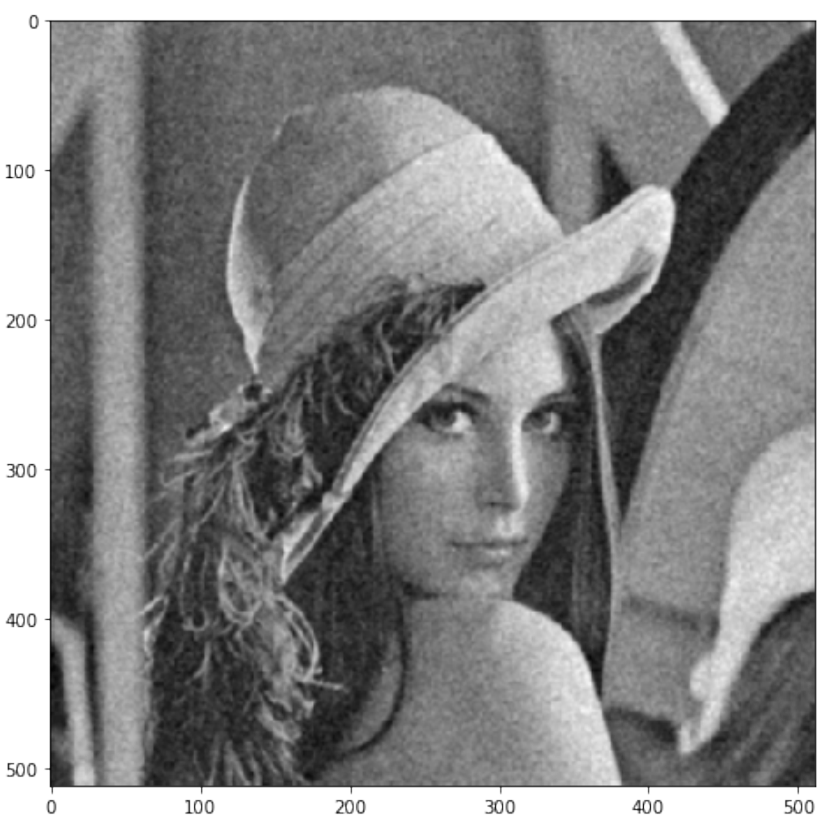
iter=100, lambda=10.0

Exercise of Noise Removing

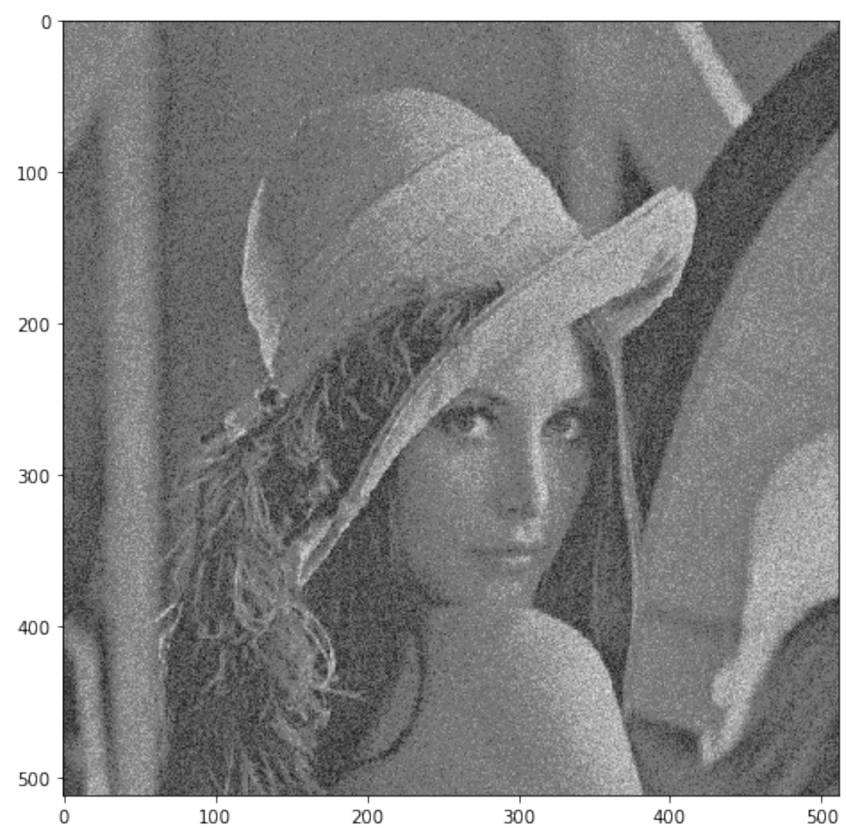
original



noised



Gaussian



LASSO

Total Variation

First Image of a black hole : center of M87

地球から5500万光年

https://alma-telescope.jp/news/press/eht-201904

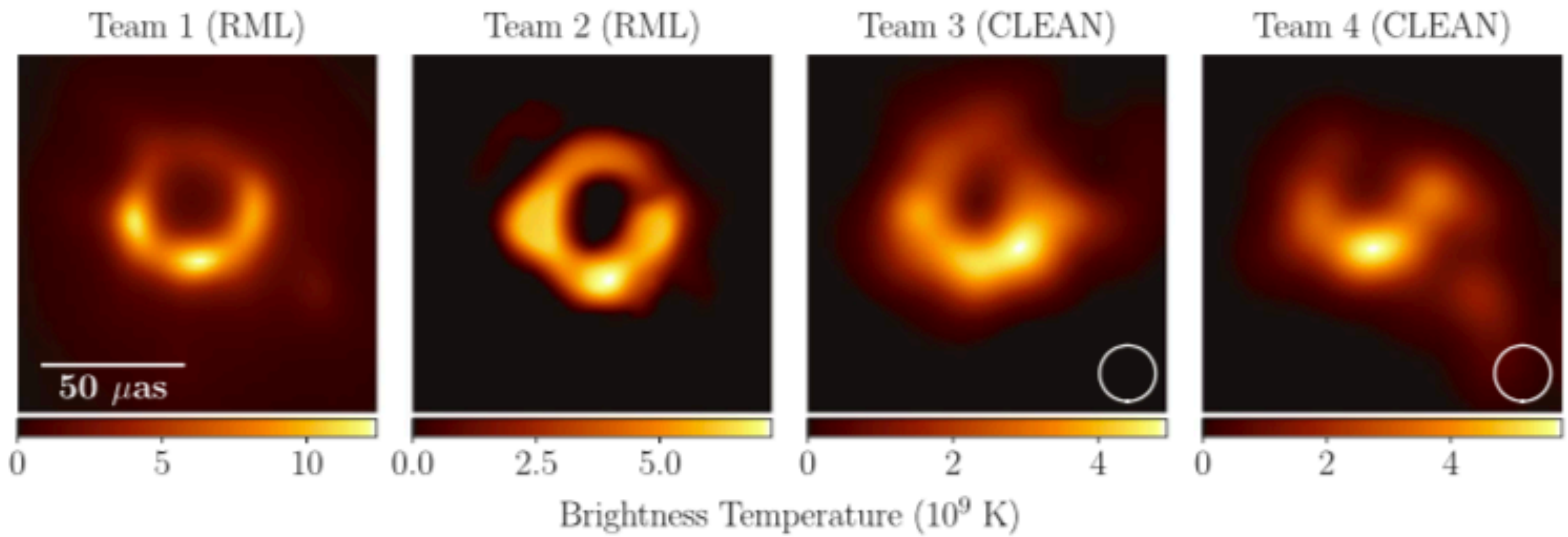


Figure 4. The first EHT images of M87, blindly reconstructed by four independent imaging teams using an early, engineering release of data from the April 11 observations. These images all used a single polarization (LCP) rather than Stokes *I*, which is used in the remainder of this Letter. Images from Teams 1 and 2 used RML methods (no restoring beam); images from Teams 3 and 4 used CLEAN (restored with a circular 20 μas beam, shown in the lower right). The images all show similar morphology, although the reconstructions show significant differences in brightness temperature because of different assumptions regarding the total compact flux density (see Table 2) and because restoring beams are applied only to CLEAN images.

