

# 自己回帰モデルを用いた重力波データ解析:

## 連星ブラックホールイベントのリングダウン波形抽出による相対論の検証

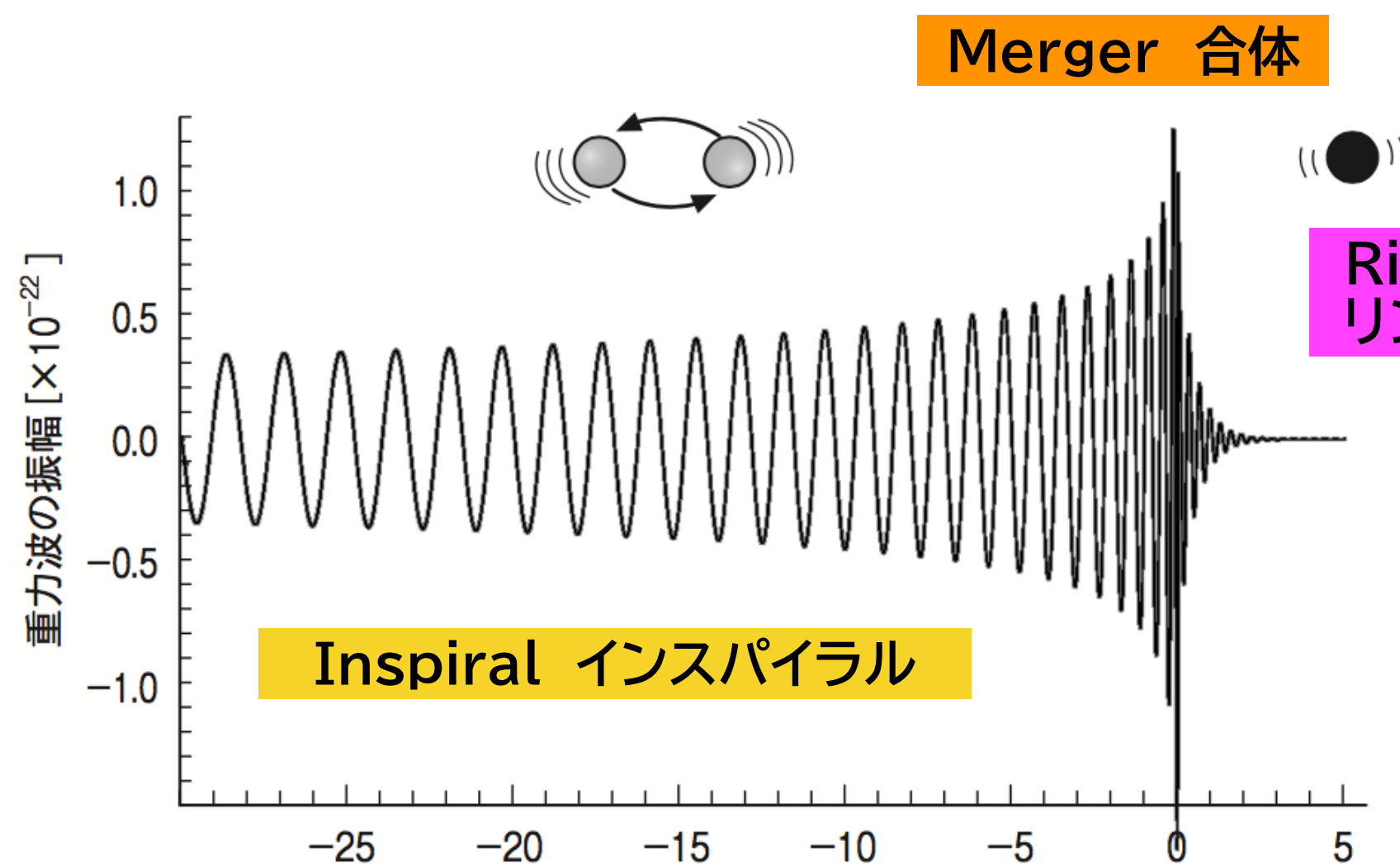
Gravitational-wave data analysis by auto-regression method:

Test of general relativity by extracting ring-down modes of binary black-hole events

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BH 準固有振動 (quasi-normal modes)

← BH 摂動 in GR

→  $(M, a)$

強い重力場の影響

→ GR検証にもっとも優れた状況

GRの予言する周波数・減衰率が見えるのか?  
オーバートーンの周波数・減衰率が見えるのか?  
高次モードの周波数・減衰率が見えるのか?

そもそも、重力波実データから直接見えるのか?

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Yes

“リングダウン部分”は極めて短時間で減衰 (減衰定数 3.7 ms for 60 Msun,  $a=0.75$ )

独立に抽出するには工夫が必要.

→ 自己回帰モデル (Auto-Regressive model)

→ 実データに応用.

今回

→ → リングダウン波の開始時刻はいつか

→ → 高次モード, 高調波モードはみつかるか

→ → GRと矛盾しないか

# 突発性重力波は, O1/O2/O3a,b 観測で90例

## 重力波

**重力波の生成機構** 一般相対性理論によれば、大質量でコンパクトな天体が加速度運動することにより、重力波が発生する。重力波源としては連星の合体や超新星爆発、非球対称な星の高速回転や、宇宙初期に起源をもつ重力波が宇宙空間を伝播していると考えられる。これらのうち、データとの相関解析を可能にする波形予測ができるのは、連星合体からの重力波である。十分に合体前はニュートン力学に相対論補正を加えたポスト・ニュートン展開により、合体前後は数値シミュレーションにより、合体後ブラックホールが生じる場合にはブラックホール時空の摂動によっても波形モデルが得られる。これらのモデルと重力波干渉計で得られる信号の相関をとることで、連星ブラックホール（以下 BBH）や連星中性子星（BNS）、および中性子星・ブラックホール連星（NSBH）の合体現象による重力波の検出、および、パラメータ推定が 2015 年以来可能になった。

**重力波の観測** これまでに、米欧のレーザー干渉計 LIGO, Virgo によって、O3b と呼ばれる観測期間終了までに、BBH 波源の重力波が 83 例、BNS 波源が 2 例、NSBH 波源が 3 例、片方が BH で相方が不明なもの 2 例の合計 90 例が報告されている。日本の KAGRA（かぐら）も O3b 観測期間の最後に共同観測に入った。次の観測期間 O4 は、2022 年 12 月開始が予定されている（2022 年 7 月現在）。

重力波イベントは、観測された年月日を用いて、GW150914 の形で命名される。O3a 期より、時分秒を加えた名称が正式となった。重力波イベントは速報体制が取られ、多波長電磁波追観測が可能になっているが、これまでに波源が特定されたのは GW170817 のみである。

重力波レーザー干渉計の位置と腕の向き (例えば N 36° W は、北から西方に 36° の向きを指す。)						
干渉計	所在地	腕長 (km)	緯度	経度	X-腕	Y-腕
LIGO Hanford	米国	4	46°27'19" N	119°24'28" W	N 36° W	W 36° S
LIGO Livingston	米国	4	30 33 46 N	90 46 27 W	W 18° S	S 18° E
Virgo	欧州	3	43 37 53 N	10 30 16 E	N 19° E	W 19° N
KAGRA	日本	3	36 24 36 N	137 18 36 E	E 28.3° N	N 28.3° W

観測期	過去の観測期間					
	Advanced LIGO		Advanced Virgo		KAGRA	
	年月日	年月日	年月日	年月日	年月日	年月日
O1	2015 9 12 - 2016 1 19	-	-	-	-	-
O2	2016 11 30 - 2017 8 25	2017 8 1 - 2017 8 25	-	-	-	-
O3a	2019 4 1 - 2019 9 30	同左	-	-	-	-
O3b	2019 11 1 - 2020 3 27	同左	-	-	-	-
O3GK	-	-	-	-	2020 4 7 - 2020 4 21	-

**観測された中で特筆すべきイベント** 突発的重力波カタログ 3 (GWTC3) として 2021 年 11 月に発表されたものが最新の重力波イベントカタログである。

GW150914 最初に報告された重力波直接観測イベント。BBH の存在を明らかにし、太陽質量 ( $M_{\odot}$ ) の 30 倍以上の BH の存在を初めて確認した。報告された BBH のイベントの中でも最もシグナル・ノイズ比 (SNR) が高い。GW170817 最初に報告された BNS イベント。直後に多くの追観測がなされ、マルチ・メッセンジャー天文学の初めての成功例となった。重力波波形から得られた中性子星の状態方程式に対する制限は核密度  $\rho_{\text{nuc}} = 2.8 \times 10^{14} \text{ g/cm}^3$  の 2 倍の密度における圧力として ( $2\rho_{\text{nuc}}$ )  $= 3.5^{+2.7}_{-1.7} \times 10^{34} \text{ dyn/cm}^2$  (90% 信頼区間) である。 $\gamma$  線が重力波のピークと 1.7 秒差で到着したことから重力波伝播速度の光速からのずれの割合は  $1 \times 10^{-15}$  以下と制限された。また、可視・赤外における追観測から鉄以上の重元

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## 重力波の項

2022年より登場

by 田中貴浩さん+HS

## 過去の観測期間

観測期	Advanced LIGO		Advanced Virgo		KAGRA	
	年月日	年月日	年月日	年月日	年月日	年月日
O1	2015 9 12	- 2016 1 19	-	-	-	-
O2	2016 11 30	- 2017 8 25	2017 8 1	- 2017 8 25	-	-
O3a	2019 4 1	- 2019 9 30	同左	-	-	-
O3b	2019 11 1	- 2020 3 27	同左	-	-	-
O3GK	-	-	-	-	2020 4 7	- 2020 4 21

## Gravitational Wave Transient Catalog

		released	arXiv	ref.	BHBH	NSNS	NSBH	BH+?	total
<b>GWTC-1</b>	O1+O2	2018/12/3	1811.12907	PRX 9 (2019) 031040	10	1			11
<b>GWTC-2</b>	O3a	2020/10/28	2010.14527	PRX 11 (2011) 021053	36	1		2	39
<b>GWTC-2.1</b>	+	2021/8/2	2108.01045		+8-3				5
<b>GWTC-3</b>	O3b	2021/11/5	2111.03606		32		3		35
				total	83	2	3	2	90





# 突発性重力波は, O1/O2/O3a,b 観測で90例

報告されたおもな重力波 (2022年7月現在)

連星の質量を  $M_1, M_2$  としたときの, チャープ質量  $M_c = (M_1 M_2)^{3/5} / (M_1 + M_2)^{1/5}$ , 質量比 (中央値の比)  $M_2/M_1$ , 有効スピン  $\chi_{\text{eff}}$ , 最終的に形成された BH の質量  $M_{\text{final}}$  (NS を含む場合は全質量  $M_{\text{全}} = M_1 + M_2$ ), 距離, 波源特定精度 (平方度)  $(\Delta\theta)^2$ , シグナル・ノイズ比を示す. 幅のある量は 90% の信頼区間. (種類ごとに日付順. BBH については, GW190521 と SNR が 17.3 より大きいもののみ.)

イベント (BBH)	$M_c (M_\odot)$	質量比	$\chi_{\text{eff}}$	$M_{\text{final}} (M_\odot)$	距離 (Mpc)	$(\Delta\theta)^2$	SNR	
GW150914	$28.6^{+1.7}_{-1.5}$	0.86	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.4}_{-3.0}$	$440^{+150}_{-170}$	182	24.4	✔ good event for test
GW190412	$13.3^{+0.5}_{-0.5}$	0.32	$0.21^{+0.12}_{-0.13}$	$35.6^{+4.8}_{-4.5}$	$720^{+240}_{-220}$	240	19.8	✔
GW190521	$63.3^{+19.6}_{-14.6}$	0.58	$-0.14^{+0.5}_{-0.45}$	$147.4^{+40.0}_{-16.0}$	$3310^{+2790}_{-1800}$	1000	14.3	✔ too heavy, low freq.
GW190521_074359	$32.8^{+3.2}_{-2.8}$	0.77	$0.1^{+0.13}_{-0.13}$	$72.6^{+6.5}_{-5.4}$	$1080^{+580}_{-530}$	470	25.9	✔
GW190814	$6.11^{+0.06}_{-0.05}$	0.11	$0^{+0.07}_{-0.07}$	$25.7^{+1.3}_{-1.3}$	$230^{+40}_{-50}$	22	25.3	✔ too light, high freq.
GW191109_010717	$47.5^{+9.6}_{-7.5}$	0.72	$-0.29^{+0.42}_{-0.31}$	$107^{+18.0}_{-15.0}$	$1290^{+1130}_{-650}$	1600	17.3	✔ too heavy, low freq.
GW191204_171526	$8.55^{+0.38}_{-0.27}$	0.69	$0.16^{+0.08}_{-0.05}$	$19.21^{+1.79}_{-0.95}$	$650^{+190}_{-250}$	350	17.5	✔ too light, high freq.
GW191216_213338	$8.33^{+0.22}_{-0.19}$	0.64	$0.11^{+0.13}_{-0.06}$	$18.87^{+2.8}_{-0.94}$	$340^{+120}_{-130}$	490	18.6	✔ too light, high freq.
GW200112_155838	$27.4^{+2.6}_{-2.1}$	0.79	$0.06^{+0.15}_{-0.15}$	$60.8^{+5.3}_{-4.3}$	$1250^{+430}_{-460}$	4300	19.8	✔
GW200129_065458	$27.2^{+2.1}_{-2.3}$	0.84	$0.11^{+0.11}_{-0.16}$	$60.3^{+4.0}_{-3.3}$	$900^{+290}_{-380}$	130	26.8	✔
GW200224_222234	$31.1^{+3.2}_{-2.6}$	0.81	$0.1^{+0.15}_{-0.15}$	$68.6^{+6.6}_{-4.7}$	$1710^{+490}_{-640}$	50.0	20	✔
GW200311_115853	$26.6^{+2.4}_{-2.0}$	0.81	$-0.02^{+0.16}_{-0.2}$	$59^{+4.8}_{-3.9}$	$1170^{+280}_{-400}$	35	17.8	✔
イベント (BNS)	$M_c (M_\odot)$	質量比	$\chi_{\text{eff}}$	$M_{\text{全}} (M_\odot)$	距離 (Mpc)	$(\Delta\theta)^2$	SNR	
GW170817	$1.186^{+0.001}_{-0.001}$	0.87	$0^{+0.02}_{-0.01}$	—	$40^{+7.0}_{-15.0}$	16	33	
GW190425	$1.44^{+0.02}_{-0.02}$	0.62	$0.07^{+0.07}_{-0.05}$	$3.4^{+0.3}_{-0.1}$	$150^{+80}_{-60}$	8700	12.4	
イベント (NSBH)	$M_c (M_\odot)$	質量比	$\chi_{\text{eff}}$	$M_{\text{全}} (M_\odot)$	距離 (Mpc)	$(\Delta\theta)^2$	SNR	
GW190917_114630	$3.7^{+0.2}_{-0.2}$	0.22	$-0.08^{+0.21}_{-0.43}$	$11.6^{+3.1}_{-2.9}$	$720^{+300}_{-310}$	2100	8.3	
GW200105_162426	$3.42^{+0.08}_{-0.08}$	0.21	$0.0^{+0.13}_{-0.18}$	$10.7^{+1.5}_{-1.4}$	$270^{+120}_{-110}$	7900	13.7	
GW200115_042309	$2.43^{+0.05}_{-0.07}$	0.24	$-0.15^{+0.24}_{-0.42}$	$7.2^{+1.8}_{-1.7}$	$290^{+150}_{-100}$	370	11.3	





1. residuals test
2. inspiral–merger–ringdown consistency test
3. parametrized tests of GW generation
4. spin-induced moments
5. modified GW dispersion relation
6. polarization content
7. ringdown
8. echoes searches

Event	Inst.	Properties					SNR	Tests performed							
		$D_L$ [Gpc]	$(1+z)M$ [ $M_\odot$ ]	$(1+z)M_f$ [ $M_\odot$ ]	$(1+z)M_r$ [ $M_\odot$ ]	$\chi_f$		RT	IMR	PAR	SIM	MDR	POL	RD	ECH
GW191109.010717	HL	1.29 <sup>+1.13</sup> <sub>-0.65</sub>	140 <sup>+21</sup> <sub>-17</sub>	60.1 <sup>+9.8</sup> <sub>-9.3</sub>	135 <sup>+19</sup> <sub>-15</sub>	0.61 <sup>+0.18</sup> <sub>-0.19</sub>	17.3 <sup>+0.5</sup> <sub>-0.5</sub>	✓	-	-	-	-	✓	✓	✓
GW191129.134029	HL	0.79 <sup>+0.26</sup> <sub>-0.33</sub>	20.10 <sup>+2.94</sup> <sub>-0.64</sub>	8.49 <sup>+0.06</sup> <sub>-0.05</sub>	19.19 <sup>+3.07</sup> <sub>-0.67</sub>	0.69 <sup>+0.03</sup> <sub>-0.05</sub>	13.1 <sup>+0.2</sup> <sub>-0.3</sub>	✓	-	-	-	-	✓	✓	✓
GW191204.171526	HL	0.65 <sup>+0.19</sup> <sub>-0.25</sub>	22.74 <sup>+1.94</sup> <sub>-0.44</sub>	9.70 <sup>+0.05</sup> <sub>-0.05</sub>	21.60 <sup>+2.05</sup> <sub>-0.50</sub>	0.73 <sup>+0.03</sup> <sub>-0.03</sub>	17.5 <sup>+0.2</sup> <sub>-0.2</sub>	✓	-	✓	✓	✓	✓	✓	✓
GW191215.223052	HLV	1.93 <sup>+0.89</sup> <sub>-0.86</sub>	58.4 <sup>+4.8</sup> <sub>-3.7</sub>	24.9 <sup>+1.5</sup> <sub>-1.4</sub>	55.8 <sup>+4.4</sup> <sub>-3.3</sub>	0.68 <sup>+0.07</sup> <sub>-0.07</sub>	11.2 <sup>+0.3</sup> <sub>-0.4</sub>	✓	-	-	-	-	✓	✓	✓
GW191216.213338	HV	0.34 <sup>+0.12</sup> <sub>-0.13</sub>	21.17 <sup>+2.93</sup> <sub>-0.66</sub>	8.94 <sup>+0.05</sup> <sub>-0.05</sub>	20.18 <sup>+3.06</sup> <sub>-0.70</sub>	0.70 <sup>+0.03</sup> <sub>-0.04</sub>	18.6 <sup>+0.2</sup> <sub>-0.2</sub>	✓	-	✓	✓	✓	✓	✓	✓
GW191222.033537	HL	3.0 <sup>+1.7</sup> <sub>-1.7</sub>	119 <sup>+16</sup> <sub>-13</sub>	51.0 <sup>+7.2</sup> <sub>-6.5</sub>	114 <sup>+14</sup> <sub>-12</sub>	0.67 <sup>+0.08</sup> <sub>-0.11</sub>	12.5 <sup>+0.2</sup> <sub>-0.3</sub>	✓	-	-	-	-	✓	✓	✓
GW200115.042309	HLV	0.29 <sup>+0.15</sup> <sub>-0.10</sub>	7.8 <sup>+1.9</sup> <sub>-1.4</sub>	2.58 <sup>+0.01</sup> <sub>-0.01</sub>	7.7 <sup>+1.9</sup> <sub>-1.8</sub>	0.42 <sup>+0.09</sup> <sub>-0.05</sub>	11.3 <sup>+0.3</sup> <sub>-0.3</sub>	✓	-	✓	-	-	-	-	✓
GW200129.065458	HLV	0.90 <sup>+0.29</sup> <sub>-0.38</sub>	74.6 <sup>+4.5</sup> <sub>-3.8</sub>	32.1 <sup>+1.8</sup> <sub>-2.6</sub>	70.9 <sup>+4.2</sup> <sub>-3.4</sub>	0.73 <sup>+0.06</sup> <sub>-0.05</sub>	26.8 <sup>+0.2</sup> <sub>-0.2</sub>	✓	✓	✓	✓	✓	✓	✓	✓
GW200202.154313	HLV	0.41 <sup>+0.15</sup> <sub>-0.16</sub>	19.01 <sup>+1.99</sup> <sub>-0.31</sub>	8.15 <sup>+0.05</sup> <sub>-0.05</sub>	18.12 <sup>+2.09</sup> <sub>-0.35</sub>	0.69 <sup>+0.03</sup> <sub>-0.04</sub>	10.8 <sup>+0.2</sup> <sub>-0.4</sub>	✓	-	✓	-	-	✓	-	✓
GW200208.130117	HLV	2.23 <sup>+1.00</sup> <sub>-0.85</sub>	91 <sup>+11</sup> <sub>-10</sub>	38.8 <sup>+5.2</sup> <sub>-2.8</sub>	87.5 <sup>+10.3</sup> <sub>-9.1</sub>	0.66 <sup>+0.09</sup> <sub>-0.13</sub>	10.8 <sup>+0.3</sup> <sub>-0.4</sub>	✓	✓	-	-	-	✓	✓	✓
GW200219.094415	HLV	3.4 <sup>+1.7</sup> <sub>-1.5</sub>	103 <sup>+14</sup> <sub>-12</sub>	43.7 <sup>+6.3</sup> <sub>-6.2</sub>	98 <sup>+13</sup> <sub>-11</sub>	0.66 <sup>+0.10</sup> <sub>-0.13</sub>	10.7 <sup>+0.3</sup> <sub>-0.5</sub>	✓	-	-	-	-	✓	✓	✓
GW200224.222234	HLV	1.71 <sup>+0.49</sup> <sub>-0.64</sub>	94.9 <sup>+5.3</sup> <sub>-7.2</sub>	40.9 <sup>+3.5</sup> <sub>-3.8</sub>	90.2 <sup>+7.5</sup> <sub>-6.4</sub>	0.73 <sup>+0.07</sup> <sub>-0.07</sub>	20.0 <sup>+0.2</sup> <sub>-0.2</sub>	✓	✓	-	-	-	✓	✓	✓
GW200225.060421	HL	1.15 <sup>+0.51</sup> <sub>-0.53</sub>	41.2 <sup>+4.0</sup> <sub>-4.0</sub>	17.65 <sup>+0.98</sup> <sub>-1.97</sub>	39.4 <sup>+2.9</sup> <sub>-3.6</sub>	0.66 <sup>+0.07</sup> <sub>-0.13</sub>	12.5 <sup>+0.3</sup> <sub>-0.4</sub>	✓	✓	✓	-	-	✓	✓	✓
GW200311.115853	HLV	1.17 <sup>+0.28</sup> <sub>-0.40</sub>	75.9 <sup>+5.2</sup> <sub>-5.7</sub>	32.7 <sup>+2.7</sup> <sub>-2.8</sub>	72.4 <sup>+5.6</sup> <sub>-5.1</sub>	0.69 <sup>+0.07</sup> <sub>-0.08</sub>	17.8 <sup>+0.2</sup> <sub>-0.2</sub>	✓	✓	✓	-	-	✓	✓	✓
GW200316.215756	HLV	1.12 <sup>+0.47</sup> <sub>-0.44</sub>	25.5 <sup>+8.7</sup> <sub>-1.1</sub>	10.68 <sup>+0.12</sup> <sub>-0.12</sub>	24.3 <sup>+9.0</sup> <sub>-1.1</sub>	0.70 <sup>+0.04</sup> <sub>-0.04</sub>	10.3 <sup>+0.4</sup> <sub>-0.7</sub>	✓	-	✓	✓	-	-	-	✓

