Poster 21
A01

Identifying Ringdown Modes of GWTC-3 events using Auto-Regressive method

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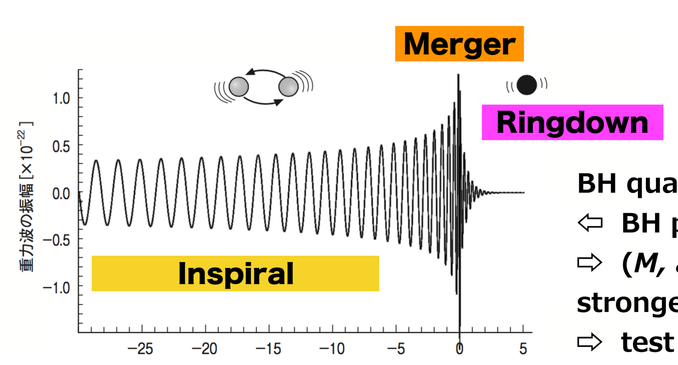
Outline & Summary

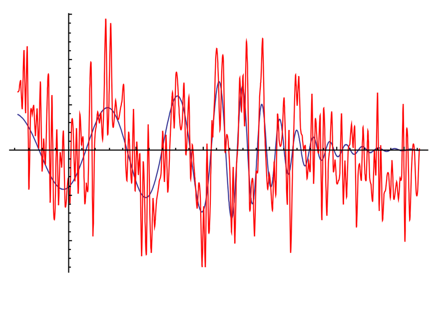
The ring-down part of gravitational waves in the final stage of merger of compact objects tells us the nature of strong gravity which can be used for testing the theories of gravity. The ring-down wave, however, fades out in a very short time with a few cycles, and hence it is challenging for gravitational wave data analysis to extract the ringdown frequency and its damping time scale.
We develop a new method, the autoregressive modeling (AR) approach, which extracts waveform by fitting a linear function from bare data. It works well for small number of data points, and does not require any templates. After obtaining the best parameters using mockdata, we applied this method for black-hole merger events of the LIGO/Virgo/KAGRA O3 catalog (GWTC-3). We find that for high SNR events, we can extract ring-down waves properly.
The identified ringdown modes are around those reported in GWTC-3, i.e. no significant deviations from the modes predicted by general relativity.
This method should work for extracting higher modes of ring-down waves, but we do not find them yet.

Motivation & O3 data

Towards testing gravity theories ⇨ Ringdown-part extraction is a key



BH quasi-normal modes
⇦ BH perturbation theory
⇨ *(M, a)*
strongest gravity we can observe
⇨ test of gravity theories