Independent methods:

>>inverse modeling (CLEAN) the standard deconvolution method

>>forward modeling (RML) regularized maximum likelifood

(classical maximum entropy method)

The Astrophysical Journal Letters, 875:L4 (52pp), 2019 April 10 https://doi.org/10.3847/2041-8213/ab0e85

First M87 Event Horizon Telescope Results. IV. Imaging the Central Supermassive Black Hole

The Event Horizon Telescope Collaboration
(See the end matter for the full list of authors.)

Received 2019 February 11; revised 2019 March 5; accepted 2019 March 6; published 2019 April 10

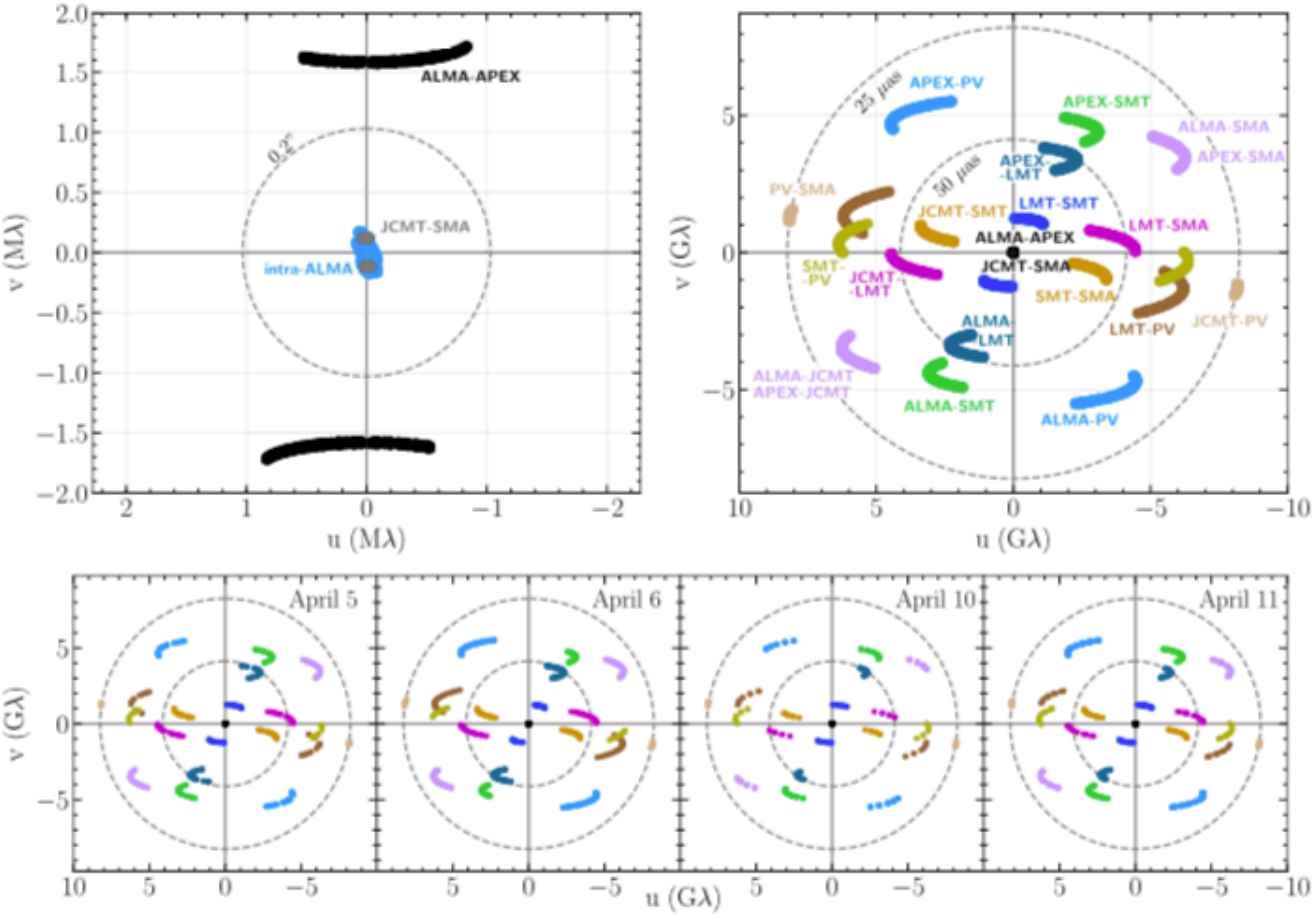


Figure 1. Top panels: aggregate baseline coverage for EHT observations of M87, combining observations on all four days. The left panel shows short-baseline coverage, comprised of ALMA interferometer baselines and intra-site EHT baselines (SMA–JCMT and ALMA–APEX). These short baselines probe angular scales larger than 0.1". The right panel shows long-baseline coverage, comprised of all inter-site EHT baselines. These long baselines span angular scales from 