## 7. ringdown test

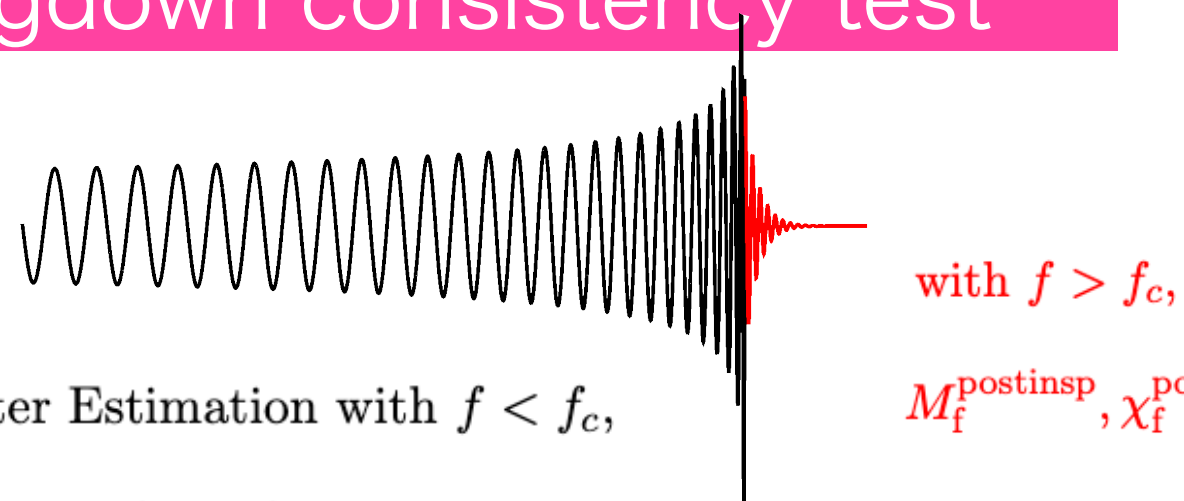
$$h_+(t) - ih_\times(t) = \sum_{\ell=2}^{+\infty} \sum_{m=-\ell}^{\ell} \sum_{n=0}^{+\infty} \mathcal{A}_{\ell mn} \exp\left[-\frac{t-t_0}{(1+z)\tau_{\ell mn}}\right] \exp\left[\frac{2\pi i f_{\ell mn}(t-t_0)}{1+z}\right] {}_{-2}S_{\ell mn}(\theta, \phi, \chi_f)$$

time-domain ringdown analysis pyRing, based on damped sinusoids,

parametrized ringdown analysis pSEOBNR, based on the SEOBNRv4HM waveform model.

Event	Redshifted final mass (1+z)M <sub>f</sub> [M <sub>⊙</sub> ]				Final spin χ <sub>f</sub>				Higher modes	Overtones	
	IMR	Kerr <sub>220</sub>	Kerr <sub>221</sub>	Kerr <sub>HM</sub>	IMR	Kerr <sub>220</sub>	Kerr <sub>221</sub>	Kerr <sub>HM</sub>	log <sub>10</sub> B <sub>220</sub> <sup>HM</sup>	log <sub>10</sub> B <sub>220</sub> <sup>221</sup>	log <sub>10</sub> O <sub>GR</sub> <sup>mod</sup>
GW191109.010717	132.7 <sup>+21.9</sup> <sub>-13.8</sub>	181.7 <sup>+28.5</sup> <sub>-30.6</sub>	179.0 <sup>+23.7</sup> <sub>-21.7</sub>	174.5 <sup>+38.1</sup> <sub>-30.1</sub>	0.60 <sup>+0.22</sup> <sub>-0.19</sub>	0.81 <sup>+0.10</sup> <sub>-0.14</sub>	0.81 <sup>+0.08</sup> <sub>-0.14</sub>	0.77 <sup>+0.11</sup> <sub>-0.21</sub>	-0.11	1.03	-0.27
GW191222.033537	114.2 <sup>+14.3</sup> <sub>-11.7</sub>	111.4 <sup>+69.3</sup> <sub>-29.7</sub>	110.3 <sup>+36.2</sup> <sub>-23.8</sub>	118.3 <sup>+97.0</sup> <sub>-46.2</sub>	0.67 <sup>+0.08</sup> <sub>-0.10</sub>	0.46 <sup>+0.41</sup> <sub>-0.41</sub>	0.52 <sup>+0.31</sup> <sub>-0.43</sub>	0.60 <sup>+0.28</sup> <sub>-0.66</sub>	0.08	-0.83	-0.20
GW200129.065458	71.8 <sup>+4.4</sup> <sub>-3.9</sub>	60.0 <sup>+16.7</sup> <sub>-8.9</sub>	77.0 <sup>+14.4</sup> <sub>-14.2</sub>	219.1 <sup>+110.4</sup> <sub>-140.0</sub>	0.75 <sup>+0.06</sup> <sub>-0.06</sub>	0.31 <sup>+0.43</sup> <sub>-0.28</sub>	0.74 <sup>+0.17</sup> <sub>-0.59</sub>	0.54 <sup>+0.35</sup> <sub>-0.59</sub>	-0.00	-0.47	-0.09
GW200224.222234	90.3 <sup>+6.4</sup> <sub>-6.3</sub>	84.4 <sup>+23.2</sup> <sub>-20.3</sub>	88.6 <sup>+15.5</sup> <sub>-15.2</sub>	119.4 <sup>+142.6</sup> <sub>-34.3</sub>	0.73 <sup>+0.06</sup> <sub>-0.07</sub>	0.61 <sup>+0.27</sup> <sub>-0.42</sub>	0.60 <sup>+0.23</sup> <sub>-0.42</sub>	0.64 <sup>+0.27</sup> <sub>-0.59</sub>	0.20	0.95	-0.11
GW200311.115853	72.1 <sup>+5.4</sup> <sub>-4.7</sub>	68.5 <sup>+23.6</sup> <sub>-13.5</sub>	72.2 <sup>+28.6</sup> <sub>-16.3</sub>	213.2 <sup>+167.8</sup> <sub>-141.5</sub>	0.68 <sup>+0.07</sup> <sub>-0.08</sub>	0.30 <sup>+0.44</sup> <sub>-0.28</sub>	0.58 <sup>+0.30</sup> <sub>-0.47</sub>	0.56 <sup>+0.32</sup> <sub>-0.54</sub>	0.02	-1.16	-0.15

## 2. inspiral–merger–ringdown consistency test



Parameter Estimation with  $f < f_c$ ,

with  $f > f_c$ ,

$M_f^{\text{postinsp}}, \chi_f^{\text{postinsp}}$

$M_f^{\text{insp}}, \chi_f^{\text{insp}}$

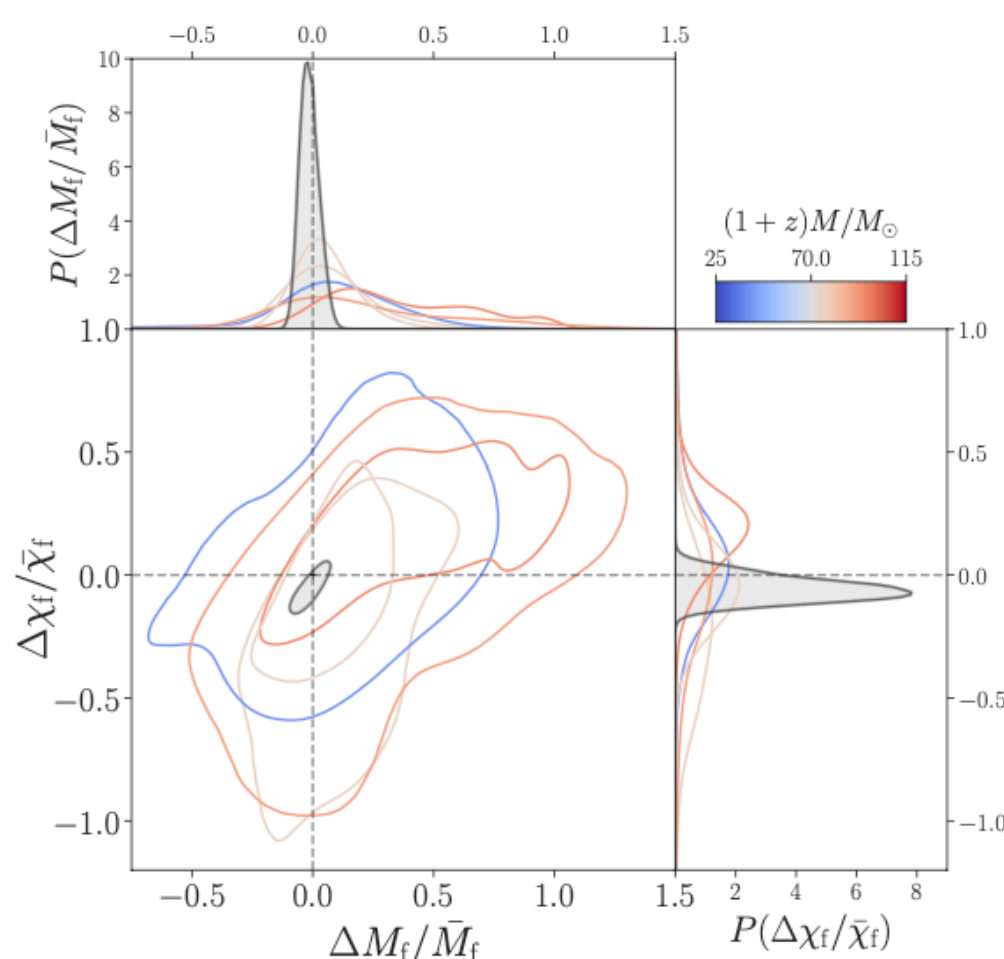
Waveform models

IMRPhenomXPHM - phenomenological PN-based models, calibrated to NR

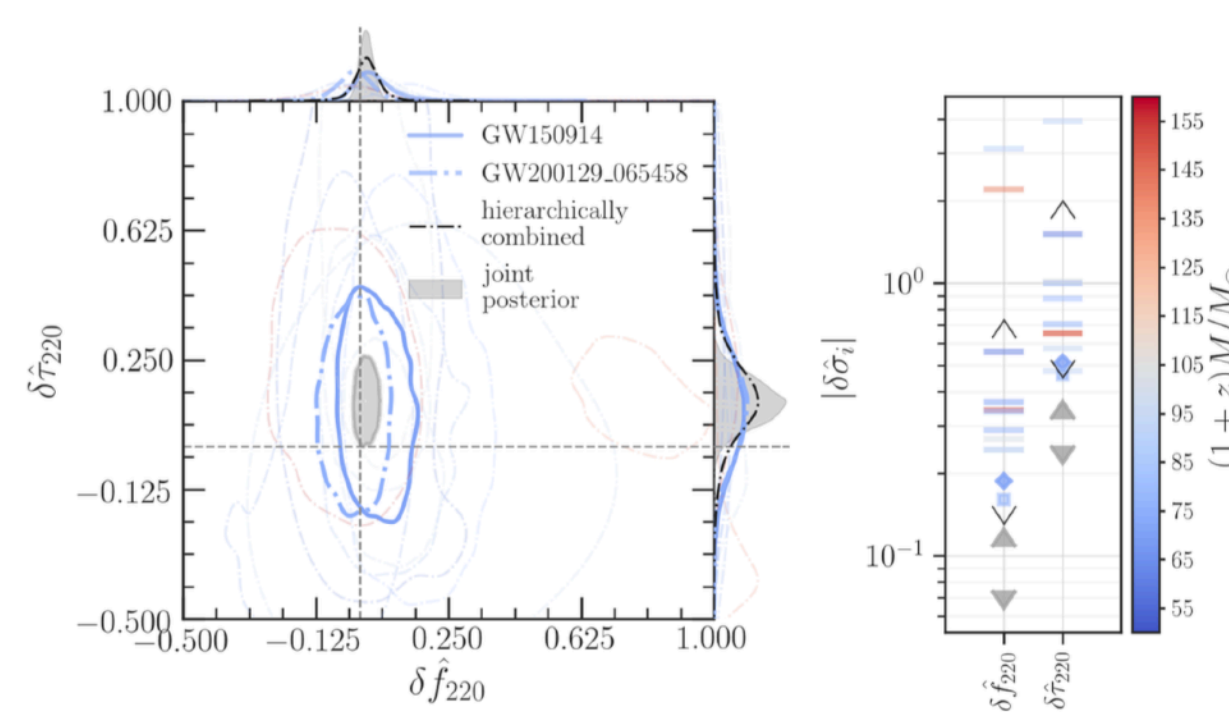
Event	$f_c^{\text{IMR}}$ [Hz]	$\rho_{\text{IMR}}$	$\rho_{\text{insp}}$	$\rho_{\text{postinsp}}$	$Q_{\text{GR}}^{2D}$ [%]
GW200129.065458	136	25.7	20.1	16.0	1.5
GW200208.130117	98	9.9	7.2	6.8	10.5
GW200224.222234	107	19.4	14.3	13.1	20.7
GW200225.060421	213	12.9	11.1	6.6	1.3
GW200311.115853	122	17.5	13.5	11.0	15.2

the fraction of the posterior enclosed by the isoprobability contour that passes through (0, 0)  
[smaller values indicate better consistency]

No statistically significant deviations from GR



$$\frac{\Delta M_f}{\bar{M}_f} = 2 \frac{M_f^{\text{insp}} - M_f^{\text{postinsp}}}{M_f^{\text{insp}} + M_f^{\text{postinsp}}}, \quad \frac{\Delta \chi_f}{\bar{\chi}_f} = 2 \frac{\chi_f^{\text{insp}} - \chi_f^{\text{postinsp}}}{\chi_f^{\text{insp}} + \chi_f^{\text{postinsp}}}$$



>0 supports higher modes

>0 supports first overtone

>0 disagreement with GR

No statistically significant deviations from GR



# 自己回帰モデル

# Auto-Regressive model (Method, general)

Fitting data with linear func.

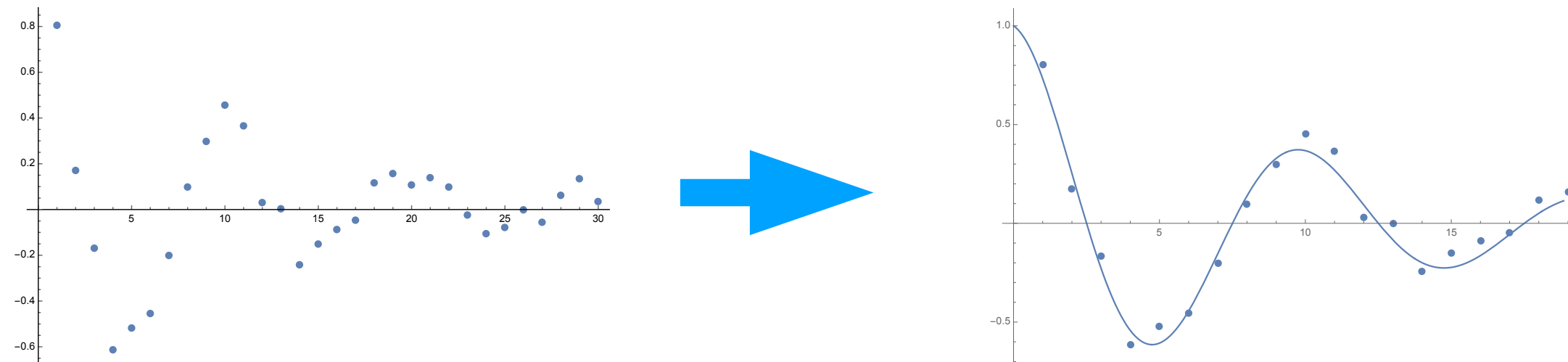
$$x_n = a_1x_{n-1} + a_2x_{n-2} + \dots + a_Mx_{n-M} + \varepsilon$$

$$= \sum_{j=1}^M a_jx_{n-j} + \varepsilon$$

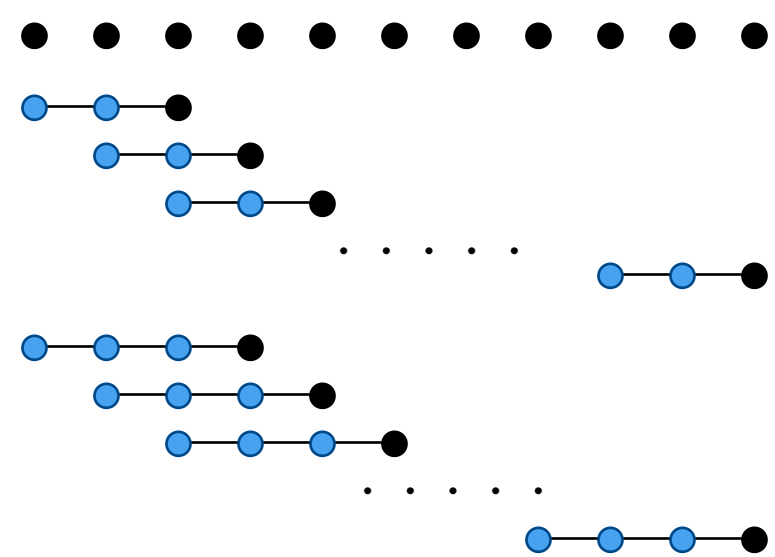
e.g.  $x_n = Ae^{-rn\Delta t} \cos(\omega n\Delta t)$

$$Z_1 = e^{-(r-j\omega)\Delta t} \rightarrow x_n = \frac{A}{2}(Z_1^n + Z_2^n) = (Z_1 + Z_2)x_{n-1} - Z_1Z_2x_{n-2}$$

$$Z_2 = e^{-(r+j\omega)\Delta t}$$



can be applied also to noisy data by adjusting  $M$



- find  $a_j$  (Burg method)
- find  $M$  (FPE final prediction error method)
- re-construct wave signal from fitted function
- apply FFT with arbitrary precision.