For *60M* BH of *a*=0.75,
frequency = 300 Hz
damping time scale = 3.7 ms

LVK arXiv:2111.03606

GWTC-3 (O3b) continued

Event	M $(M_\odot)$	$\mathcal{M}$ $(M_\odot)$	m_1 $(M_\odot)$	m_2 $(M_\odot)$	$\chi_{\rm eff}$	D_L (Gpc)	z	M_f $(M_\odot)$	χ_f	$\Delta\Omega$ (deg^2)	SNR
GW200115_042309	$7.4^{+1.8}_{-1.7}$	$2.43^{+0.05}_{-0.07}$	$5.9^{+2.0}_{-2.5}$	$1.44^{+0.85}_{-0.29}$	$-0.15^{+0.24}_{-0.42}$	$0.29^{+0.15}_{-0.10}$	$0.06^{+0.03}_{-0.02}$	$7.2^{+1.8}_{-1.7}$	$0.42^{+0.09}_{-0.05}$	370	$11.3^{+0.3}_{-0.5}$
GW200128_022011	75^{+17}_{-12}	$32.0^{+7.5}_{-5.5}$	$42.2^{+11.6}_{-8.1}$	$32.6^{+9.5}_{-9.2}$	$0.12^{+0.24}_{-0.25}$	$3.4^{+2.1}_{-1.8}$	$0.56^{+0.28}_{-0.28}$	71^{+16}_{-11}	$0.74^{+0.10}_{-0.10}$	2600	$10.6^{+0.3}_{-0.4}$
GW200129_065458	$63.4^{+4.3}_{-3.6}$	$27.2^{+2.1}_{-2.3}$	$34.5^{+9.9}_{-3.2}$	$28.9^{+3.4}_{-9.3}$	$0.11^{+0.11}_{-0.16}$	$0.90^{+0.29}_{-0.38}$	$0.18^{+0.05}_{-0.07}$	$60.3^{+4.0}_{-3.3}$	$0.73^{+0.06}_{-0.05}$	130	$26.8^{+0.2}_{-0.2}$
GW200202_154313	$17.58^{+1.78}_{-0.67}$	$7.49^{+0.24}_{-0.20}$	$10.1^{+3.5}_{-1.4}$	$7.3^{+1.1}_{-1.7}$	$0.04^{+0.13}_{-0.06}$	$0.41^{+0.15}_{-0.16}$	$0.09^{+0.03}_{-0.03}$	$16.76^{+1.87}_{-0.66}$	$0.69^{+0.03}_{-0.04}$	170	$10.8^{+0.2}_{-0.4}$
GW200208_130117	$65.4^{+7.8}_{-6.8}$	$27.7^{+3.6}_{-3.1}$	$37.8^{+9.2}_{-6.2}$	$27.4^{+6.1}_{-7.4}$	$-0.07^{+0.22}_{-0.27}$	$2.23^{+1.00}_{-0.85}$	$0.40^{+0.15}_{-0.14}$	$62.5^{+7.3}_{-6.4}$	$0.66^{+0.09}_{-0.13}$	30	$10.8^{+0.3}_{-0.4}$
GW200208_222617	63^{+100}_{-25}	$19.6^{+10.7}_{-5.1}$	51^{+104}_{-30}	$12.3^{+9.0}_{-5.7}$	$0.45^{+0.43}_{-0.44}$	$4.1^{+4.4}_{-1.9}$	$0.66^{+0.54}_{-0.28}$	61^{+100}_{-25}	$0.83^{+0.14}_{-0.27}$	2000	$7.4^{+1.4}_{-1.2}$
GW200209_085452	$62.6^{+13.9}_{-9.4}$	$26.7^{+6.0}_{-4.2}$	$35.6^{+10.5}_{-6.8}$	$27.1^{+7.8}_{-7.8}$	$-0.12^{+0.24}_{-0.30}$	$3.4^{+1.9}_{-1.8}$	$0.57^{+0.25}_{-0.26}$	$59.9^{+13.1}_{-8.9}$	$0.66^{+0.10}_{-0.12}$	730	$9.6^{+0.4}_{-0.5}$
GW200210_092254	$27.0^{+7.1}_{-4.3}$	$6.56^{+0.38}_{-0.40}$	$24.1^{+7.5}_{-4.6}$	$2.83^{+0.47}_{-0.42}$	$0.02^{+0.22}_{-0.21}$	$0.94^{+0.43}_{-0.34}$	$0.19^{+0.08}_{-0.06}$	$26.7^{+7.2}_{-4.3}$	$0.34^{+0.13}_{-0.08}$	1800	$8.4^{+0.5}_{-0.7}$
GW200216_220804	81^{+20}_{-14}	$32.9^{+9.3}_{-8.5}$	51^{+22}_{-13}	30^{+14}_{-16}	$0.10^{+0.34}_{-0.36}$	$3.8^{+3.0}_{-2.0}$	$0.63^{+0.37}_{-0.29}$	78^{+19}_{-13}	$0.70^{+0.14}_{-0.24}$	2900	$8.1^{+0.4}_{-0.5}$