25 to 170 μas. Each point denotes a single scan, which range in duration from 4 to 7 minutes. Bottom panels: the full baseline coverage on M87 for each observation. In all panels, the dashed circles show baseline lengths corresponding to the indicated fringe spacings (0.2" for the upper-left panel; 25 and 50 μas for the remaining panels).

Observed "Visibility" $\boldsymbol{V}$ is Fourier-transformed image $\boldsymbol{I}$: $\boldsymbol{V} = \boldsymbol{F}\boldsymbol{I}$.

- LASSO

$$\boldsymbol{I} = \mathrm{argmin}_{\boldsymbol{I}} \quad \left[||\boldsymbol{V} - \boldsymbol{F}\boldsymbol{I}||_2^2 + \Lambda_\ell ||\boldsymbol{I}||_1\right], \qquad \text{subject to} \qquad \boldsymbol{I} \geq 0 \tag{1}$$

- LASSO with Total Variance of the image

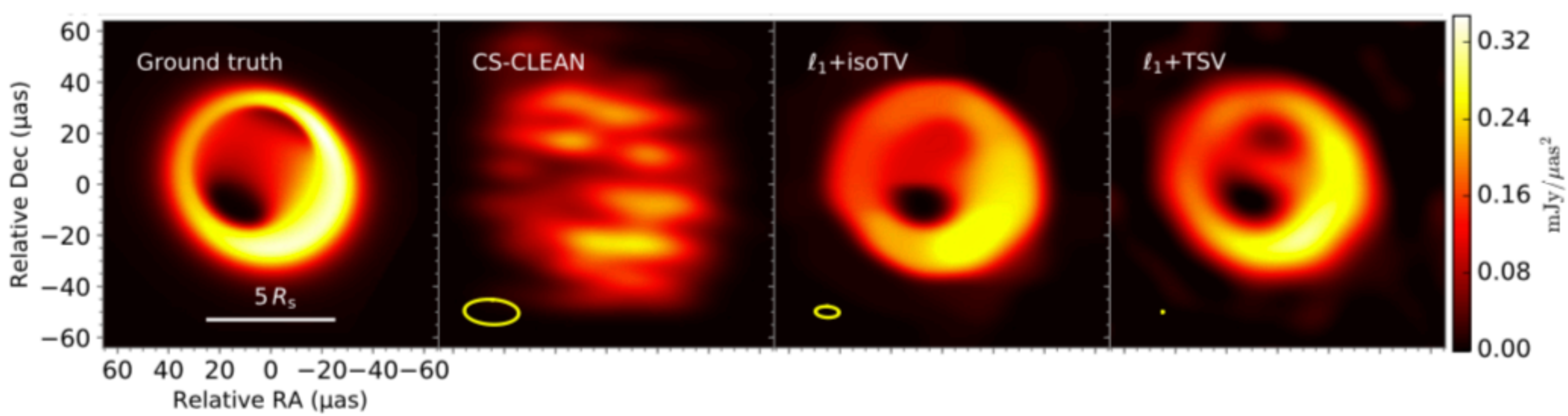
$$\boldsymbol{I} = \mathrm{argmin}_{\boldsymbol{I}} \quad \left[||\boldsymbol{V} - \boldsymbol{F}\boldsymbol{I}||_2^2 + \Lambda_\ell ||\boldsymbol{I}||_1 + \Lambda_t ||\boldsymbol{I}||_{\mathrm{TV}}\right], \qquad \text{subject to} \qquad \boldsymbol{I} \geq 0 \tag{2}$$