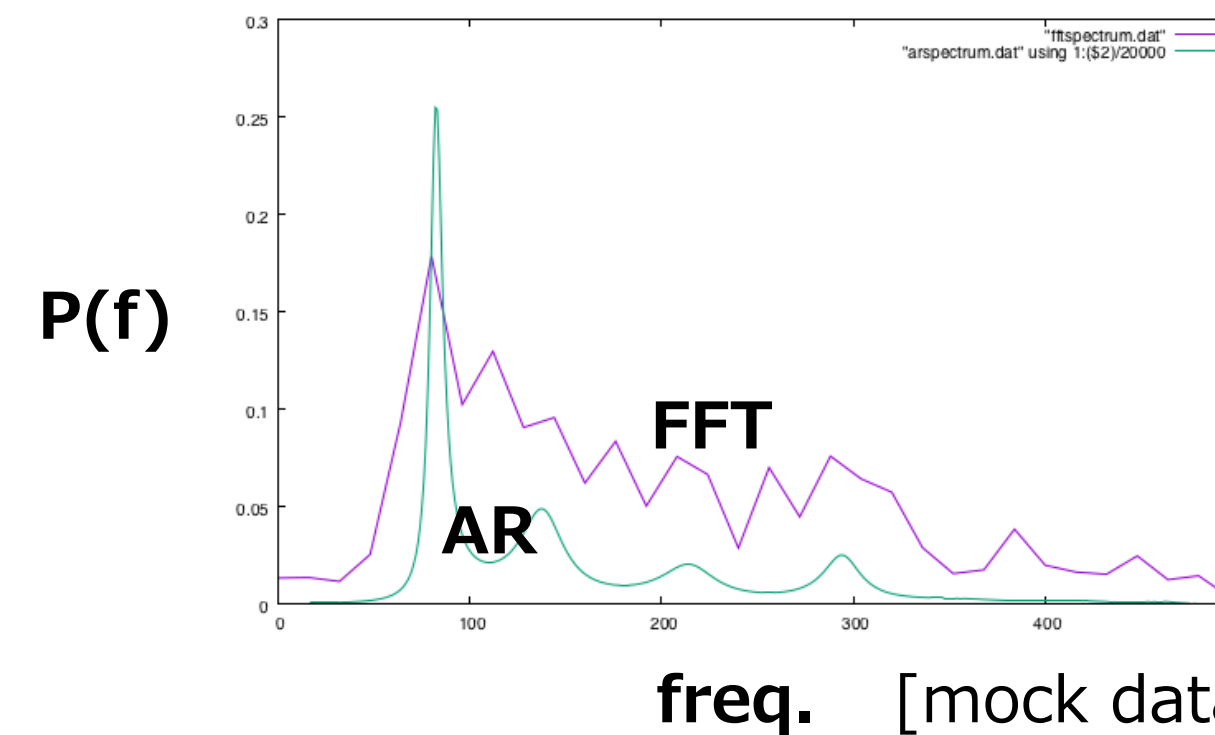
power spectrum

$$p(f) = \frac{\sigma^2}{\left| 1 - \sum_{j=1}^M a_j e^{-I2\pi j f \Delta t} \right|^2}$$

characteristic eq.

$$f(z) = 1 - \sum_{j=1}^M a_j z^j = 0$$

$|z_k|$  says amplitude,  
 $\arg(z_k)$  says frequency.



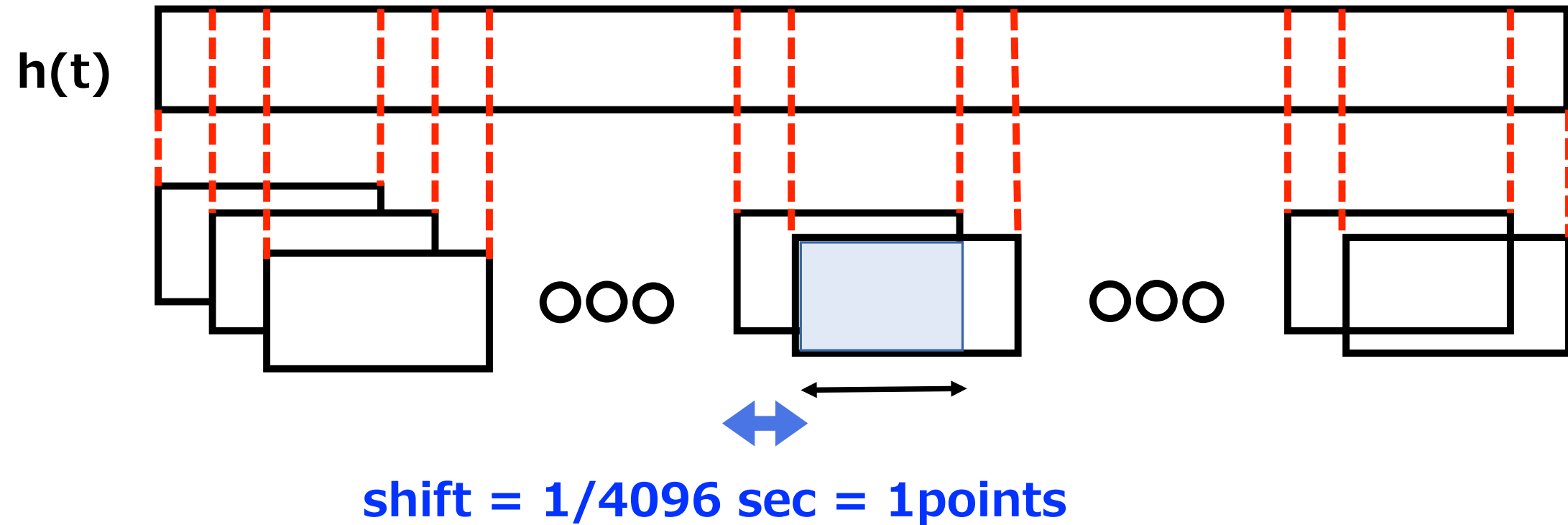
The order  $M$  can be fixed at 2~8.

Even for short segment,  
AR model shows precise  
power-spectrum.



# 自己回帰モデル(重力波解析手順) Auto-Regressive model (GW)

sampling rate=4096 segment = 1/16 sec = 256 points  
or = 1/64 sec = 64 points



干渉計のイベントが報告されたデータ(ホワイトニング後)で, 合体時刻後のデータを解析する.  
自己回帰モデルを用いて, 周波数  $f_{\text{real}}$  と振幅減衰率  $\tau (= f_{\text{imag}})$  を特定する.

モデルを仮定せずに, 実データのみに基づく 検出器ごと独立に解析

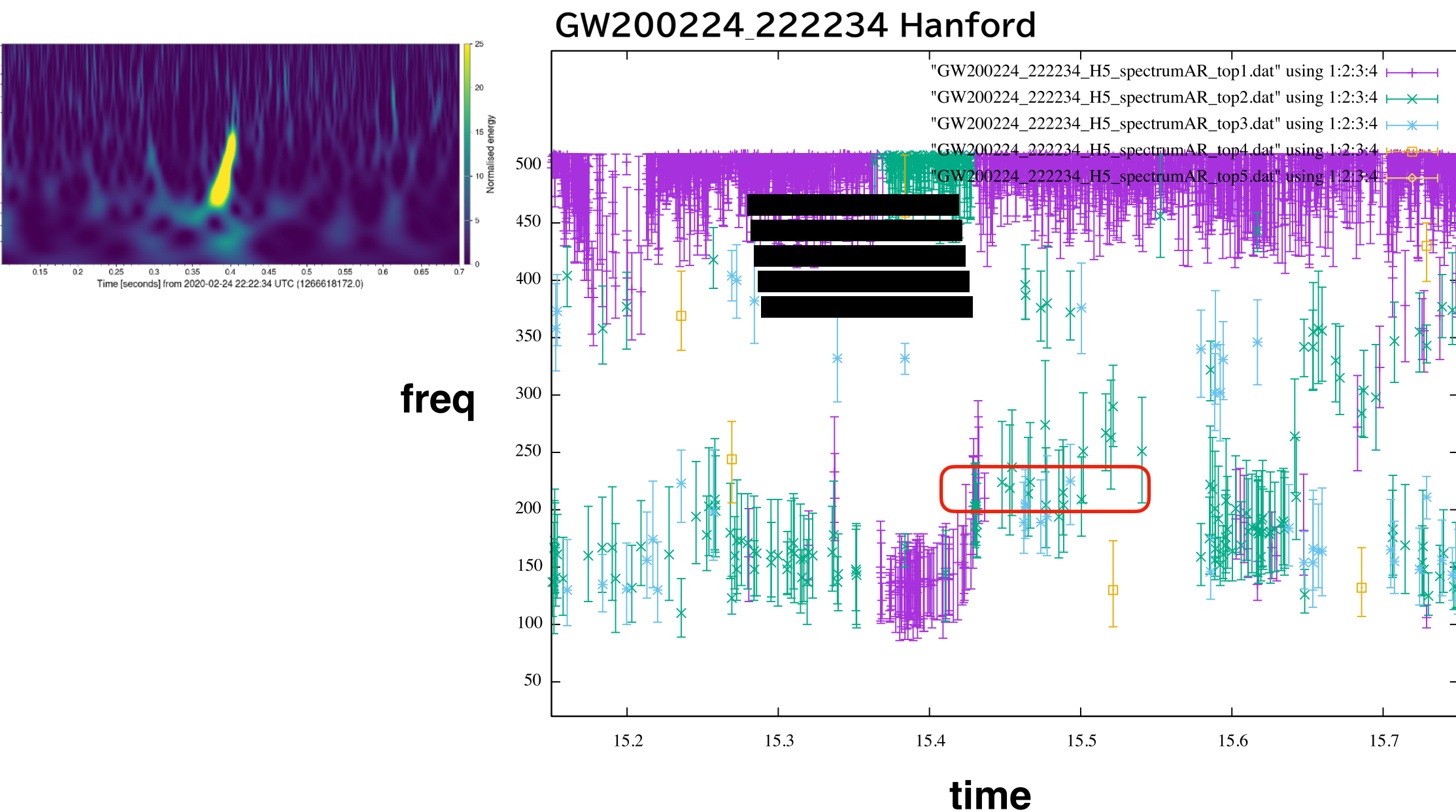
一定周波数となっているものを探す. (リングダウンモード !?)

$f_{\text{real}}$  の散らばりが  $1\sigma$  以内,  $\tau (= f_{\text{imag}})$  の散らばりが  $1.25\sigma$  以内のものを抽出

各モードの存在時間が抽出可能

各データセグメントで, 複数の  $(f, \tau)$  を抽出可能

複数のモードを抽出可能



LVK catalog の 最終的なBHの  $(M_f, a_f)$  + 赤方偏移  $z$  のデータから

検出データのリングダウンモード  $(f, \tau)$  を求め, GR値とみなす

$$f_R = f_1 + f_2(1-a)^{f_3}$$

$$Q \equiv \frac{f_R}{2f_I} = q_1 + q_2(1-a)^{q_3}$$

$$f_{\text{qnm}}[\text{Hz}] = \frac{c^3}{2\pi GM} f_R \sim 32314.1 \left(\frac{M_\odot}{M}\right) f_R.$$

$$a = 1 - \left(\frac{Q - q_1}{q_2}\right)^{1/q_3}$$

$$M[M_\odot] = 32314.1 \times \frac{f_1 + f_2(1-a)^{f_3}}{f_{\text{qnm}}[\text{Hz}]}$$

Berti, Cardoso & Will PRD 73, 064030 (2006).

AR法はリングダウン波を抽出しているか

3干渉計(Hanford, Livingston, Virgo) で合致した値になるか  
LVK catalogと合致した値になるか

リングダウン波の開始時刻はいつか

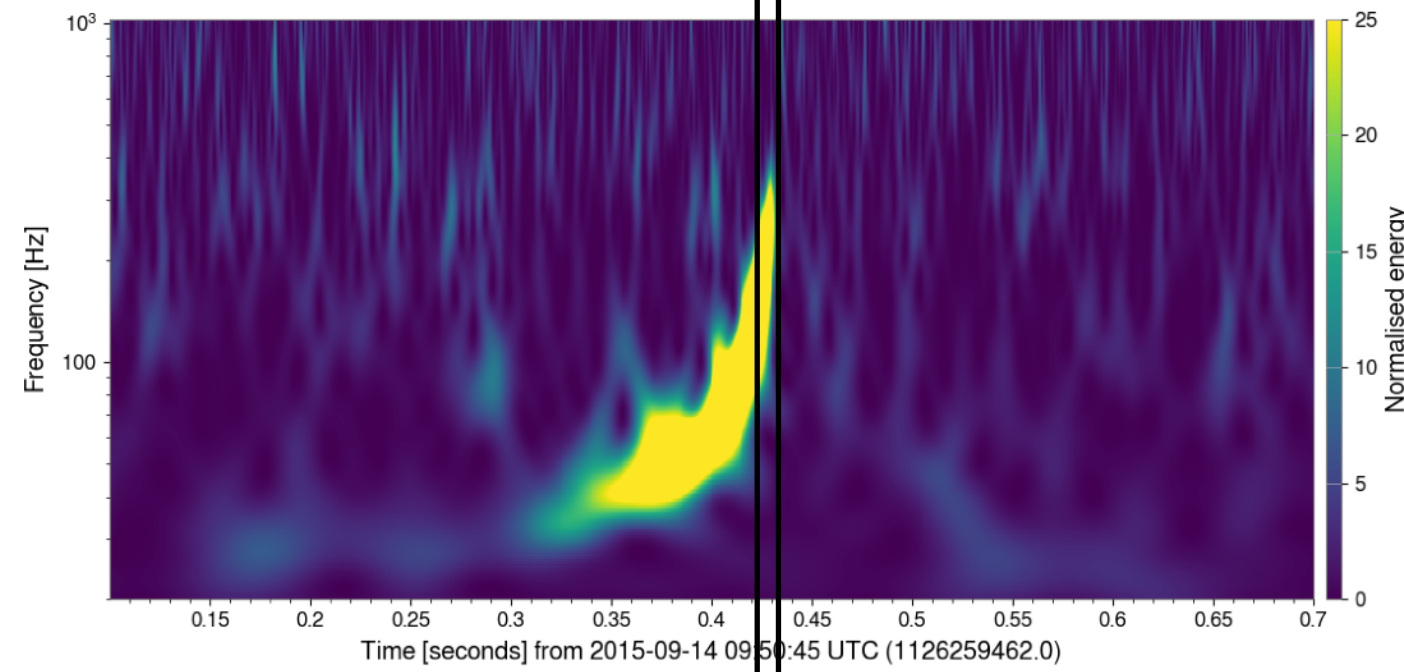
高次モード, 高調波モードはみつかるか  
GRと矛盾しないか



# 重力波到来方向の特定から、各検出器での連星「合体」重力波通過時刻を補正

## GW150914の例

Hanford



Livingston

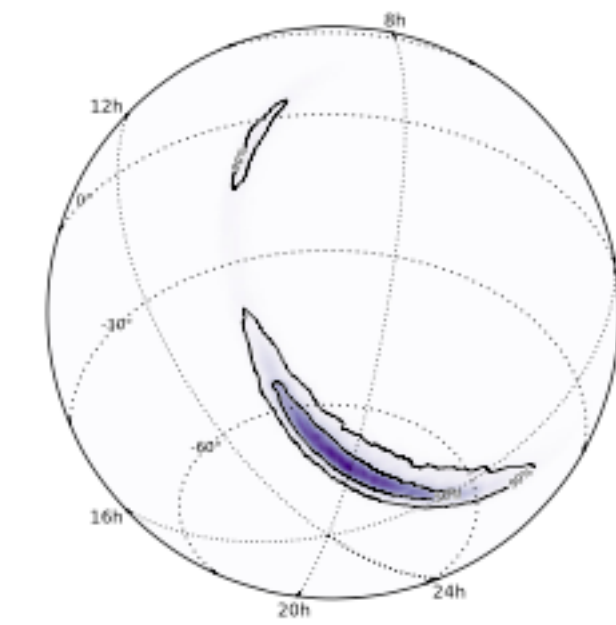
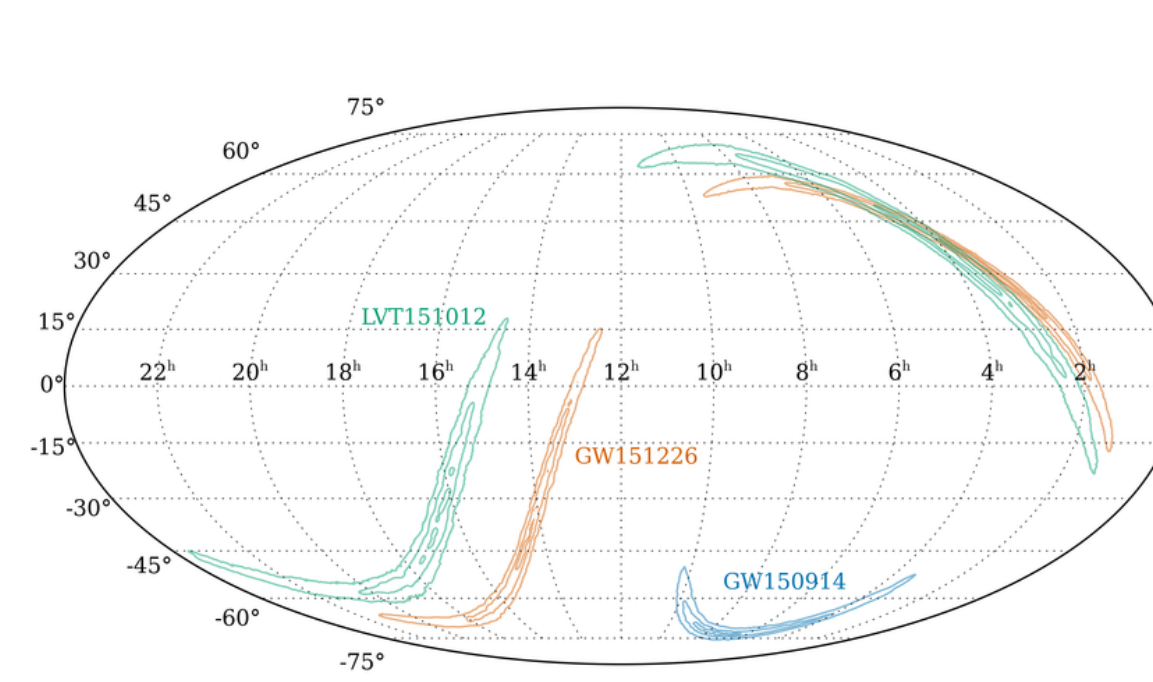
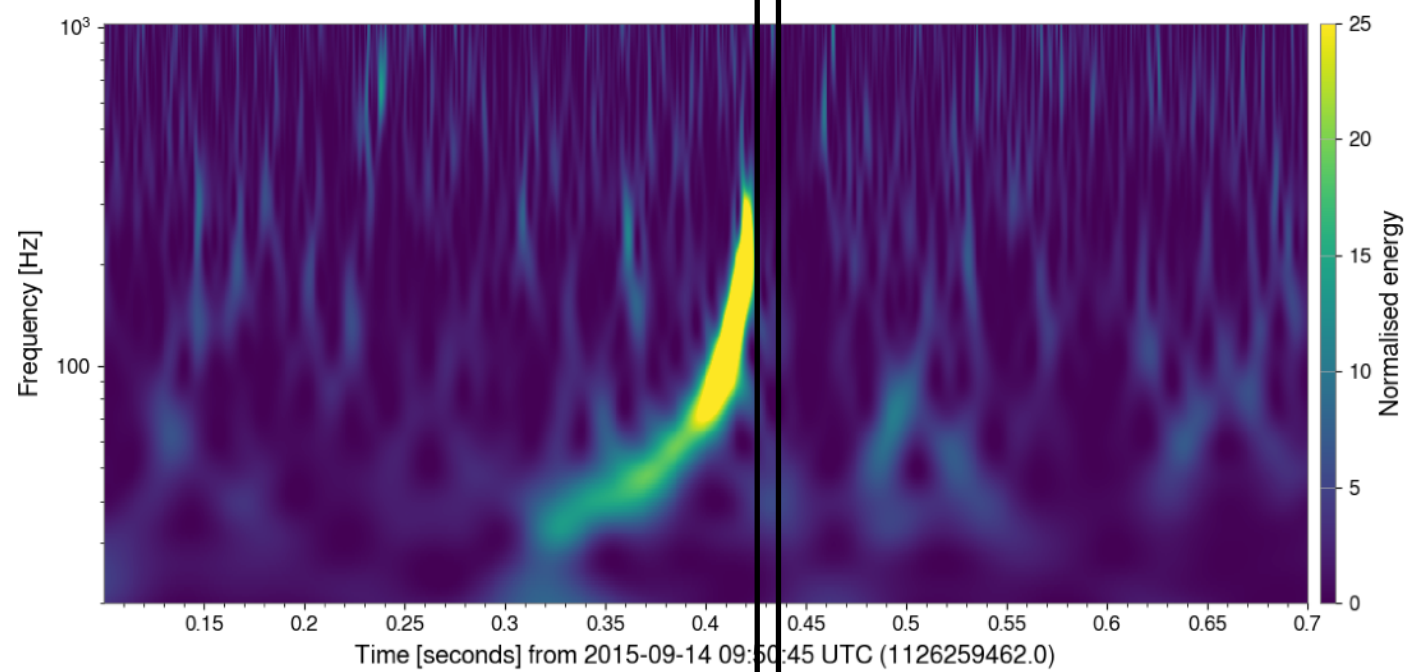
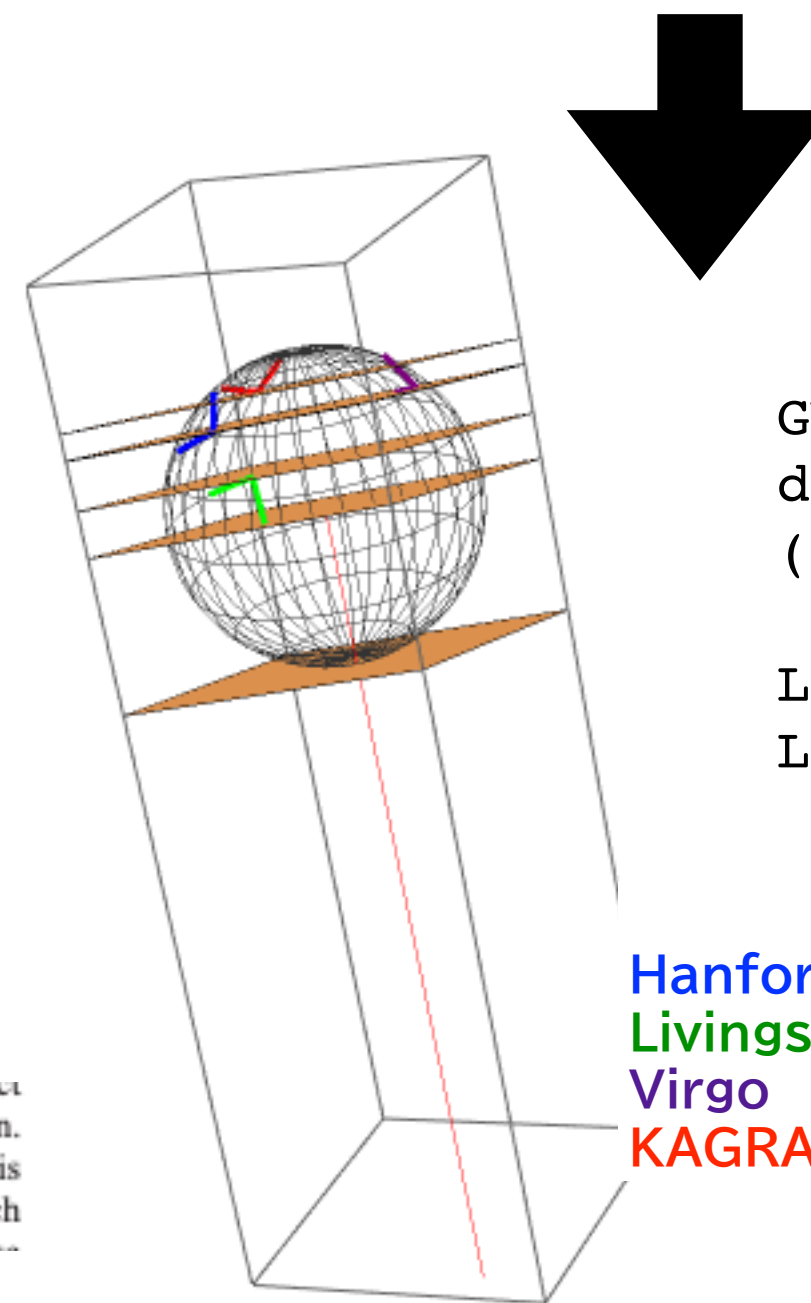
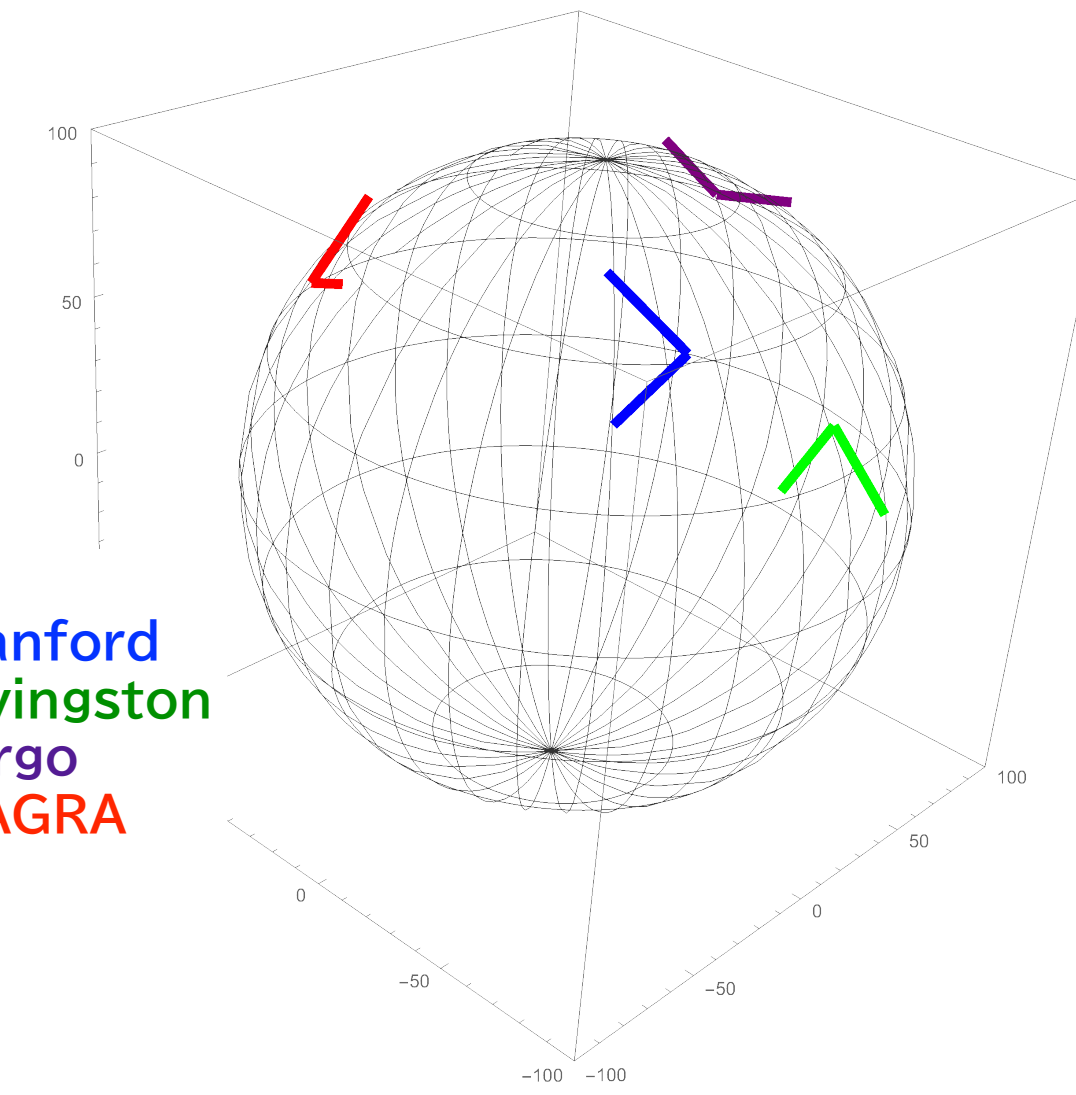