GW200219_094415	$65.0^{+12.6}_{-8.2}$	$27.6^{+5.6}_{-3.8}$	$37.5^{+10.1}_{-6.9}$	$27.9^{+7.4}_{-8.4}$	$-0.08^{+0.23}_{-0.29}$	$3.4^{+1.7}_{-1.5}$	$0.57^{+0.22}_{-0.22}$	$62.2^{+11.7}_{-7.8}$	$0.66^{+0.10}_{-0.13}$	700	$10.7^{+0.3}_{-0.5}$
GW200220_061928	148^{+55}_{-33}	62^{+23}_{-15}	87^{+40}_{-23}	61^{+26}_{-25}	$0.06^{+0.40}_{-0.38}$	$6.0^{+4.8}_{-3.1}$	$0.90^{+0.55}_{-0.40}$	141^{+51}_{-31}	$0.71^{+0.15}_{-0.17}$	3000	$7.2^{+0.4}_{-0.7}$
GW200220_124850	67^{+17}_{-12}	$28.2^{+7.3}_{-5.1}$	$38.9^{+14.1}_{-8.6}$	$27.9^{+9.2}_{-9.0}$	$-0.07^{+0.27}_{-0.33}$	$4.0^{+2.8}_{-2.2}$	$0.66^{+0.36}_{-0.31}$	64^{+16}_{-11}	$0.67^{+0.11}_{-0.14}$	3200	$8.5^{+0.3}_{-0.5}$
GW200224_222234	$72.2^{+7.2}_{-5.1}$	$31.1^{+3.2}_{-2.6}$	$40.0^{+6.9}_{-4.5}$	$32.5^{+5.0}_{-7.2}$	$0.10^{+0.15}_{-0.15}$	$1.71^{+0.49}_{-0.64}$	$0.32^{+0.08}_{-0.11}$	$68.6^{+6.6}_{-4.7}$	$0.73^{+0.07}_{-0.07}$	50	$20.0^{+0.2}_{-0.2}$
GW200225_060421	$33.5^{+3.6}_{-3.0}$	$14.2^{+1.5}_{-1.4}$	$19.3^{+5.0}_{-3.0}$	$14.0^{+2.8}_{-3.5}$	$-0.12^{+0.17}_{-0.28}$	$1.15^{+0.51}_{-0.53}$	$0.22^{+0.09}_{-0.10}$	$32.1^{+3.5}_{-2.8}$	$0.66^{+0.07}_{-0.13}$	370	$12.5^{+0.3}_{-0.4}$
GW200302_015811	$57.8^{+9.6}_{-6.9}$	$23.4^{+4.7}_{-3.0}$	$37.8^{+8.7}_{-8.5}$	$20.0^{+8.1}_{-5.7}$	$0.01^{+0.25}_{-0.26}$	$1.48^{+1.02}_{-0.70}$	$0.28^{+0.16}_{-0.12}$	$55.5^{+8.9}_{-6.6}$	$0.66^{+0.13}_{-0.15}$	6000	$10.8^{+0.3}_{-0.4}$
GW200306_093714	$43.9^{+11.8}_{-7.5}$	$17.5^{+3.5}_{-3.0}$	$28.3^{+17.1}_{-7.7}$	$14.8^{+6.5}_{-6.4}$	$0.32^{+0.28}_{-0.46}$	$2.1^{+1.7}_{-1.1}$	$0.38^{+0.24}_{-0.18}$	$41.7^{+12.3}_{-6.9}$	$0.78^{+0.11}_{-0.26}$	4600	$7.8^{+0.4}_{-0.6}$
GW200308_173609*	$50.6^{+10.9}_{-8.5}$	$19.0^{+4.8}_{-2.8}$	$36.4^{+11.2}_{-9.6}$	$13.8^{+7.2}_{-3.3}$	$0.65^{+0.17}_{-0.21}$	$5.4^{+2.7}_{-2.6}$	$0.83^{+0.32}_{-0.35}$	$47.4^{+11.1}_{-7.7}$	$0.91^{+0.03}_{-0.08}$	2000	$7.1^{+0.5}_{-0.5}$
GW200311_115853	$61.9^{+5.3}_{-4.2}$	$26.6^{+2.4}_{-2.0}$	$34.2^{+6.4}_{-3.8}$	$27.7^{+4.1}_{-5.9}$	$-0.02^{+0.16}_{-0.20}$	$1.17^{+0.28}_{-0.40}$	$0.23^{+0.05}_{-0.07}$	$59.0^{+4.8}_{-3.9}$	$0.69^{+0.07}_{-0.08}$	35	$17.8^{+0.2}_{-0.2}$
GW200316_215756	$21.2^{+7.2}_{-2.0}$	$8.75^{+0.62}_{-0.55}$	$13.1^{+10.2}_{-2.9}$	$7.8^{+1.9}_{-2.9}$	$0.13^{+0.27}_{-0.10}$	$1.12^{+0.47}_{-0.44}$	$0.22^{+0.08}_{-0.08}$	$20.2^{+7.4}_{-1.9}$	$0.70^{+0.04}_{-0.04}$	190	$10.3^{+0.4}_{-0.7}$
GW200322_091133*	55^{+37}_{-27}	$15.5^{+15.7}_{-3.7}$	34^{+48}_{-18}	$14.0^{+16.8}_{-8.7}$	$0.24^{+0.45}_{-0.51}$	$3.6^{+7.0}_{-2.0}$	$0.60^{+0.84}_{-0.30}$	53^{+38}_{-26}	$0.78^{+0.16}_{-0.17}$	6500	$6.0^{+1.7}_{-1.2}$