where $||\boldsymbol{I}||_{\mathrm{TV}}$ is either isotropic TV [Akiyama+, ApJ 838 (2017) 1]

$$||\boldsymbol{I}||_{\mathrm{isoTV}} = \sum_i \sum_j \sqrt{|I_{i+1,j} - I_{i,j}|^2 + |I_{i,j+1} - I_{i,j}|^2} \tag{3}$$

or total squared variation [Kuramochi+, ApJ 858 (2018) 56]

$$||\boldsymbol{I}||_{\mathrm{TSV}} = \sum_i \sum_j \left(|I_{i+1,j} - I_{i,j}|^2 + |I_{i,j+1} - I_{i,j}|^2\right) \tag{4}$$



Application plans of Sparse-Modeling to GW data analysis

Auto-Regressive model (Method, general)

Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon$$
$$= \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

- find a_j (Burg method)
- find M (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

power spectrum

$$p(f) = \frac{\sigma^2}{\left|1 - \sum_{j=1}^{M} a_j e^{-I 2\pi j f \Delta t}\right|^2}$$

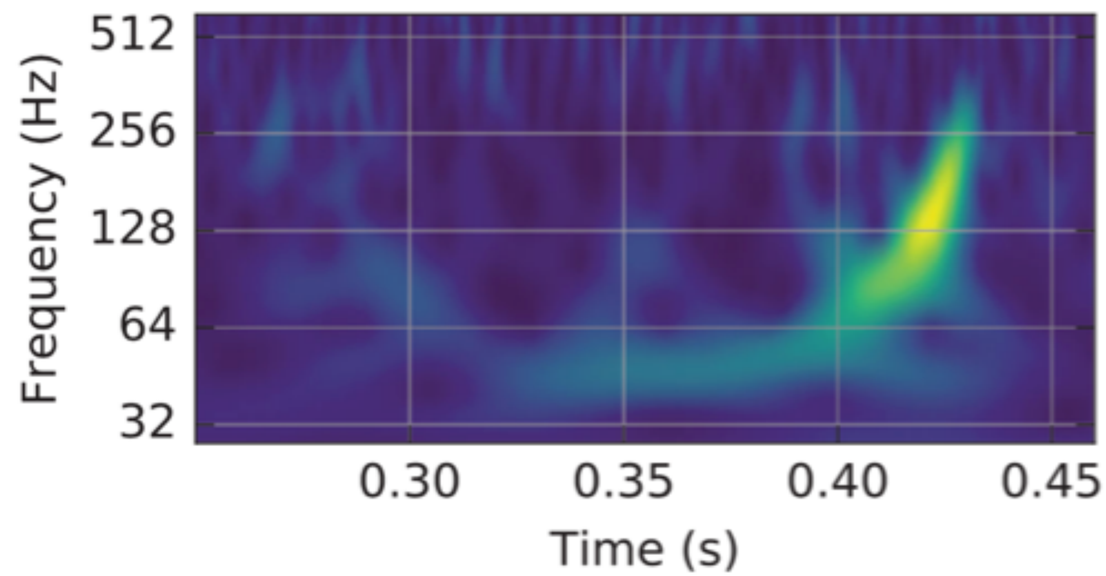
characteristic eq.