FIG. 4. An orthographic projection of the PDF for the sky location of GW150914 given in terms of right ascension  $\alpha$  (measured in hours and labeled around the edge of the figure) and declination  $\delta$  (measured in degrees and labeled inside the figure). The contours of the 50% and 90% credible regions are plotted over a color-coded PDF. The sky localization forms part of an annulus, set by the time delay of  $6.9^{+0.5}_{-0.4}$  ms between the Livingston and Hanford detectors.

GPS: 1126259462.4  
UTC Time: 2015-09-14 09:50

PRL 116 (2016) 241102

Hanford  
Livingston  
Virgo  
KAGRA



GW150914  
delay time (msec) from  $t_0$   
(+ delay, - advanced)

LHO = 12.8483  
LLO = 6.26193

Hanford  
Livingston  
Virgo  
KAGRA

PRL 116, 221101 (2016) PHYSICAL REVIEW

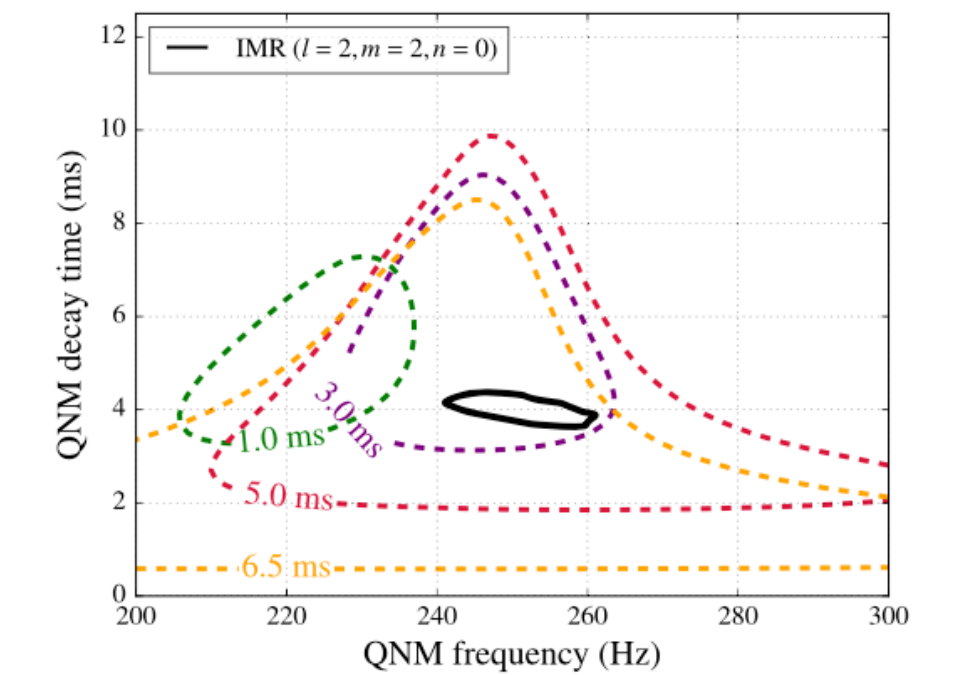


FIG. 5. 90% credible regions in the joint posterior distributions for the damped-sinusoid parameters  $f_0$  and  $\tau$  (see the main text), assuming start times  $t_0 = t_M + 1, 3, 5, 6.5$  ms, where  $t_M$  is the merger time of the MAP waveform for GW150914. The black solid line shows the 90% credible region for the frequency and decay time of the  $\ell = 2, m = 2, n = 0$  (i.e., the least-damped) QNM, as derived from the posterior distributions of the remnant mass and spin parameters.

PRL 116 (2016) 061102

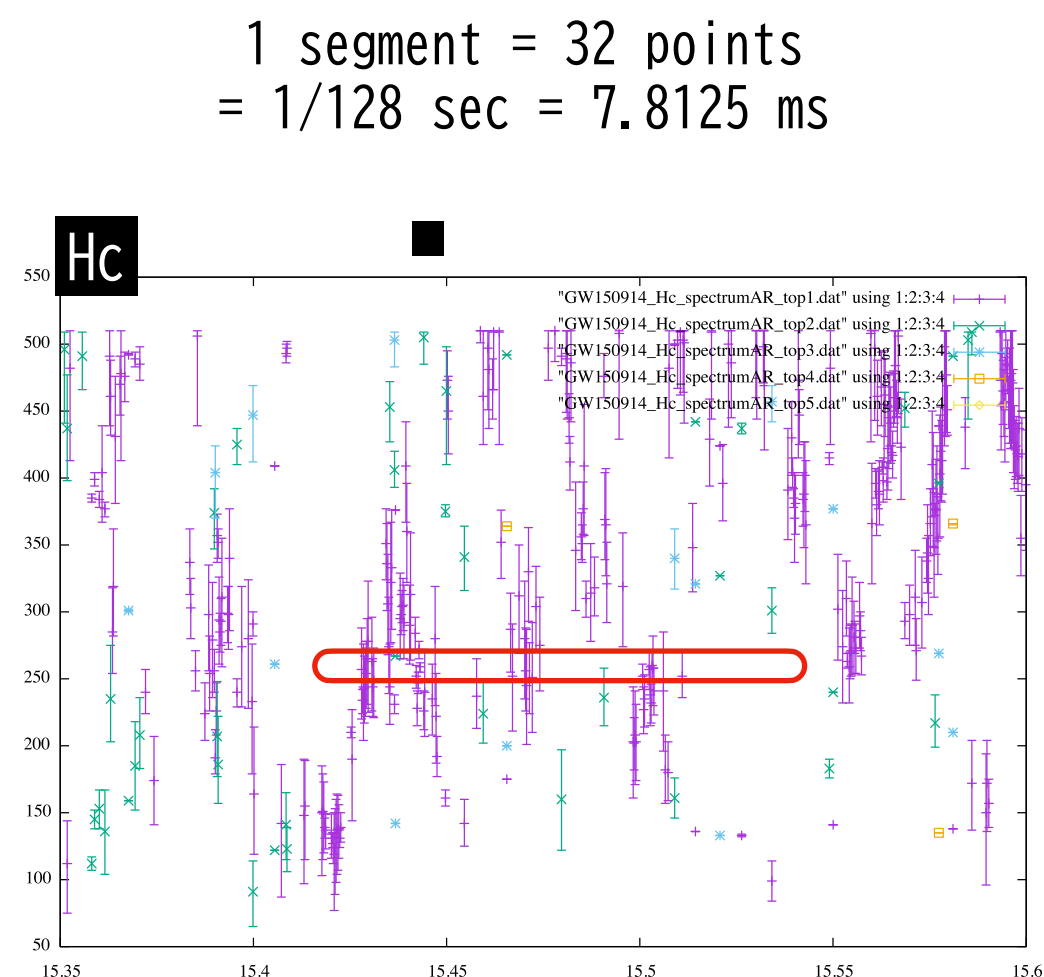
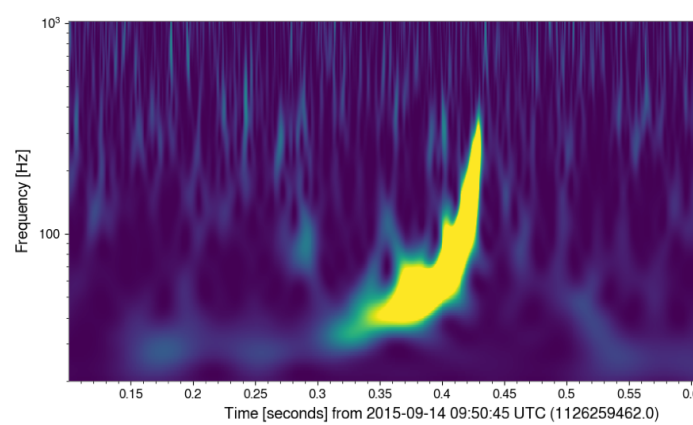


# AR法を適用するセグメント長と結果の違い

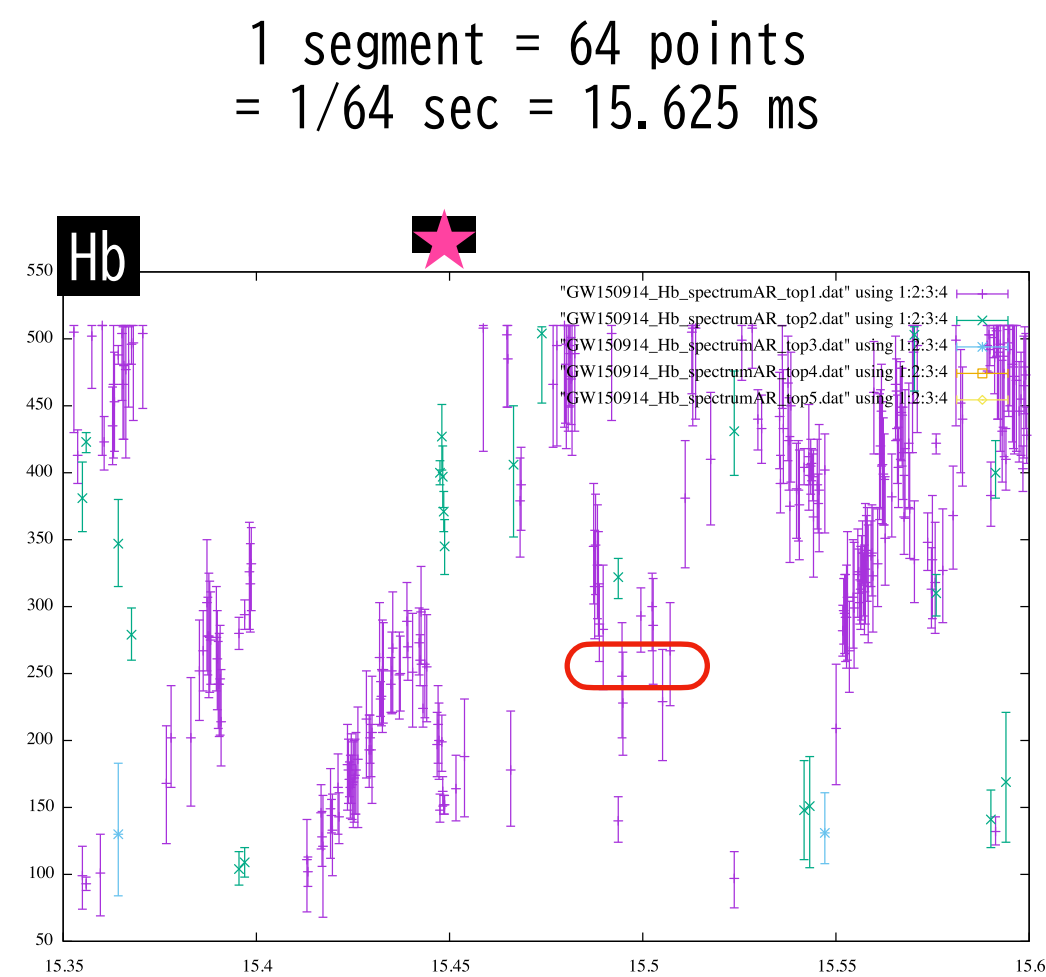
1. セグメント時刻を中央値で表示しているため、実合体時刻+セグメント長/2 が合体時刻と示される
2. セグメントが長いと、短命なリングダウン波は、周波数候補の首位に表れにくくなる
3. セグメントが短いと、突発的なノイズを多く拾うようになる

## GW150914

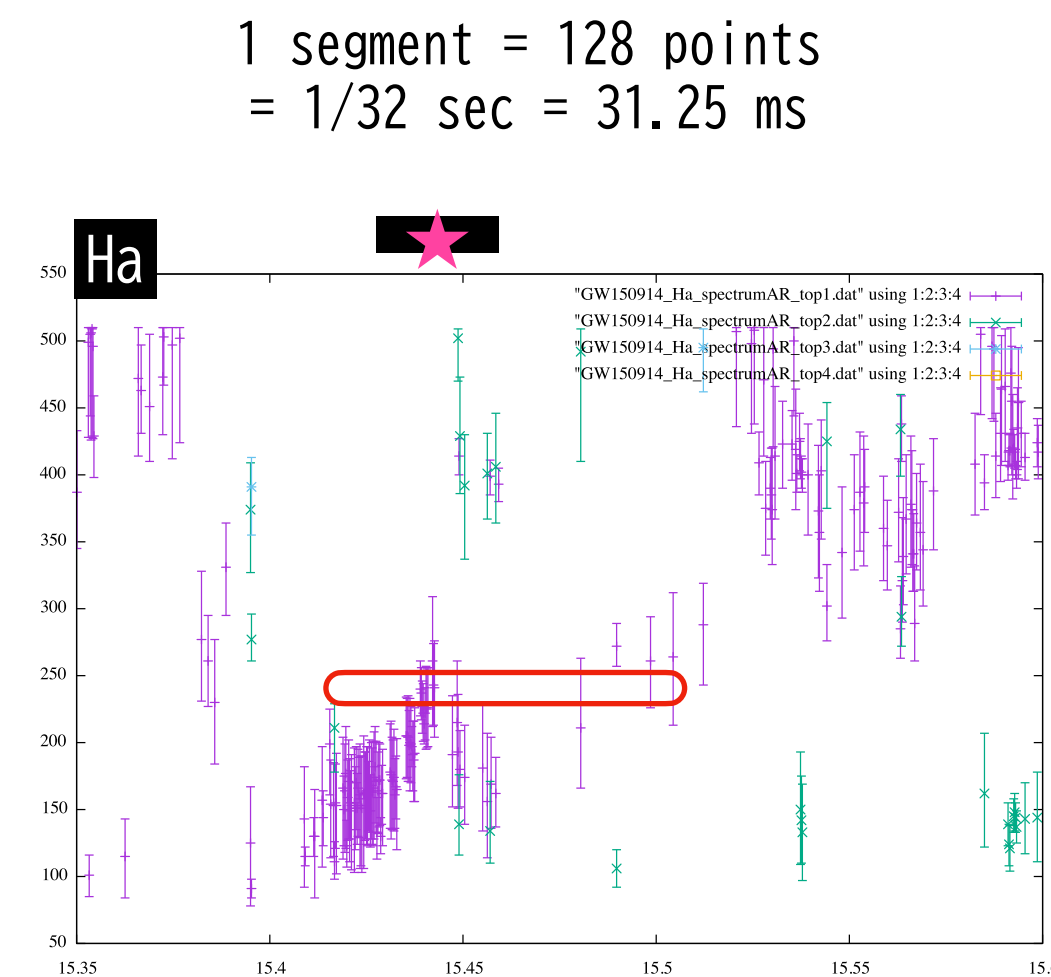
### Hanford



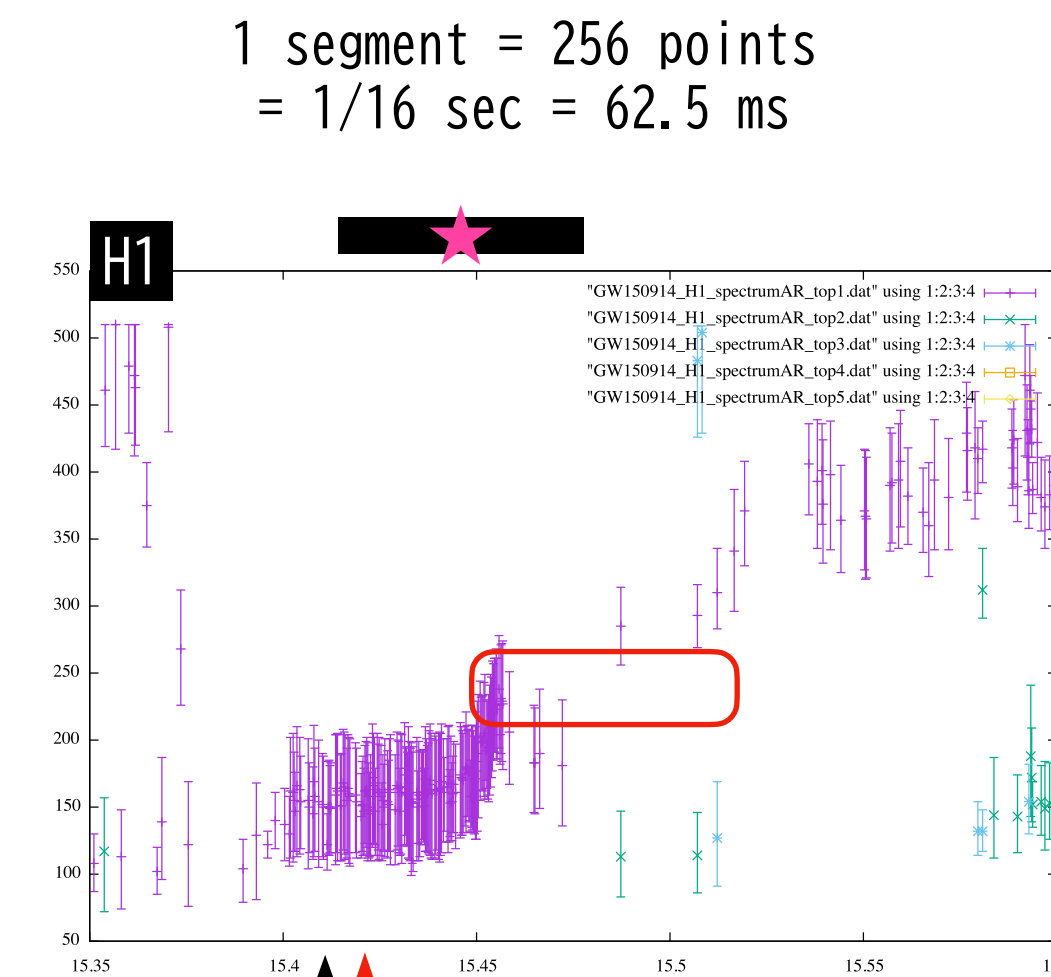
21 segments  
f=251.85 ± 9.17 Hz  
t=15.4285 -15.5449