Mockdata Comparison

Phys. Rev. D 99, 124032 (2019) [arXiv:1811.06443]

Mockdata preparation

SXS data + shifted ringdown injection + aLIGO noise
modified after t_merger (set A) 60 set
modified before/after t_merger (set B) 60 set

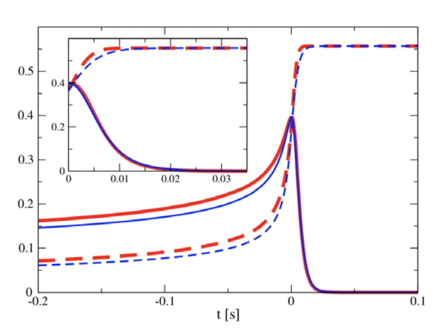


FIG. 1. Examples of sets A and B. (Inset) The ringdown part. Here, set A [(red) thick] with SXS:BBH:0174 and set B [(blue) thin] with SXS:BBH:0002 are shown. The solid lines denote the modified amplitude $A_{22}(t)$, and the dashed lines are the GW frequency $\omega_{22}(t)/(2\pi)$. The total mass is $M = 60M_\odot$, and the real and imaginary parts of the ringdown frequency are 300 and 40 Hz, respectively. The real frequency is obtained by multiplying by 538.609 Hz, and the real amplitude of set A is derived by dividing by 1.37903. The large difference in the inspiral phase is due to the difference of the binary parameters.

data	SNR		injected	
	$\rho_{\rm all}$	$\rho_{\rm rd}$	$f_R^{\rm (inj)}$	$f_I^{\rm (inj)}$
A-01	60.0	13.81	260.68	44.58
A-02	60.0	12.73	345.16	50.49
A-03	60.0	13.79	382.53	32.58
A-04	60.0	11.84	284.18	44.73
A-05	60.0	16.78	346.20	23.07
A-06	30.0	5.57	272.85	33.40
A-07	30.0	6.56	272.85	44.54
A-08	30.0	7.27	301.89	42.24
A-09	30.0	6.93	324.60	27.25
A-10	30.0	7.88	282.55	37.45
A-11	20.0	6.36	314.24	30.58
A-12	20.0	3.45	382.10	48.60
A-13	20.0	4.68	249.36	47.97
A-14	20.0	4.13	299.32	41.88
A-15	20.0	4.54	319.42	31.55
B-01	60.0	17.58	352.56	36.20
B-02	60.0	14.27	210.78	42.77
B-03	60.0	13.67	258.83	48.42
B-04	60.0	20.09	271.13	25.40
B-05	60.0	17.07	291.99	34.20
B-06	30.0	9.53	411.57	29.48
B-07	30.0	6.29	295.78	59.38
B-08	30.0	6.03	312.39	59.24
B-09	30.0	6.01	198.34	57.91
B-10	30.0	8.31	323.32	37.86
B-11	20.0	5.20	208.80	39.75
B-12	20.0	6.60	246.66	27.85
B-13	20.0	4.46	323.71	62.51
B-14	20.0	6.20	215.15	33.15
B-15	20.0	5.85	335.20	25.11

https://gw-genesis.scphys.kyoto-u.ac.jp/ilias/goto_root_fold_669.html
http://www.oit.ac.jp/is/shinkai/mockdatachallenge/

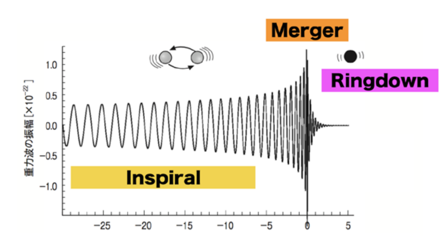
Nakano+, PRD99 (2019) 124032

mockdata-challenge comparison

PHYSICAL REVIEW D 99, 124032 (2019)

Comparison of various methods to extract ringdown frequency from gravitational wave data

Hiroyuki Nakano, Tatsuya Narikawa, Ken-ichi Oohara, Kazuki Sakai, Hisa-aki Shinkai, Hirotaka Takahashi, Takahiro Tanaka, Nami Uchikata, Shun Yamamoto, and Takahiro S. Yamamoto



ringdown search
60 mockdata

TABLE III. We show the values of $\overline{\delta \log f_R}$, $\sigma(f_R)$, $\overline{\delta \log f_I}$, and $\sigma(f_I)$ for various methods. The results limited to set A are given on the first law of each method, while those limited to set B are on the second.