$$f(z) = 1 - \sum_{j=1}^{M} a_j z^j = 0$$

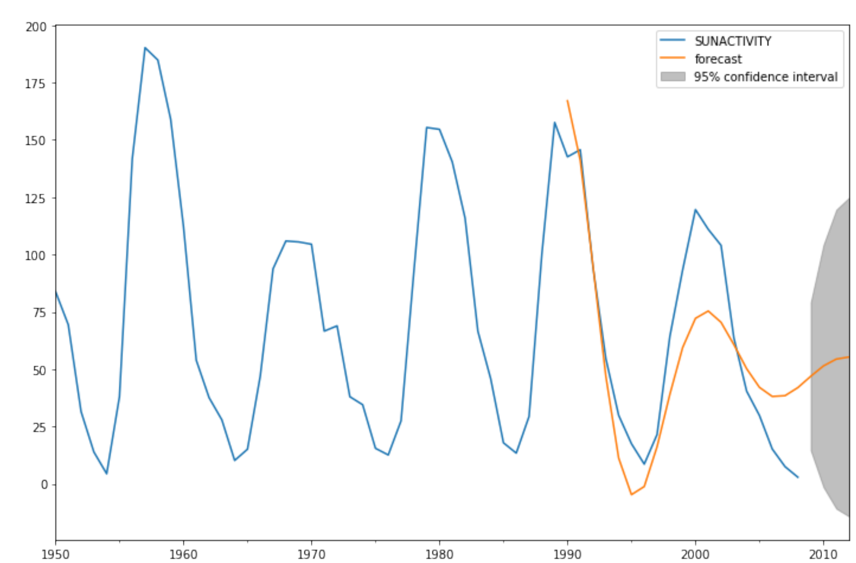
$|z_k|$ says amplitude,
$\arg(z_k)$ says frequency.

alternative linear-regression method

noise erasing method



time series online prediction



detection algorithm in neural net

- Classification problem: Set the logistic regression for probability function

$$p(y \mid \boldsymbol{x}; \boldsymbol{\beta}, \beta_0) = \frac{\exp\left[y(\beta_0 + \sum_j \beta_j \boldsymbol{x}^{(j)})\right]}{1 + \exp\left[\beta_0 + \sum_j \beta_j \boldsymbol{x}^{(j)}\right]}$$

and consider

$$\min_\beta \quad \sum_{i=1}^{N} \left[-\log p_i(y_i \mid \boldsymbol{x}_i; \boldsymbol{\beta}, \beta_0) + \lambda ||\boldsymbol{\beta}||_1\right]$$

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Spline based search method for unmodeled transient gravitational wave chirps

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(Received 28 September 2017; published 30 November 2017)

penalized spline method

$$\hat{\alpha} = \arg\min_{\bar{\alpha}} \sum_{i=0}^{N-1} (y_i - \bar{\alpha}\mathbf{A}(\bar{\tau}_a))^2 + \lambda\bar{\alpha}\bar{\alpha}^T$$

SEECR (spline enabled effectively-chirp regression)

$$\Lambda(\bar{\alpha}, \bar{\theta}, \phi_0 | \bar{y}, \lambda) = R(\bar{\alpha}, \bar{\theta}, \phi_0 | \bar{y}) + \lambda\bar{\alpha}\bar{\alpha}^T, \qquad (19)$$

||

$$\|\bar{y} - \bar{s}(\bar{\alpha}, \bar{\theta}, \phi_0)\|^2$$

▲ GWdata ▲ signal model