7 segments  
f=251.63 ± 14.6 Hz  
t=15.4705 -15.5188

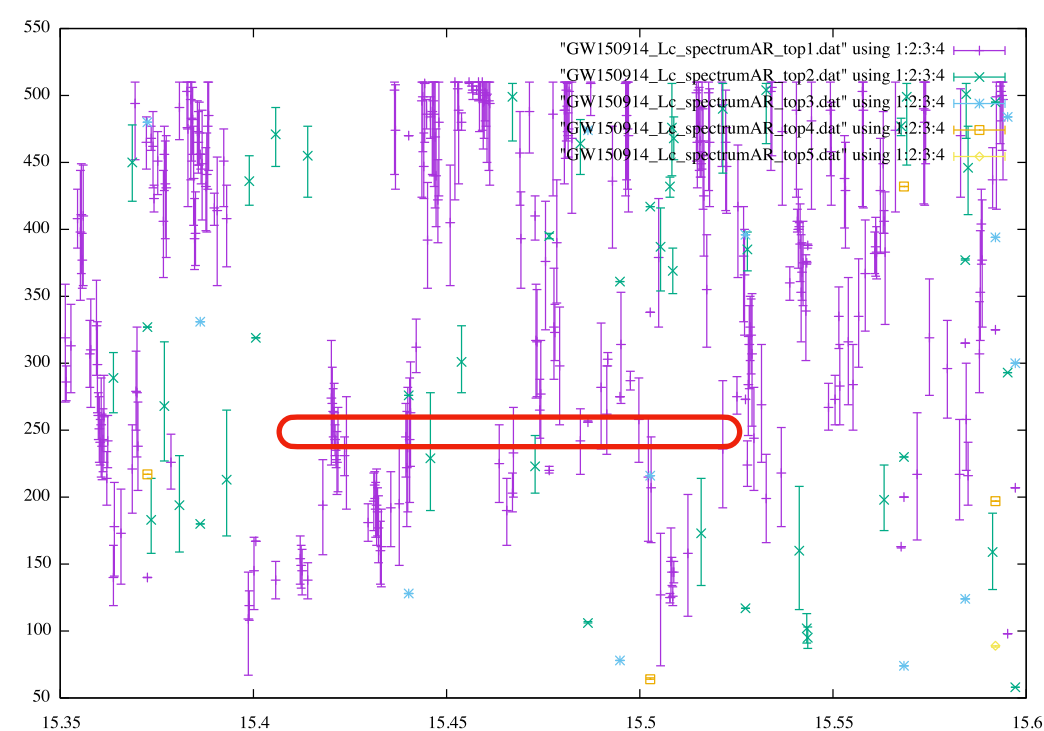
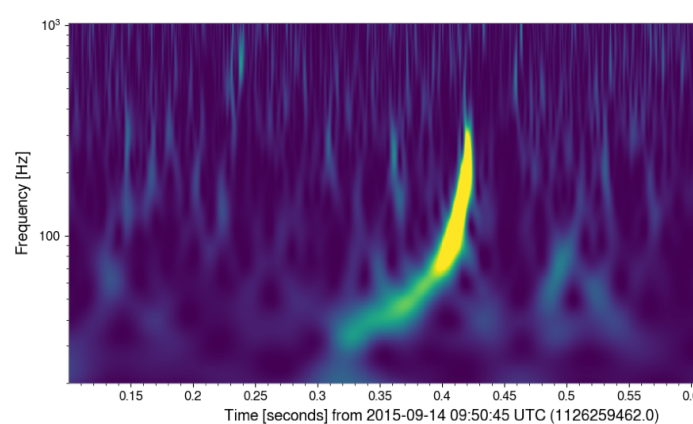


13 segments  
f=242.20 ± 8.36 Hz  
t=15.4351 -15.5132

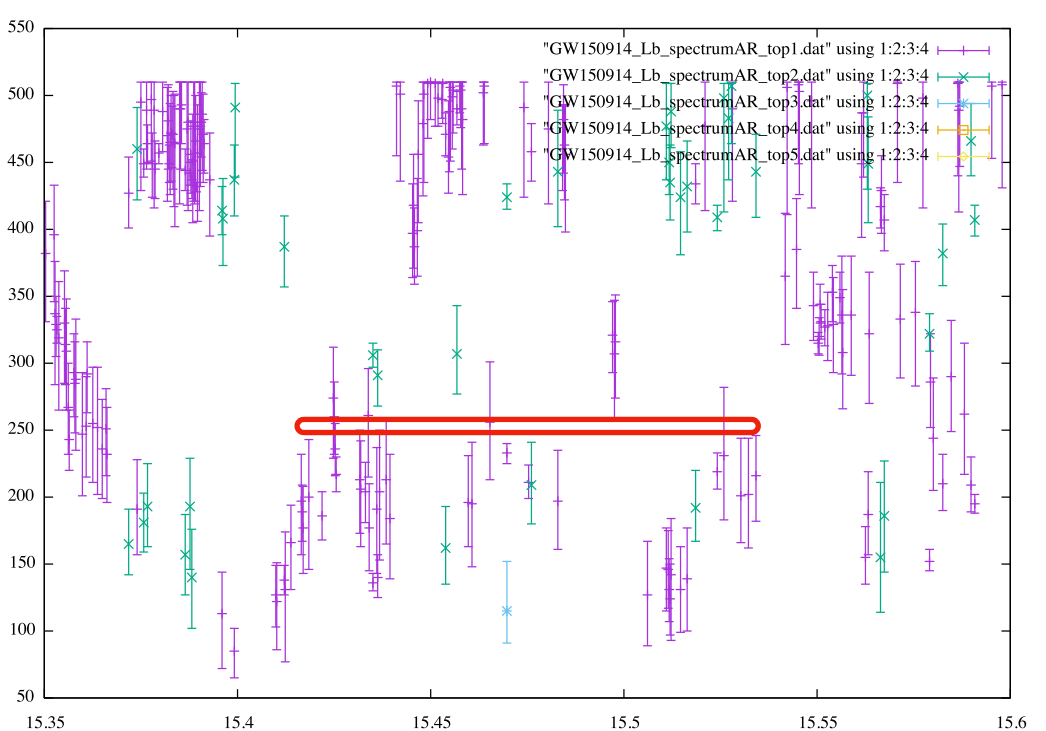


18 segments  
f=249.07 ± 28.19 Hz  
t=15.4578 -15.5286

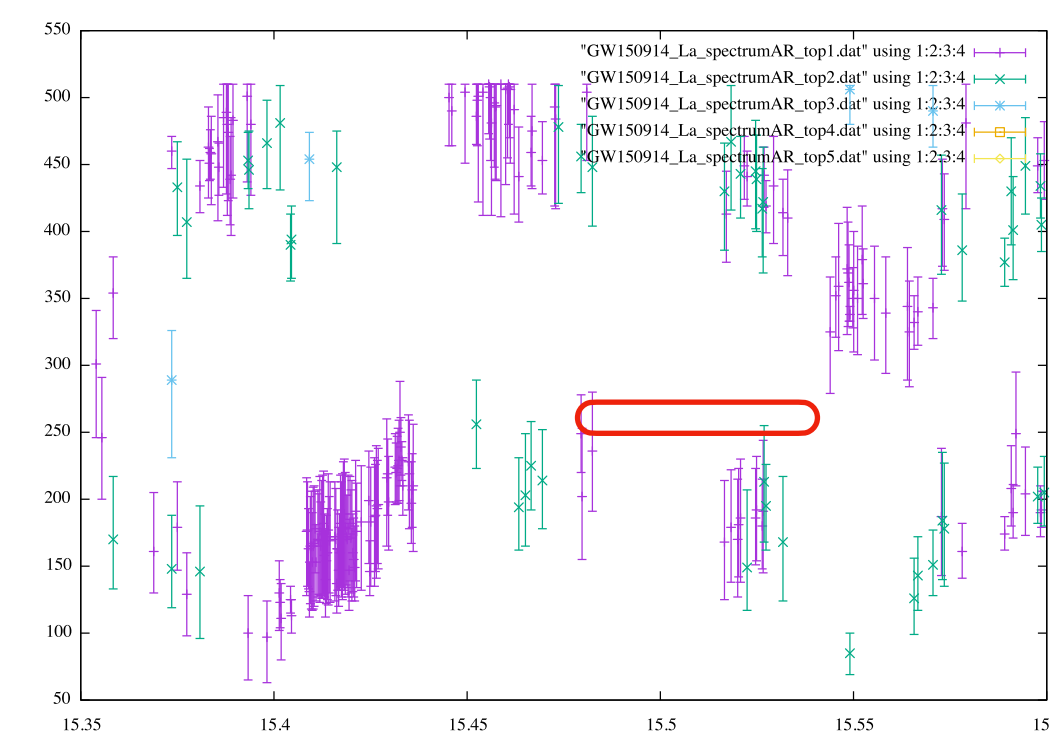
### Livingston



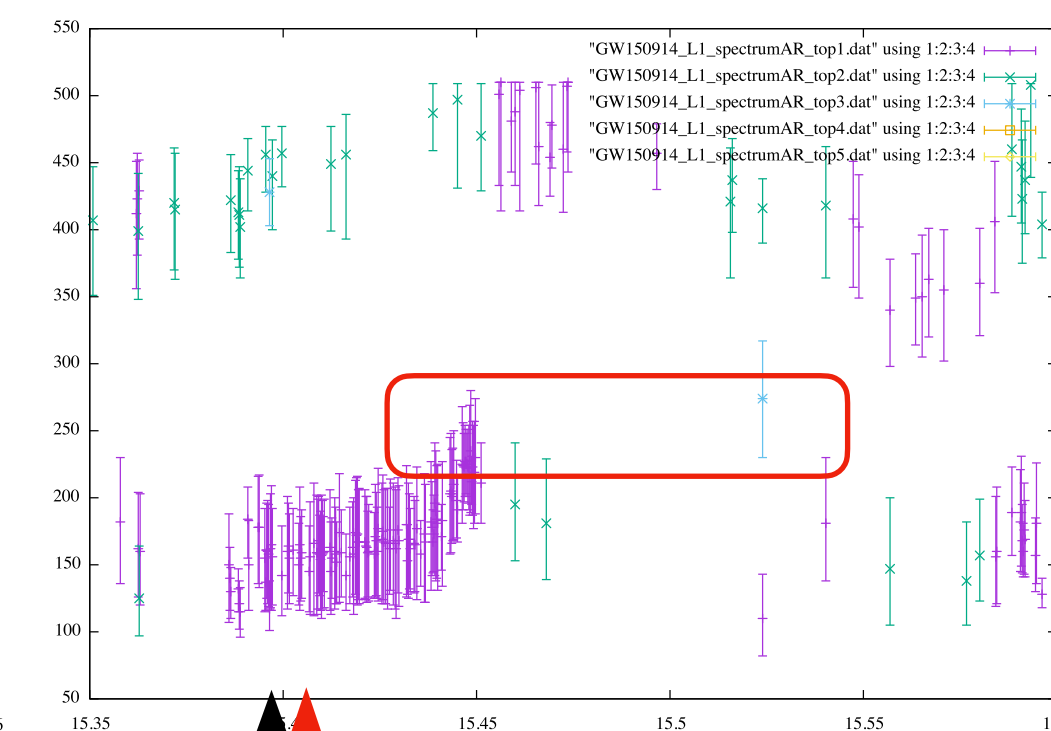
16 segments  
f=251.57 ± 11.43 Hz  
t=15.4177 -15.5295



15 segments  
f=248.52 ± 8.12 Hz  
t=15.4199 -15.5352



13 segments  
f=254.29 ± 12.32 Hz  
t=15.4763 -15.5447



12 segments  
f=248.64 ± 41.95 Hz  
t=15.4443 -15.5432

# GW150914

1 segment = 256 points  
= 1/16 sec = 62.5 ms

LV paper ▶

$$(M, a, z) = (63.1_{-3.4}^{+3.4}, 0.69_{-0.04}^{+0.05}, 0.09_{-0.03}^{+0.03})$$

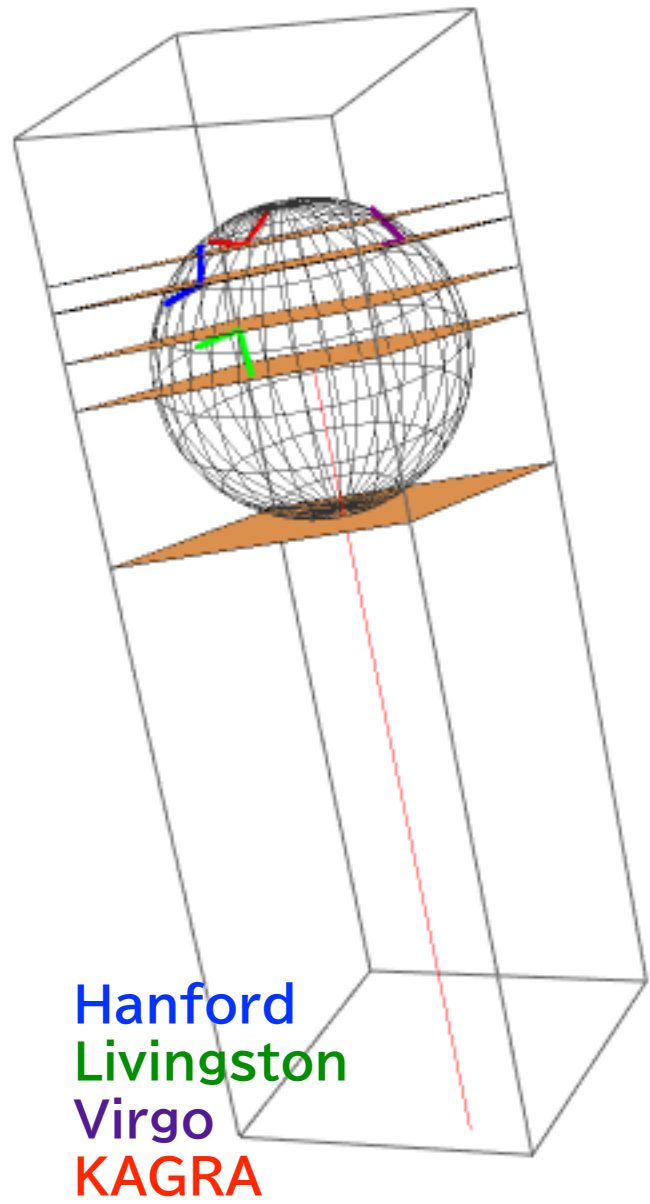
$f_{\text{QNM}}$   
@Earth ▶

$$f_{220} = 249.4 + i 38.42 \text{ Hz}, f_{221} = 244.0 + i 116.6 \text{ Hz}, f_{222} = 233.7 + i 197.2 \text{ Hz}$$

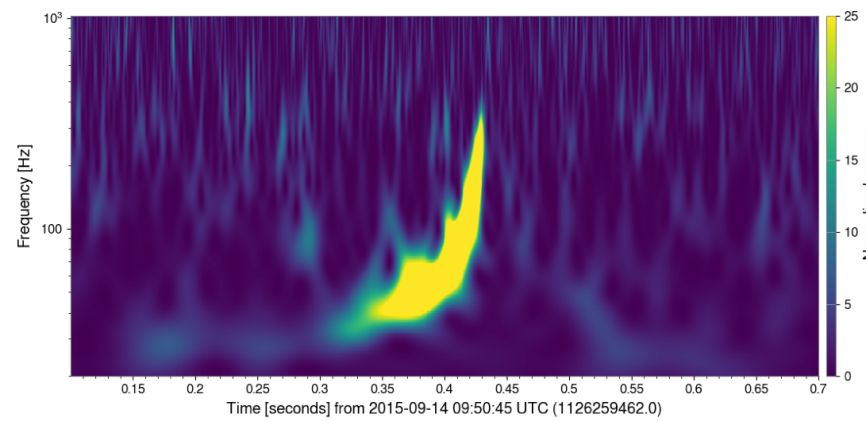
$$f_{210} = 349.3 + i 62.91 \text{ Hz}, f_{211} = 207.1 + i 117.5 \text{ Hz}, f_{200} = 231.9 + i 49.46 \text{ Hz}$$

$$f_{330} = 395.3 + i 39.41 \text{ Hz}, f_{331} = 392.1 + i 121.2 \text{ Hz}, f_{332} = 386.3 + i 217.9 \text{ Hz}$$

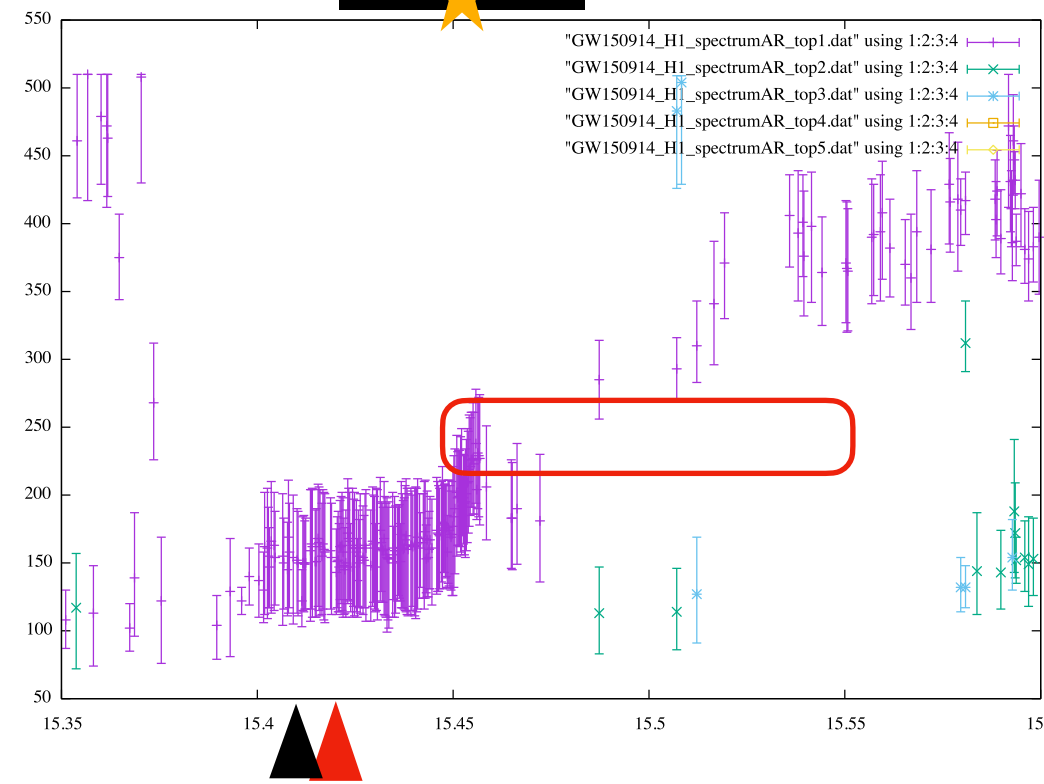
$$f_{320} = 355.9 + i 39.73 \text{ Hz}, f_{310} = 322.1 + i 53.2 \text{ Hz}, f_{300} = 293.9 + i 41.37 \text{ Hz}$$