			$\overline{\delta \log f_R}(\%)$	$\sigma(f_R)(\%)$	$\overline{\delta \log f_I}(\%)$	$\sigma(f_I)(\%)$
matched filtering	MF-R	A	−12.88	28.36	−71.51	97.79
		B	−0.82	27.53	−46.11	75.48
	MF-MR	A	6.25 (Very good)	17.27 (Good)	−12.62 (Very good)	37.9 (Very good)
		B	2.47	10.41	7.18	27.61
Hilbert-Huan Transformation	HHT	A	−13.38	21.91	−44.11	61.58
		B	−8.08	19.81	−28.78	49.61
Auto-Regression Method	AR	A	0.2 (Excellent)	9.93 (Excellent)	4.88 (Excellent)	38.75 (Good)
		B	1.91	8.57	6.2	34.64
Neural Network method	NN	A	−6.64 (Good)	16.48 (Very good)	−15.23 (Good)	33.96 (Excellent)
		B	−6.65	11.97	9.96	23.76

Method

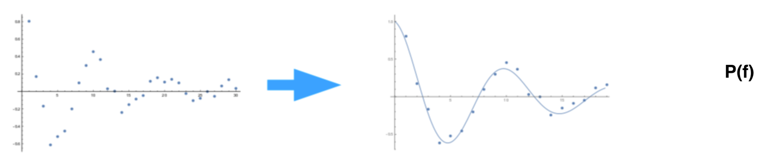
Auto-Regressive model (idea)

Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon = \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

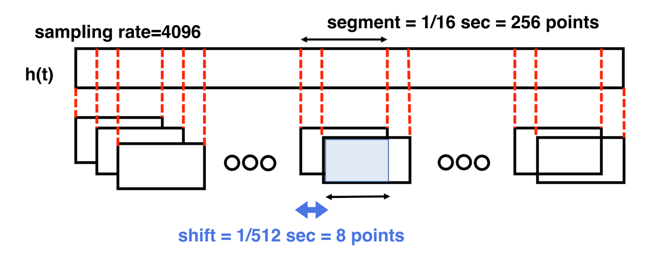
e.g. $x_n = Ae^{-rn\Delta t}\cos(\omega n\Delta t)$

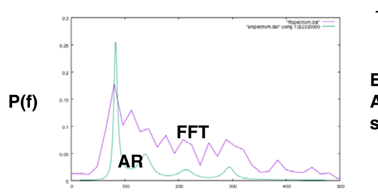
$$Z_1 = e^{-(r-j\omega)\Delta t}, \quad Z_2 = e^{-(r+j\omega)\Delta t} \quad \Rightarrow \quad x_n = \frac{A}{2}(Z_1^n + Z_2^n) = (Z_1+Z_2)x_{n-1} - Z_1 Z_2 x_{n-2}$$



can be applied also to noisy data by adjusting *M*

Auto-Regressive model vs Short FFT





[mock data, SNR=40, inspiral part]

The order *M* can be fixed at 2∼8.

Even for short segment, AR model shows precise power-spectrum.

Auto-Regressive model (Method, general)

Fitting data with linear func.

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \cdots + a_M x_{n-M} + \varepsilon = \sum_{j=1}^{M} a_j x_{n-j} + \varepsilon$$

- find a_j (Burg method)
- find M (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

power spectrum

$$p(f) = \frac{\sigma^2}{\left|1 - \sum_{j=1}^{M} a_j e^{-I2\pi jf\Delta t}\right|^2}$$

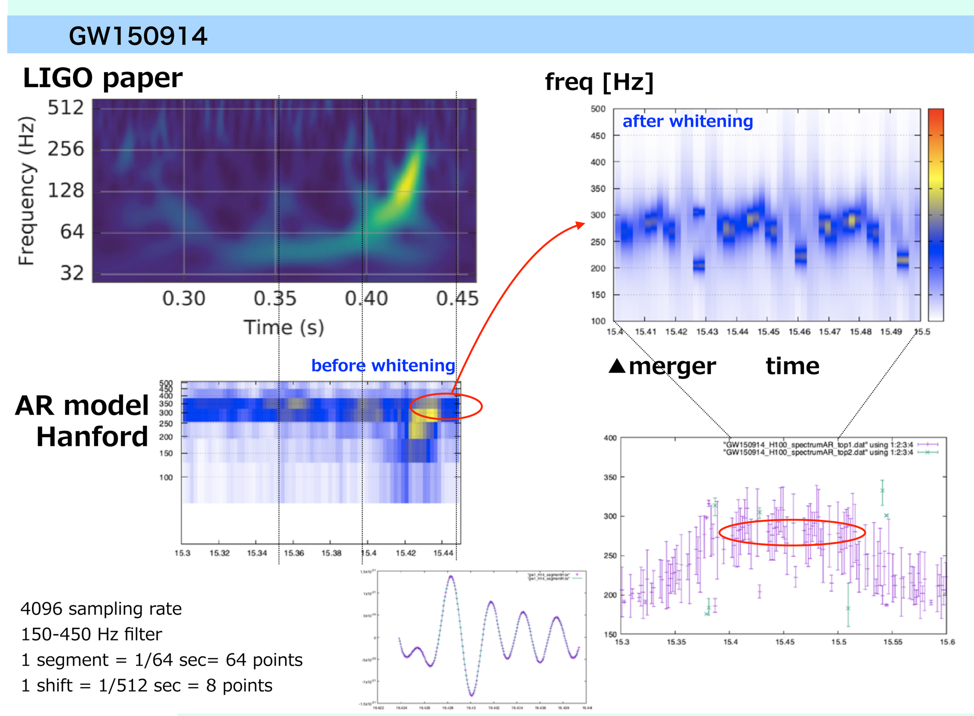
characteristic eq.

$$f(z) = 1 - \sum_{j=1}^{M} a_j z^j = 0$$

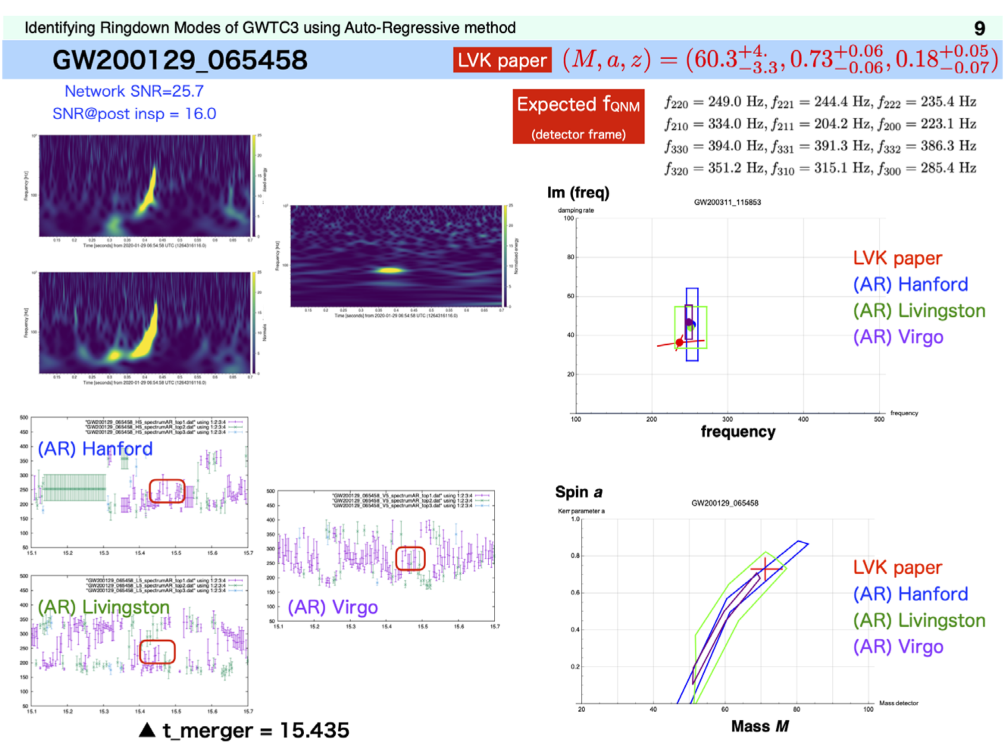
$|z_k|$ says amplitude,
$\arg(z_k)$ says frequency.