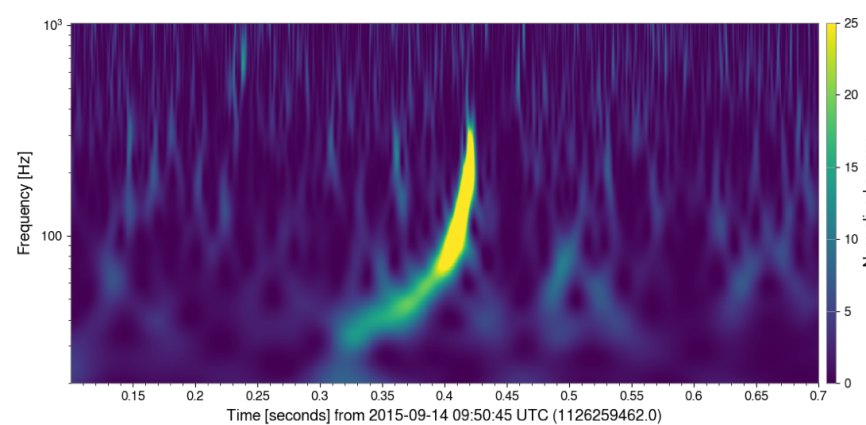
Hanford



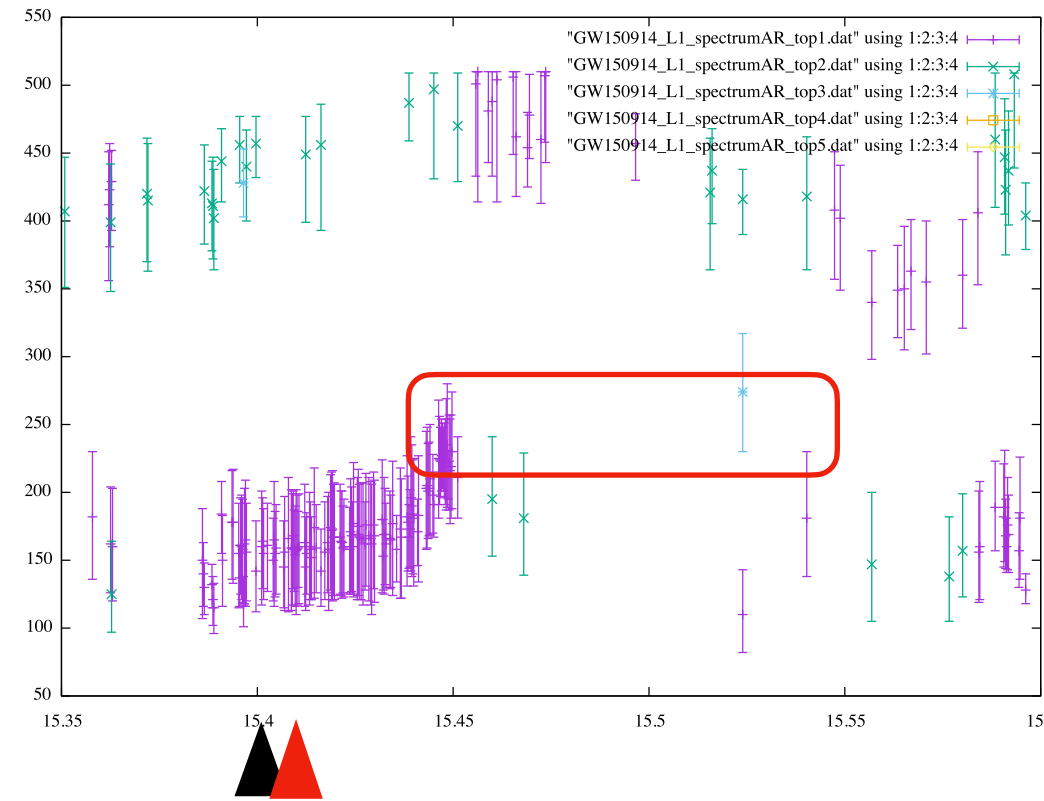
Hanford (SNR=20.6)



Livingston



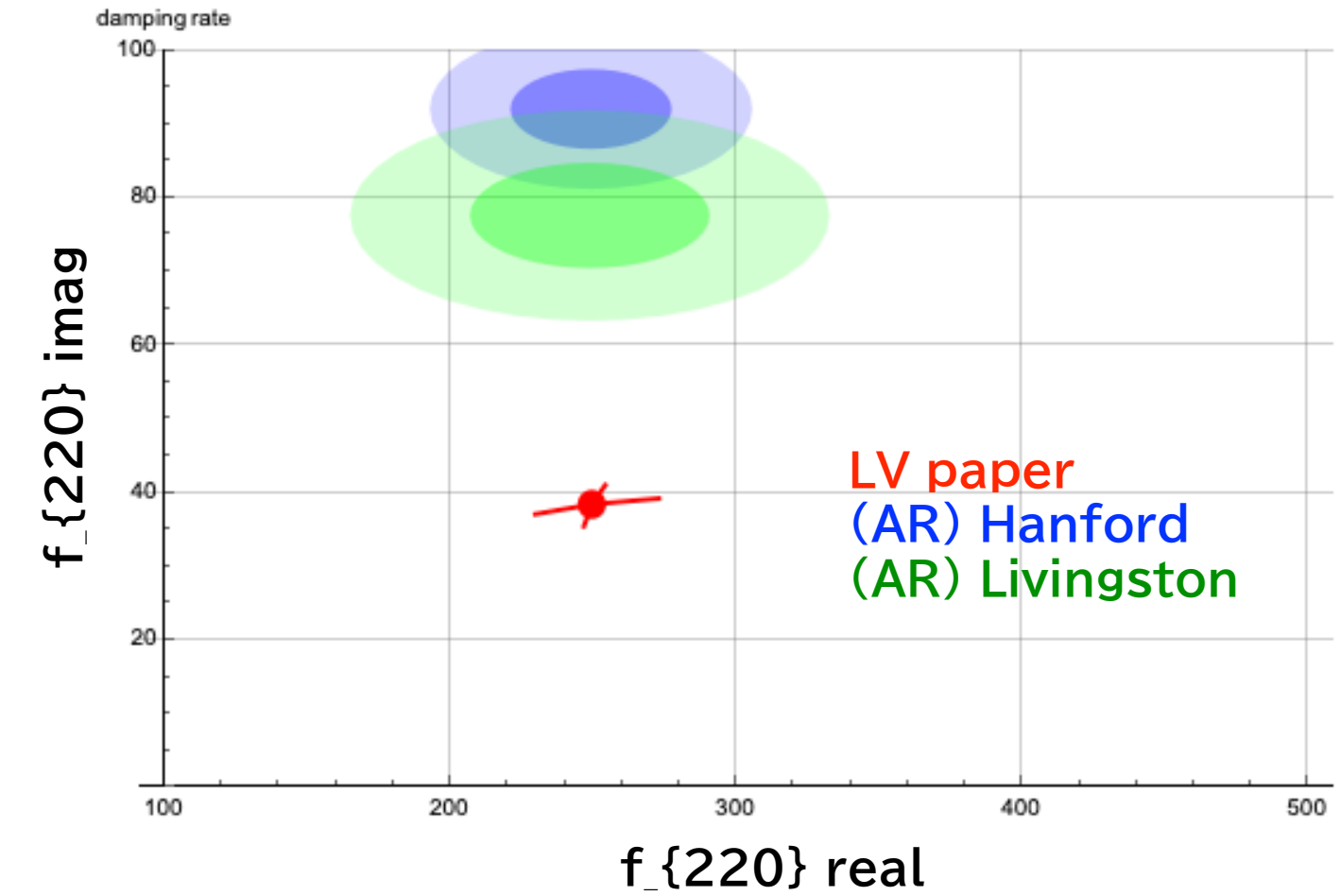
Livingston (SNR=14.2)



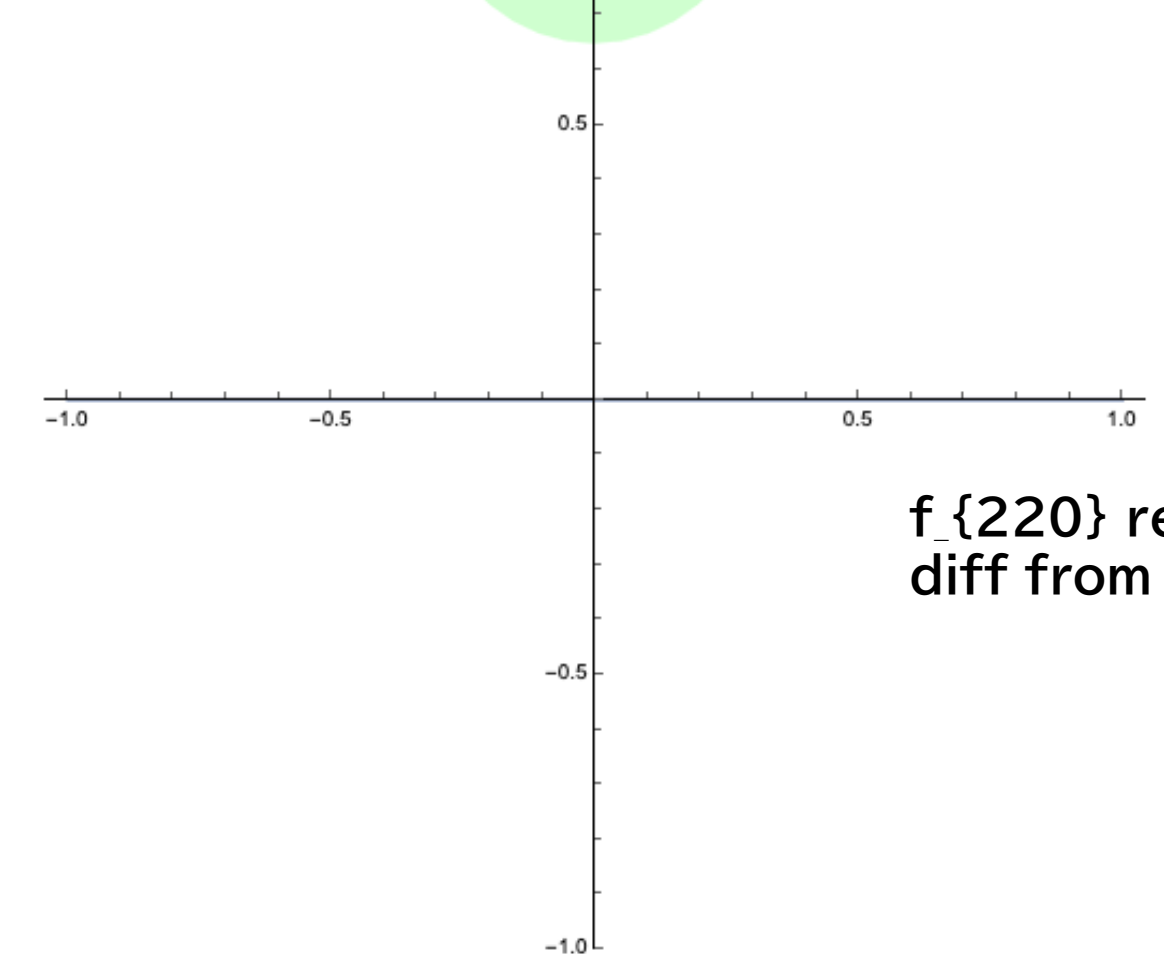
18 segments  
 $f=249.07 \pm 28.19 \text{ Hz}$   
 $t=15.4578 - 15.5286$

12 segments  
 $f=248.64 \pm 41.95 \text{ Hz}$   
 $t=15.4443 - 15.5432$

1 segment = 256 points  
= 1/16 sec = 62.5 ms



$f_{\{220\}} \text{ imag}$   
diff from GR



GW150914  
delay time (msec) from  $t_0$   
(+ delay, - advanced)

LHO = 12.8483  
LLO = 6.26193

$t_{\text{merger}} = 15.4 \text{ s}$   
 $t_{\text{mergerH}} = 15.413$   
 $t_{\text{mergerL}} = 15.406$



# GW150914

1 segment = 32 points  
= 1/128 sec = 7.8125 ms

LV paper ▶

$$(M, a, z) = (63.1_{-3.4}^{+3.4}, 0.69_{-0.04}^{+0.05}, 0.09_{-0.03}^{+0.03})$$

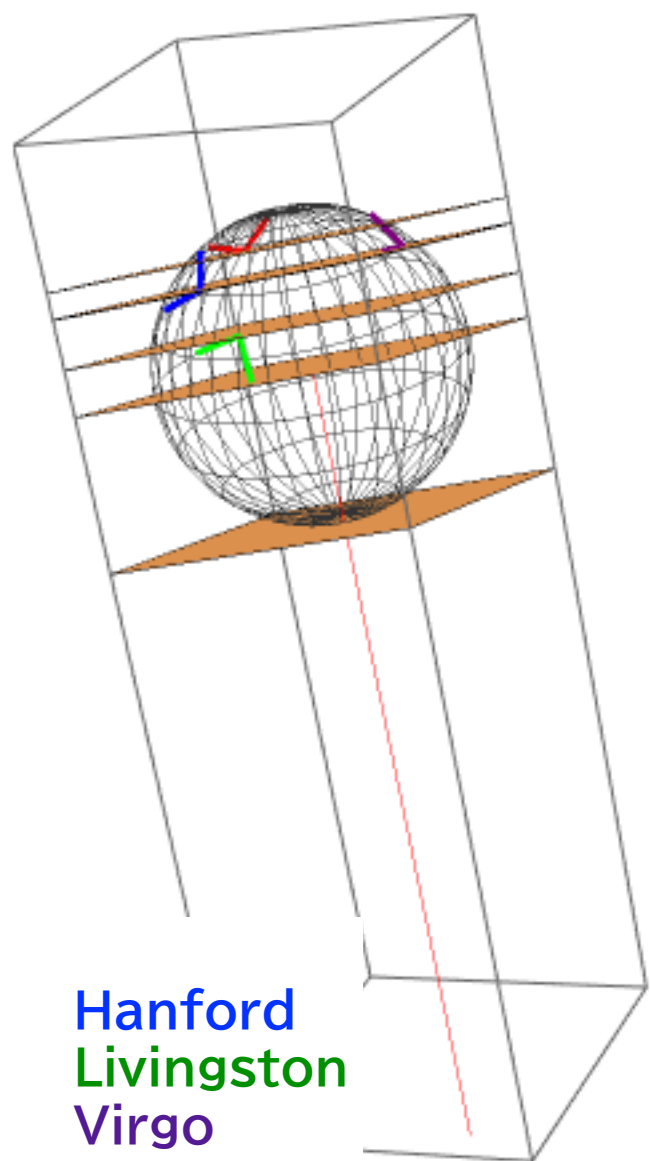
$f_{QNM}$   
@Earth ▶

$$f_{220} = 249.4 + i 38.42 \text{ Hz}, f_{221} = 244.0 + i 116.6 \text{ Hz}, f_{222} = 233.7 + i 197.2 \text{ Hz}$$

$$f_{210} = 349.3 + i 62.91 \text{ Hz}, f_{211} = 207.1 + i 117.5 \text{ Hz}, f_{200} = 231.9 + i 49.46 \text{ Hz}$$

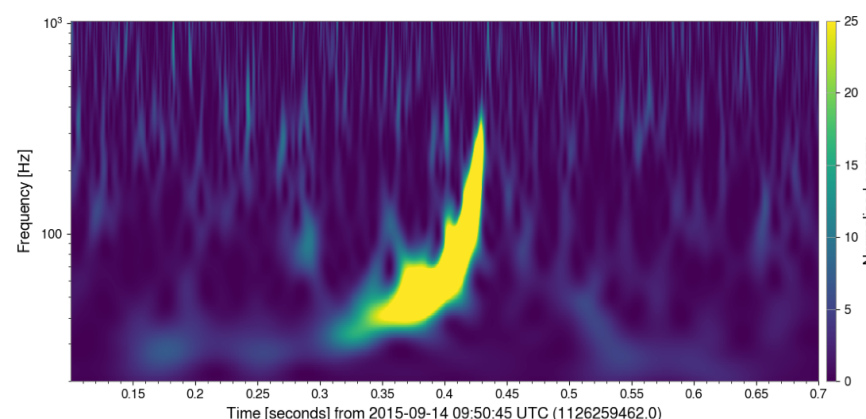
$$f_{330} = 395.3 + i 39.41 \text{ Hz}, f_{331} = 392.1 + i 121.2 \text{ Hz}, f_{332} = 386.3 + i 217.9 \text{ Hz}$$

$$f_{320} = 355.9 + i 39.73 \text{ Hz}, f_{310} = 322.1 + i 53.2 \text{ Hz}, f_{300} = 293.9 + i 41.37 \text{ Hz}$$

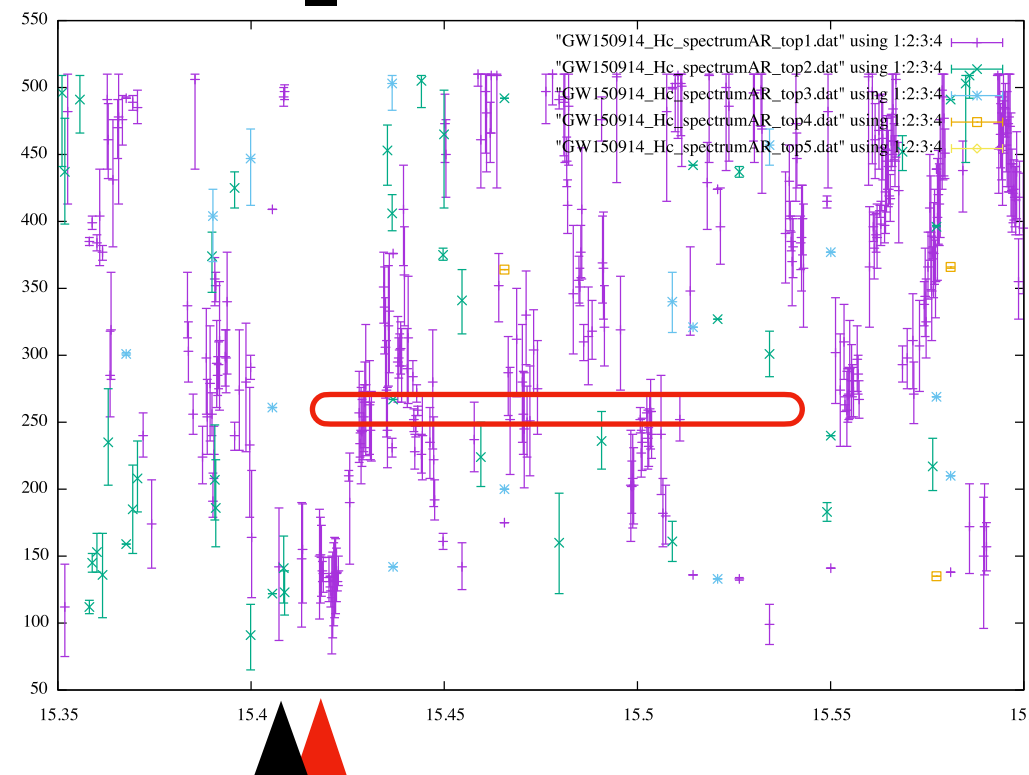


Hanford  
Livingston  
Virgo  
KAGRA

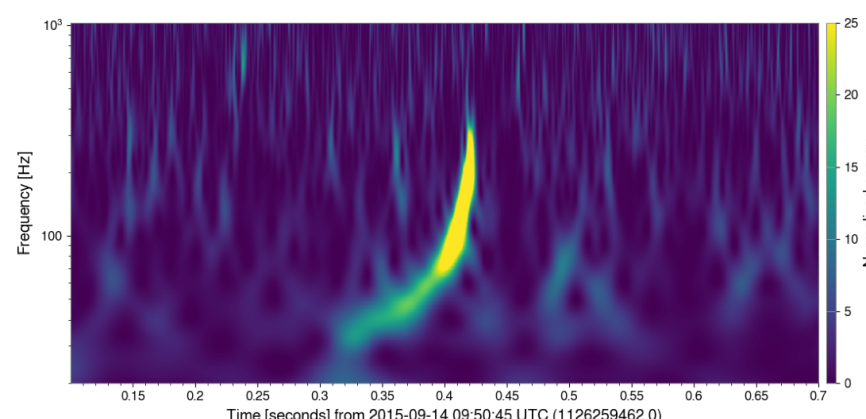
Hanford



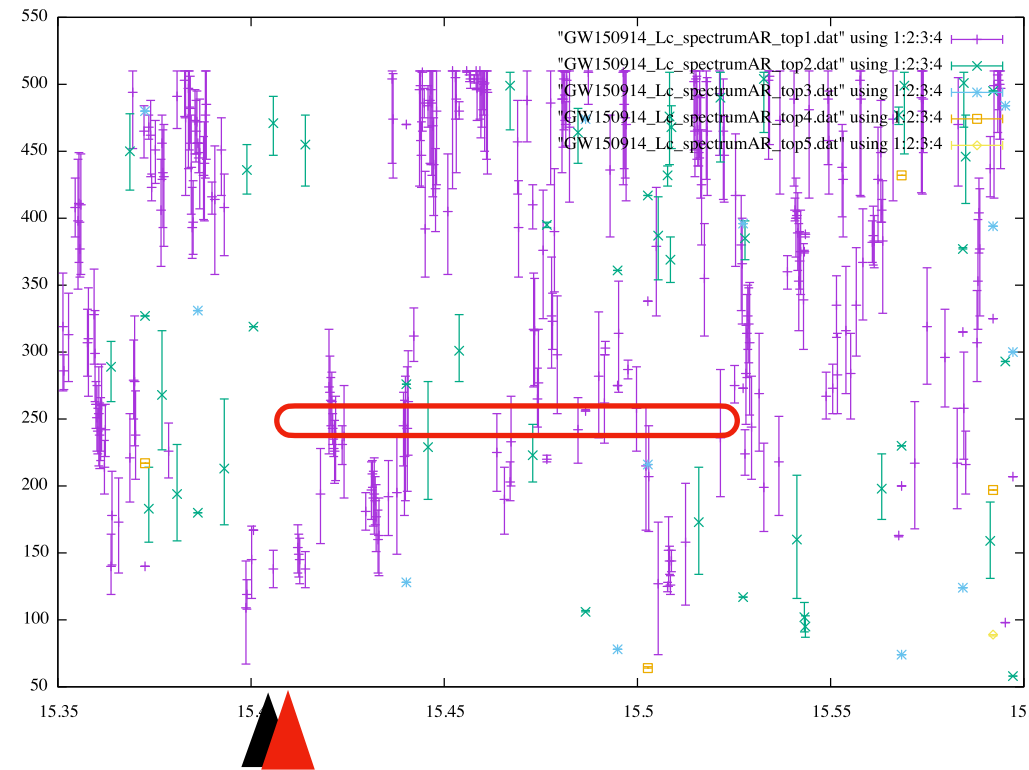
Hanford (SNR=20.6)