Results

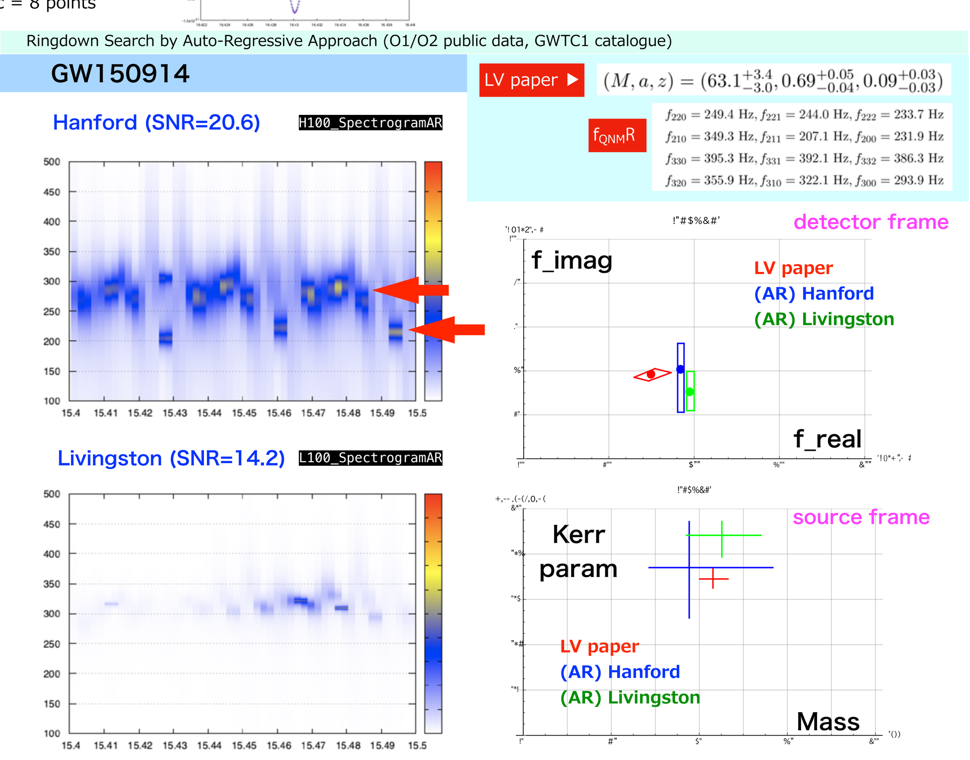
GW150914



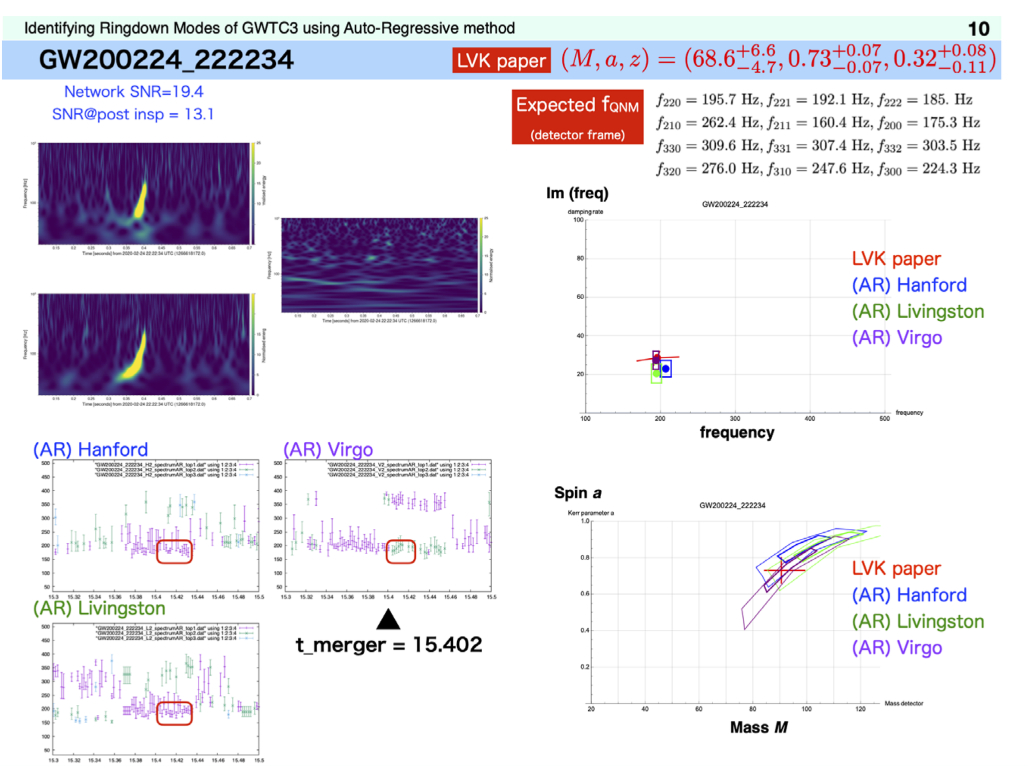
4096 sampling rate
150-450 Hz filter
1 segment = 1/64 sec= 64 points
1 shift = 1/512 sec = 8 points



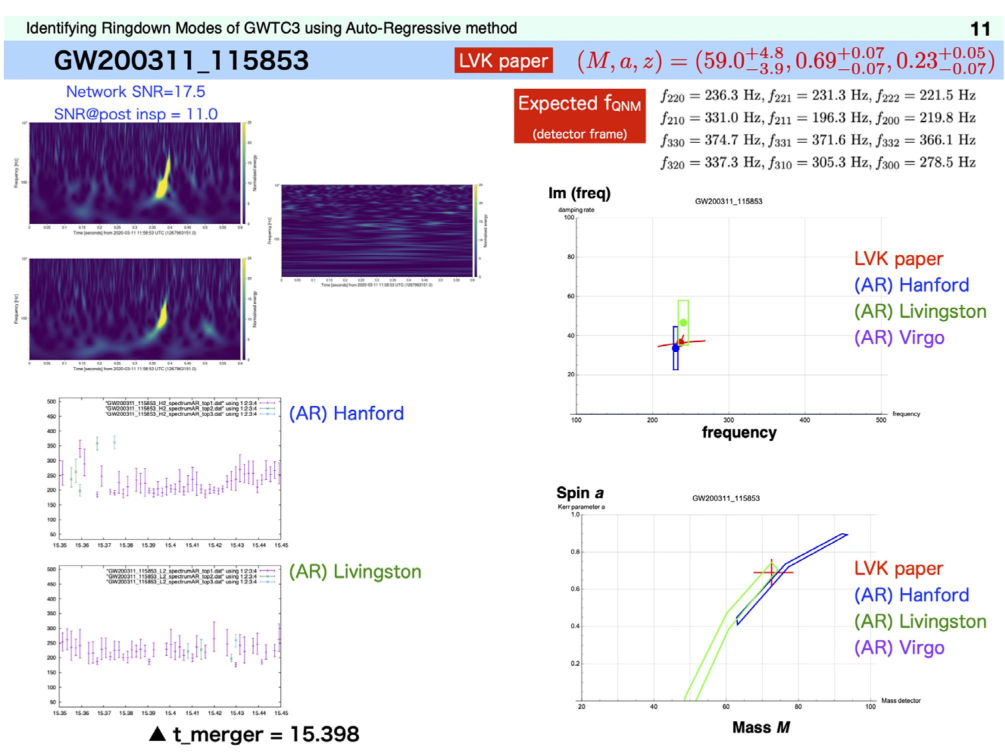
$(M, a, z) = (60.3^{+4.0}_{-3.3}, 0.73^{+0.06}_{-0.05}, 0.18^{+0.05}_{-0.07})$



$(M, a, z) = (63.1^{+3.4}_{-3.0}, 0.69^{+0.05}_{-0.04}, 0.09^{+0.03}_{-0.03})$



$(M, a, z) = (68.6^{+6.6}_{-4.7}, 0.73^{+0.07}_{-0.07}, 0.32^{+0.08}_{-0.11})$



$(M, a, z) = (59.0^{+4.8}_{-3.9}, 0.69^{+0.07}_{-0.08}, 0.23^{+0.05}_{-0.07})$

ACKNOWLEDGMENTS
This work was supported by JSPS KAKENHI Grant No. JP17H06358 (and also JP17H06357) [A01: Testing gravity theories using gravitational waves, as a part of the innovative research area, "Gravitational wave physics and astronomy: Genesis"], by JSPS KAKENHI Grant No. JP18K03630 [Non-linear dynamics in the modified gravity theories as a test of string theory] and by No. 19H01901 [New directions in gravitational-wave data analysis: both in computing algorithms and hardwares including its outreach activities].