Livingston



Livingston (SNR=14.2)

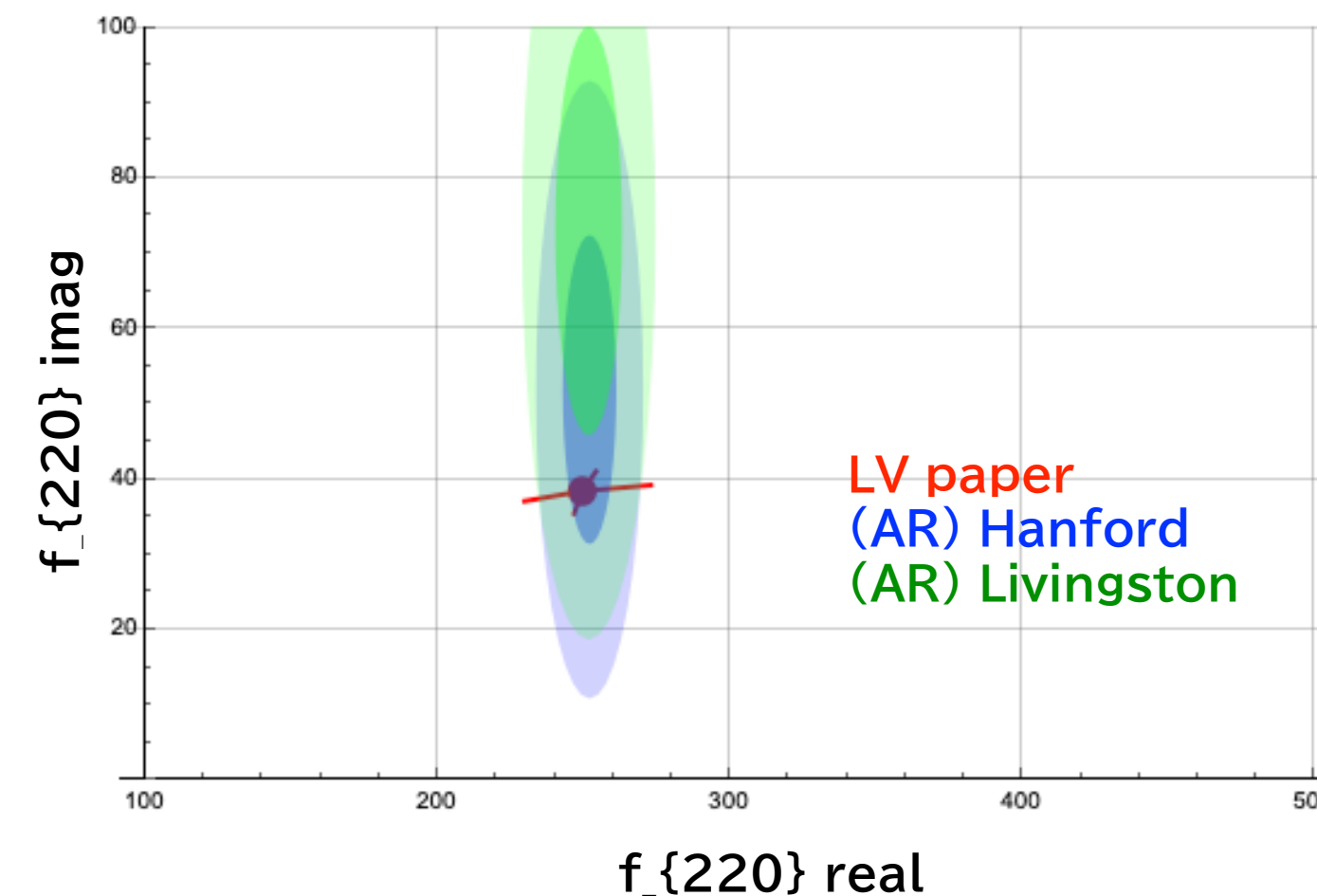


21 segments  
f=251.85 ± 9.17 Hz  
t=15.4285 -15.5449

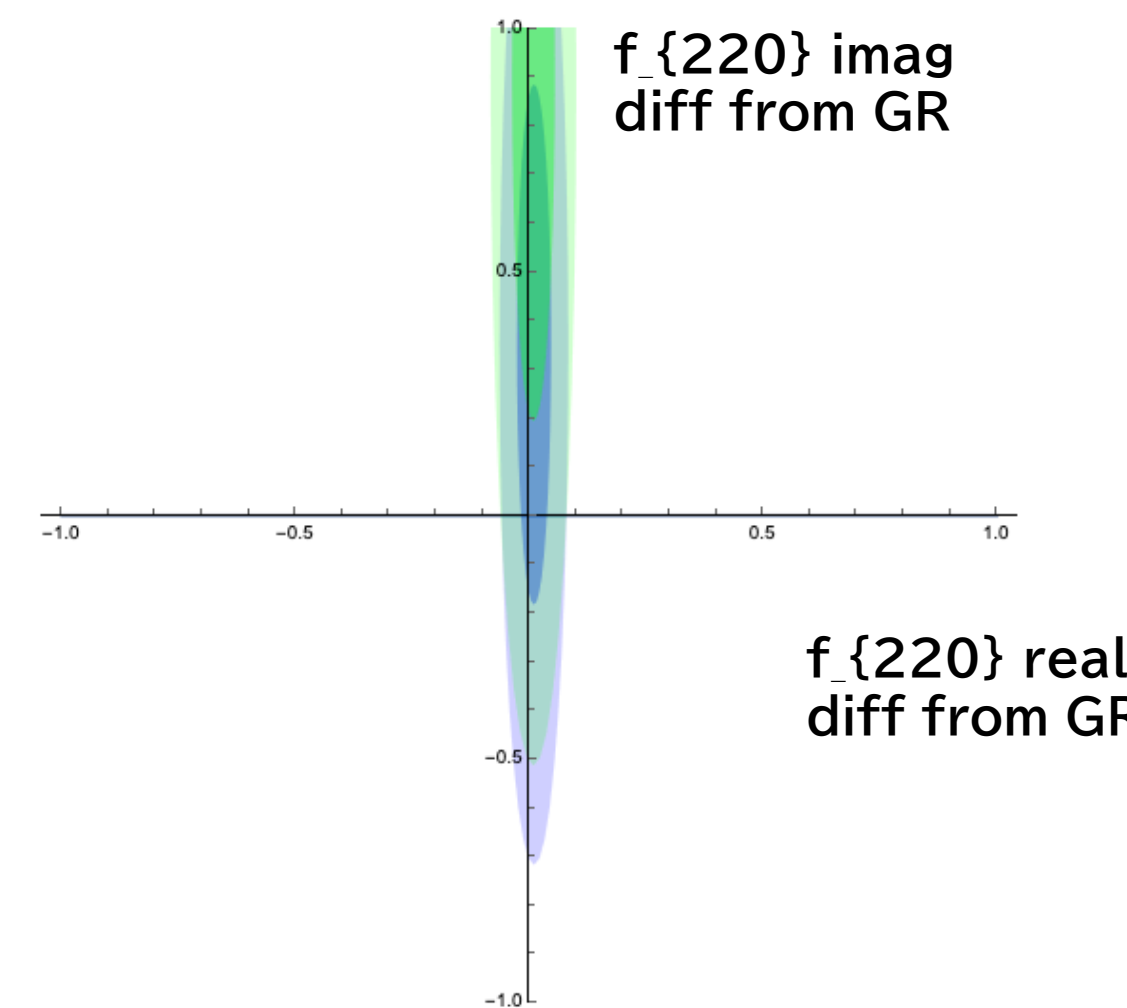
$f_{\{220\}}$  starts 8ms  
after  $t_{\text{merger}}$

16 segments  
f=251.57 ± 11.43 Hz  
t=15.4177 -15.5295

$f_{\{220\}}$  starts 11ms  
after  $t_{\text{merger}}$



LV paper  
(AR) Hanford  
(AR) Livingston



GW150914  
delay time (msec) from  $t_0$   
(+ delay, - advanced)

LHO = 12.8483  
LLO = 6.26193

$t_{\text{merger}} = 15.4$  s  
 $t_{\text{mergerH}} = 15.413$   
 $t_{\text{mergerL}} = 15.406$

+3.9ms  $t_{\text{mergerH}} = 15.417$   
 $t_{\text{mergerL}} = 15.410$

どちらも 251.6 Hz のリングダウン波形を拾っている  
Livingston は Hanford よりも 11ms はやく拾っている  
合体から Livingston は 8ms 後, Hanford は 11ms 後から拾っている



# GW190412

Network SNR=18.9

1 segment = 64 points  
= 1/64 sec = 15.625 ms

LV paper ▶

$$(M, a, z) = (37.3^{+3.9}_{-3.8}, 0.67^{+0.06}_{-0.07}, 0.15^{+0.03}_{-0.03})$$

$f_{\text{QNM}}$   
@Earth ▶

$$f_{220} = 393.7 + i 62.17 \text{ Hz}, f_{221} = 384.7 + i 188.9 \text{ Hz}, f_{222} = 367.4 + i 319.9 \text{ Hz}$$

$$f_{210} = 562.9 + i 103.0 \text{ Hz}, f_{211} = 328.9 + i 190.0 \text{ Hz}, f_{200} = 372.7 + i 79.98 \text{ Hz}$$

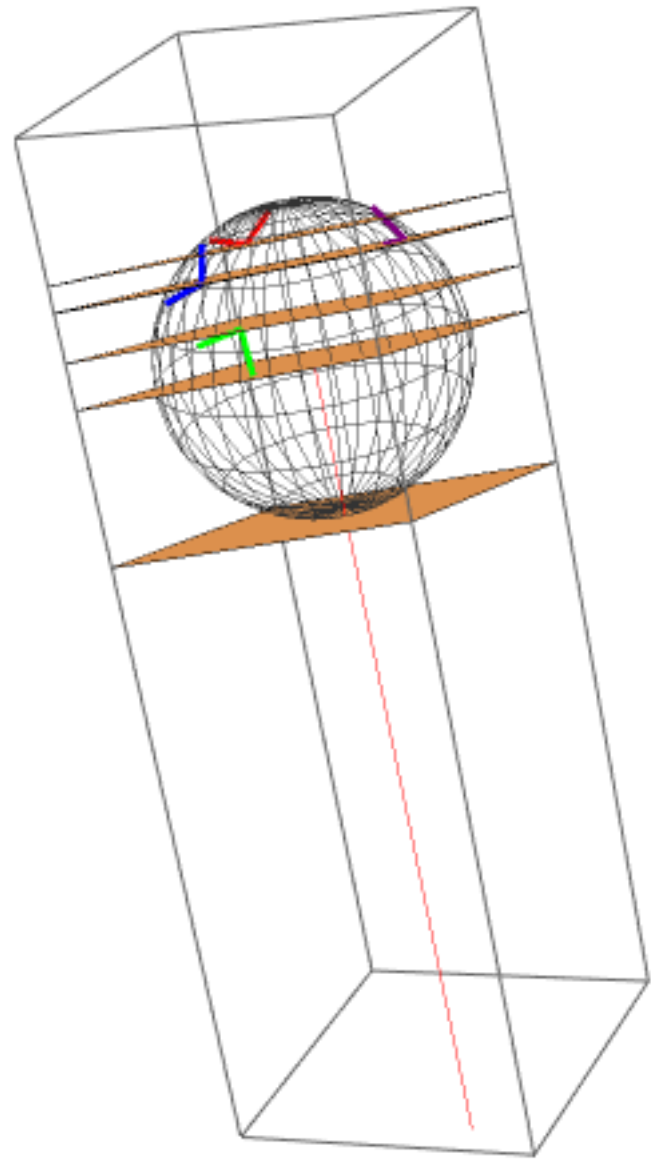
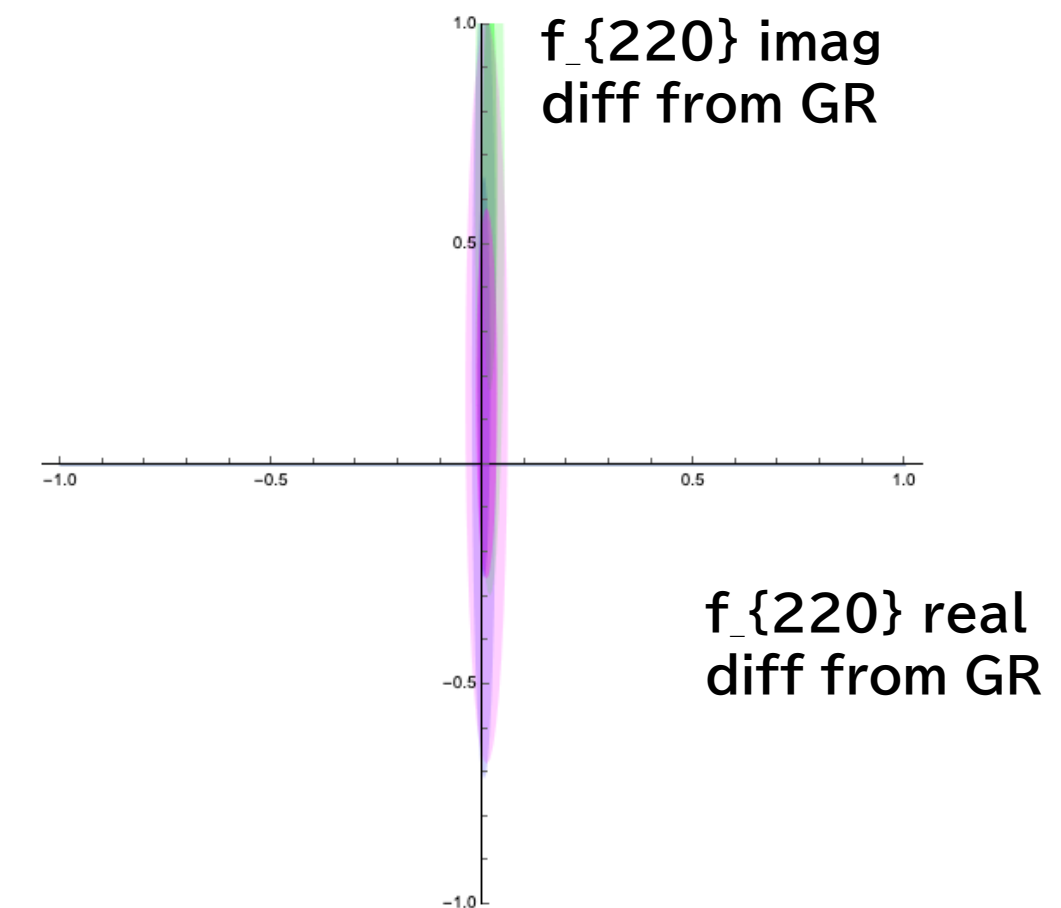
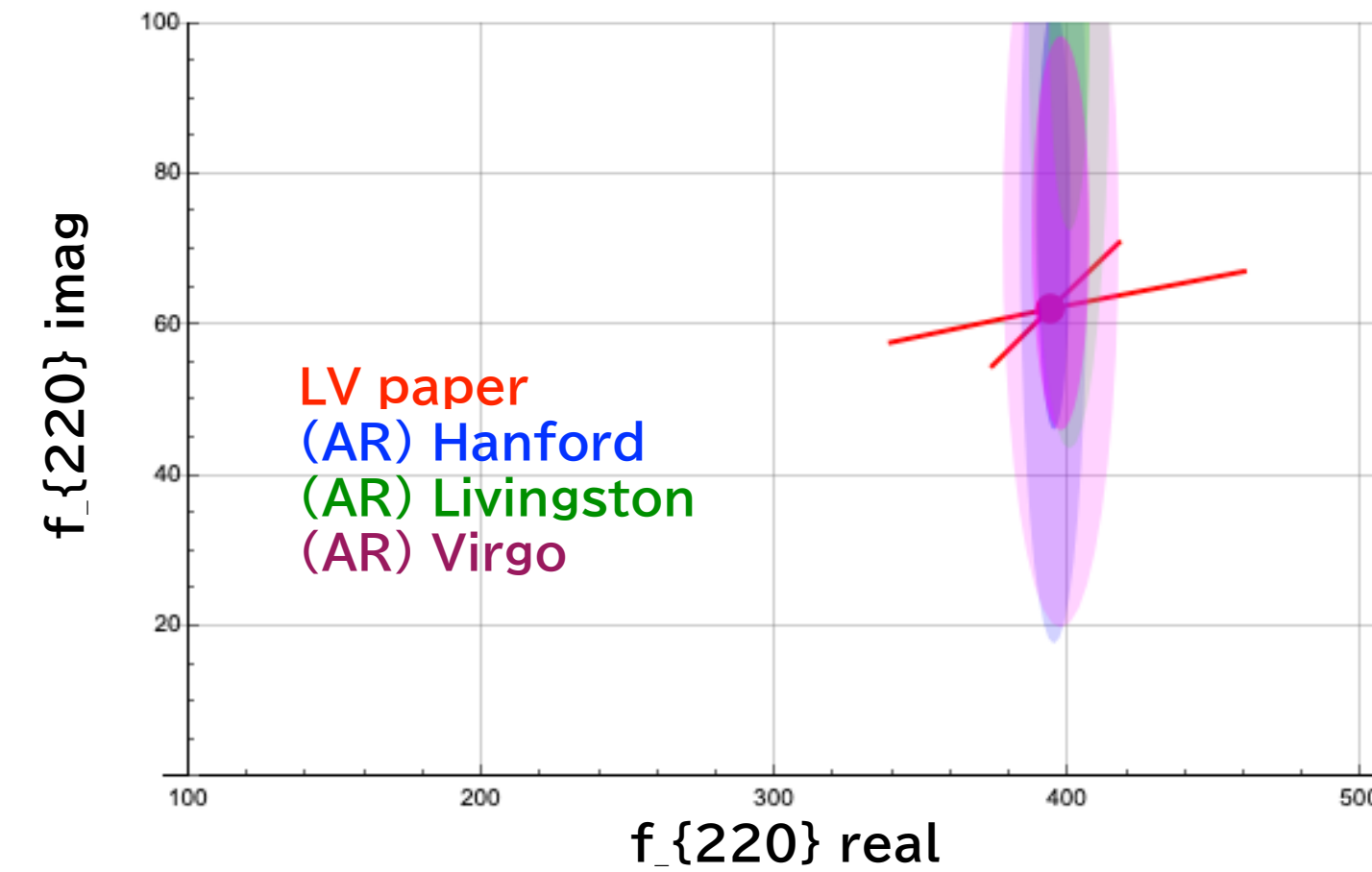
$$f_{330} = 624.8 + i 63.84 \text{ Hz}, f_{331} = 619.4 + i 196.1 \text{ Hz}, f_{332} = 609.5 + i 354.1 \text{ Hz}$$

$$f_{320} = 565.1 + i 64.26 \text{ Hz}, f_{310} = 513.6 + i 84.68 \text{ Hz}, f_{300} = 470.2 + i 66.54 \text{ Hz}$$

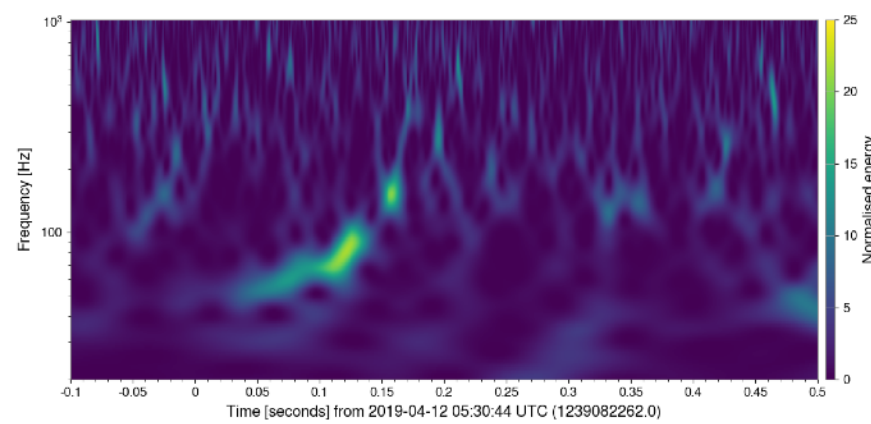
21 segments  
 $f=394.97 \pm 5.74 \text{ Hz}$   
 $t=15.1675 - 15.3018$

19 segments  
 $f=400.28 \pm 6.93 \text{ Hz}$   
 $t=15.1660 - 15.3159$

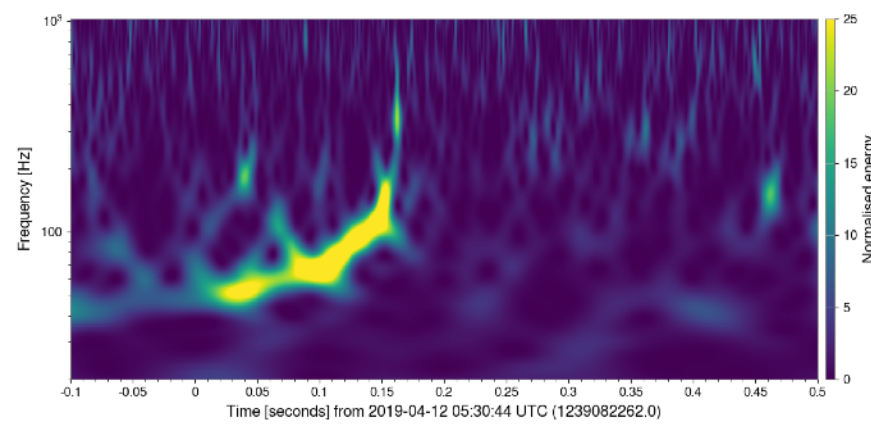
12 segments  
 $f=397.33 \pm 9.90 \text{ Hz}$   
 $t=15.1699 - 15.2835$



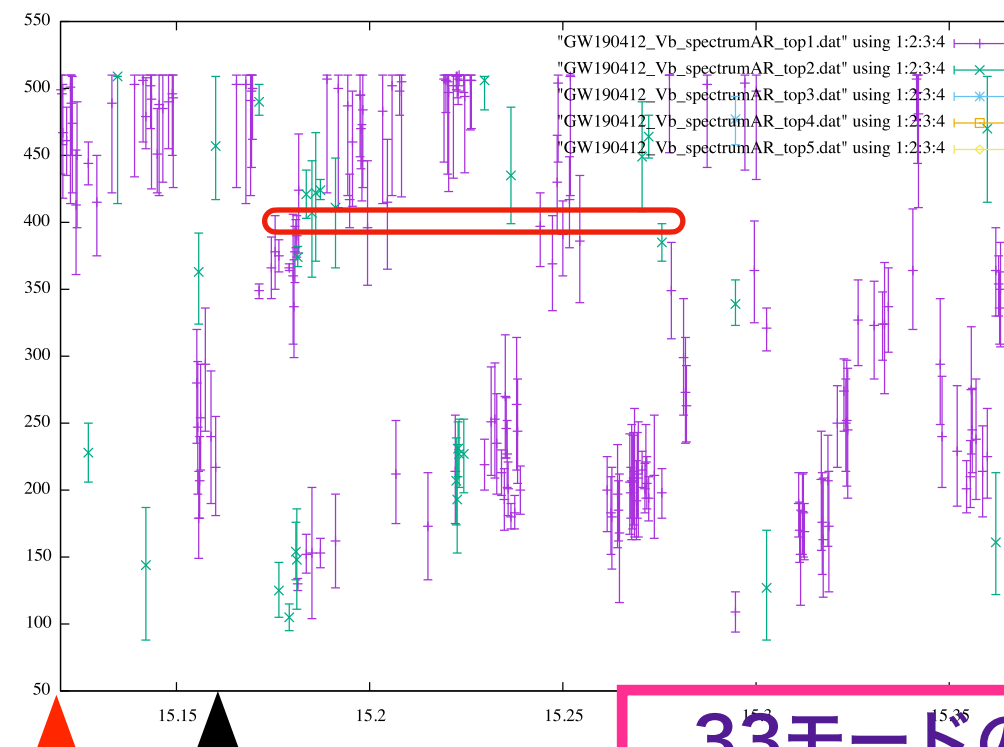
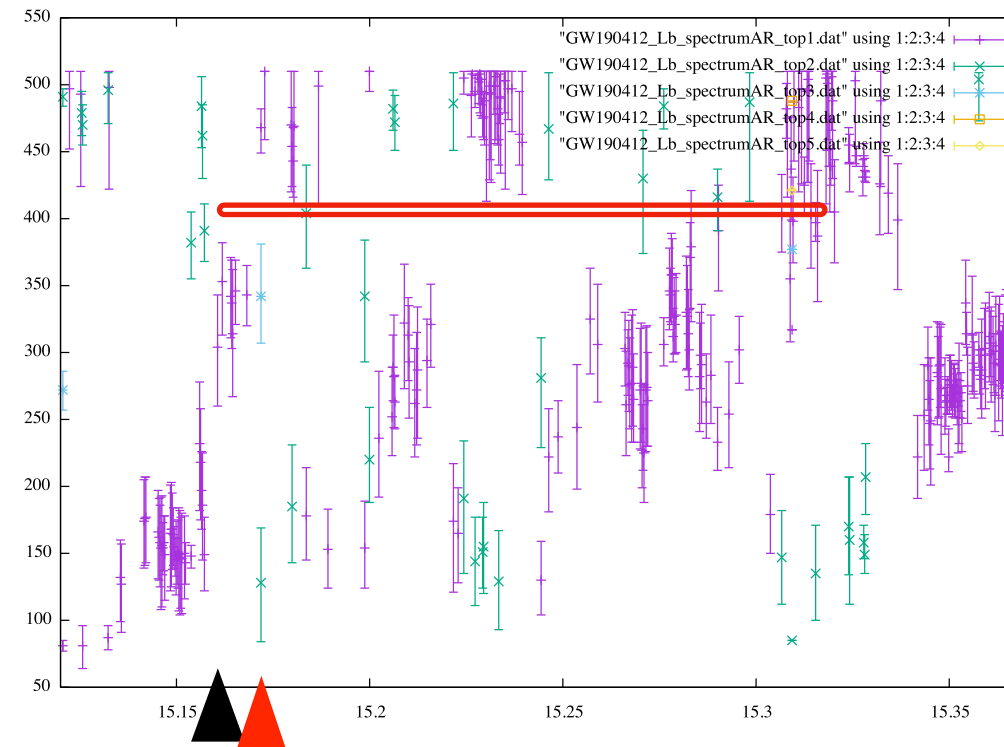
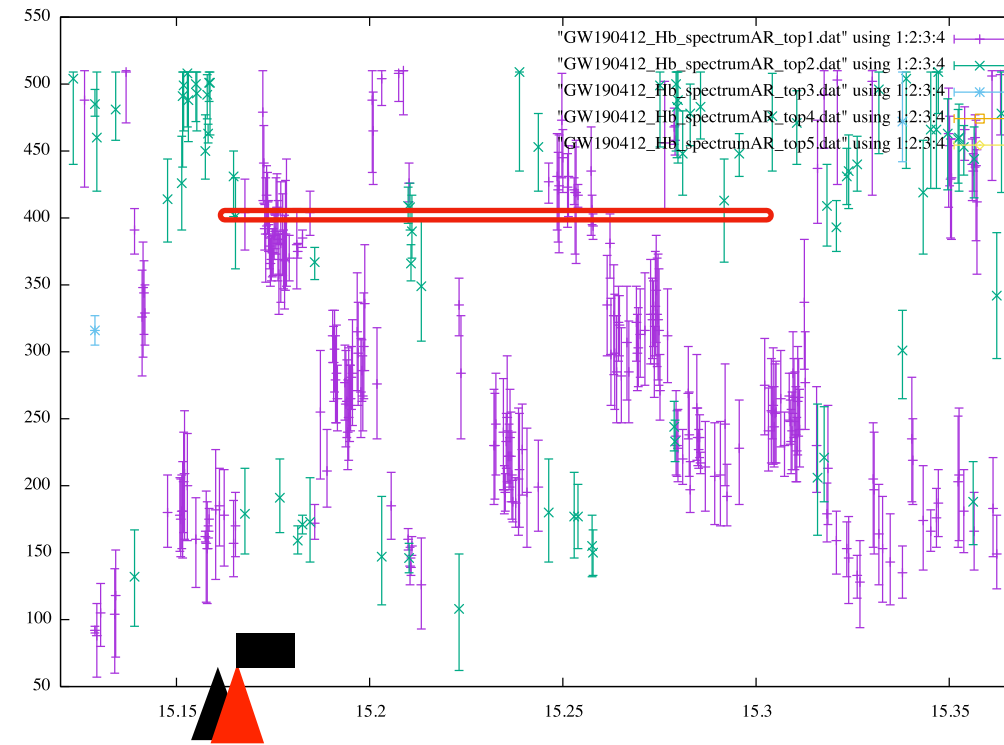
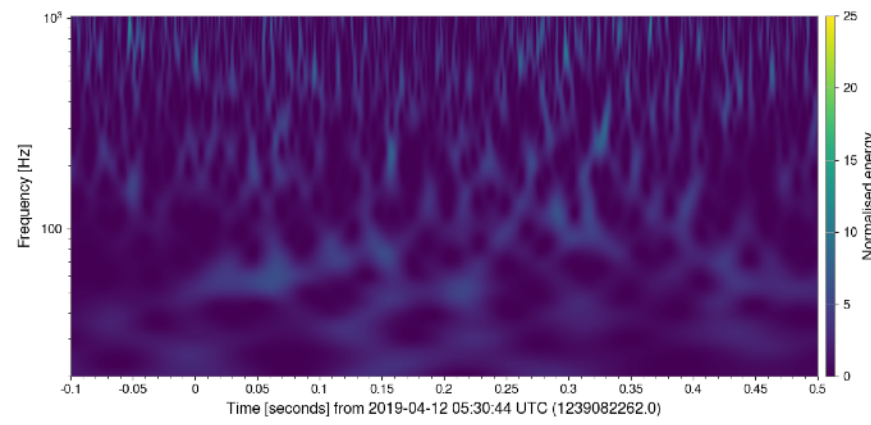
Hanford



Livingston



Virgo



33モードの存在が指摘されているが、高振動数はノイズまみれで無理だった

GW190412  
delay time (msec) from  $t_0$   
(+ delay, - advanced)

LHO = 0.87046  
LLO = 7.94444  
Virgo = -11.2685

$t_{\text{merger}} = 15.166$   
 $t_{\text{mergerH}} = 15.167$   
 $t_{\text{mergerL}} = 15.174$   
 $t_{\text{mergerV}} = 15.105$



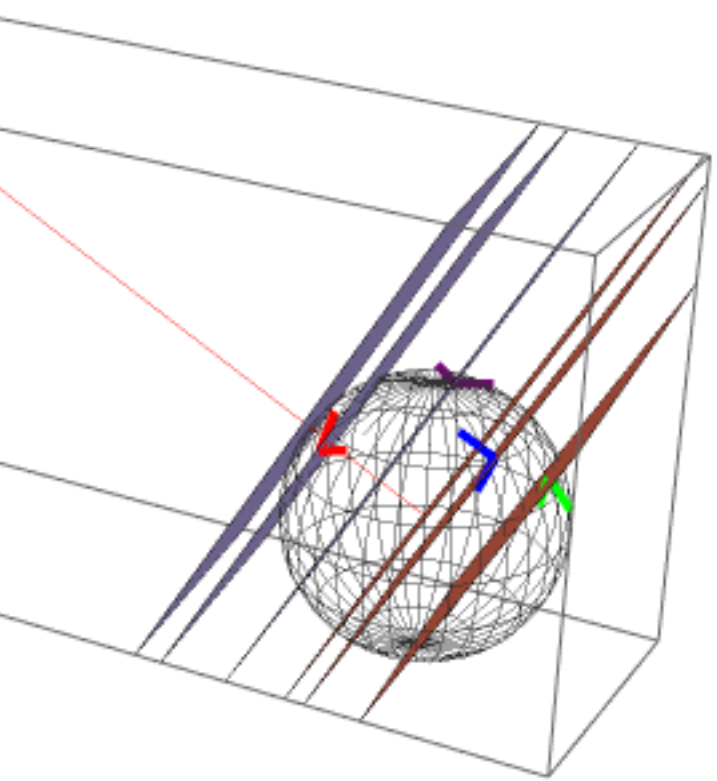
# GW190521

Network SNR=14.2

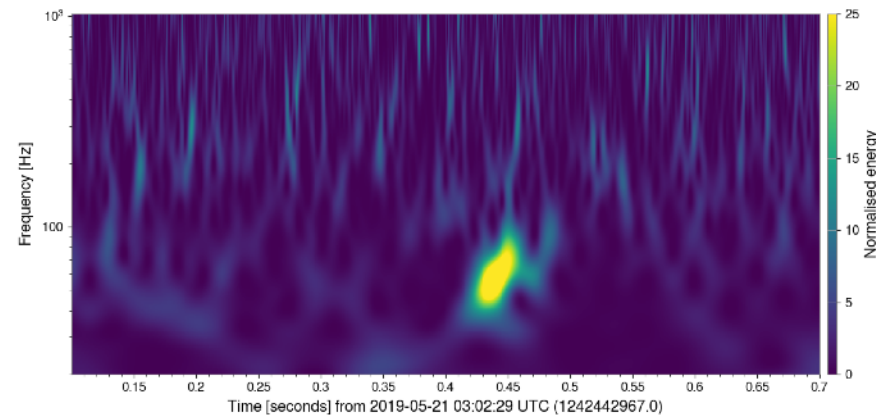
1 segment = 64 points  
= 1/64 sec = 15.625 ms

LV paper ▶

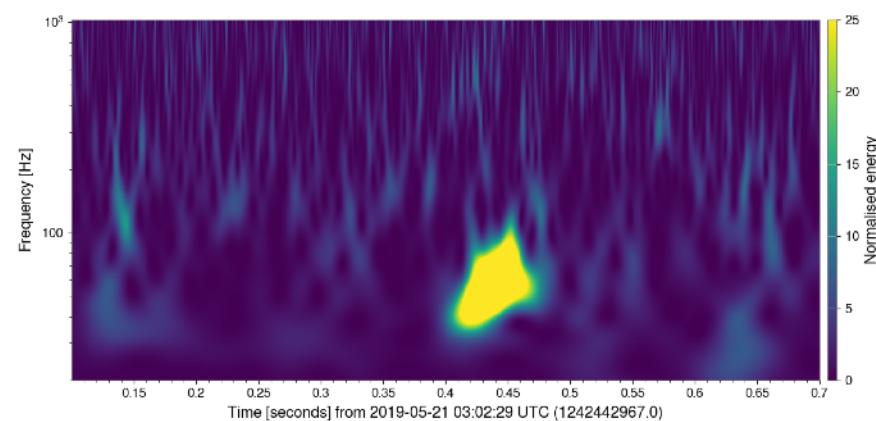
$$(M, a, z) = (150.3_{-20.}^{+35.8}, 0.73_{-0.14}^{+0.11}, 0.72_{-0.29}^{+0.29})$$



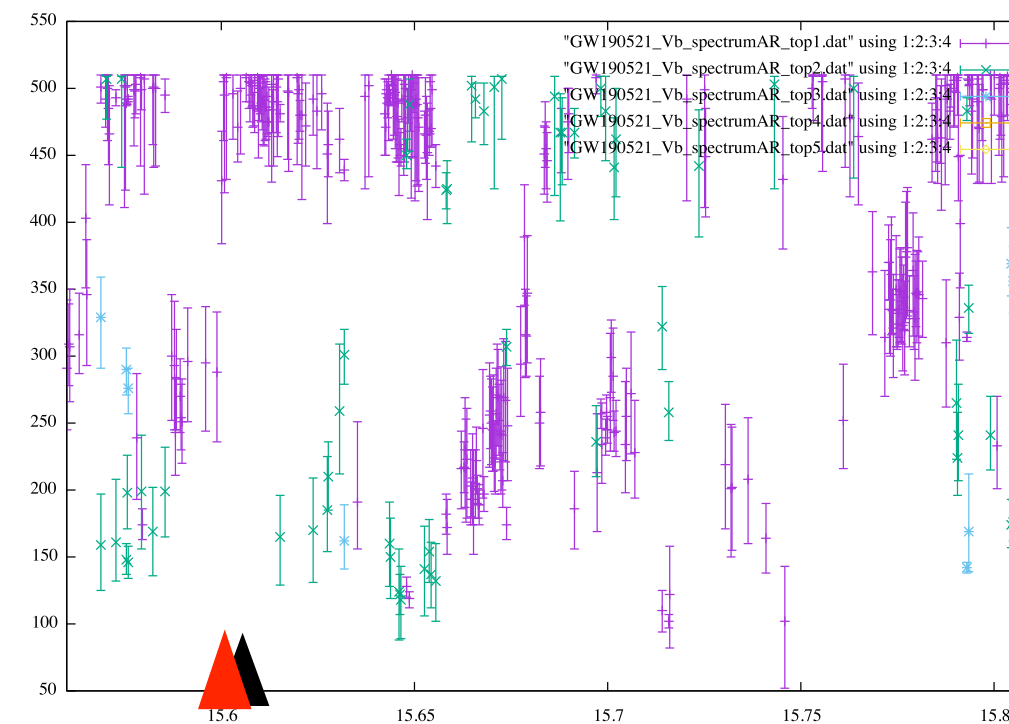
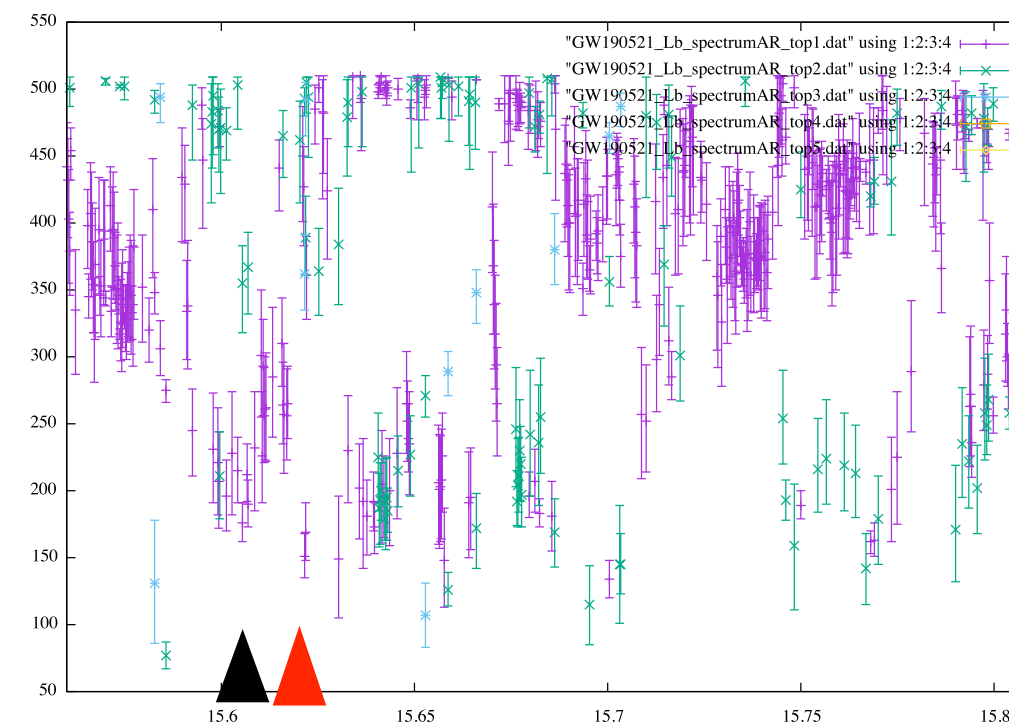
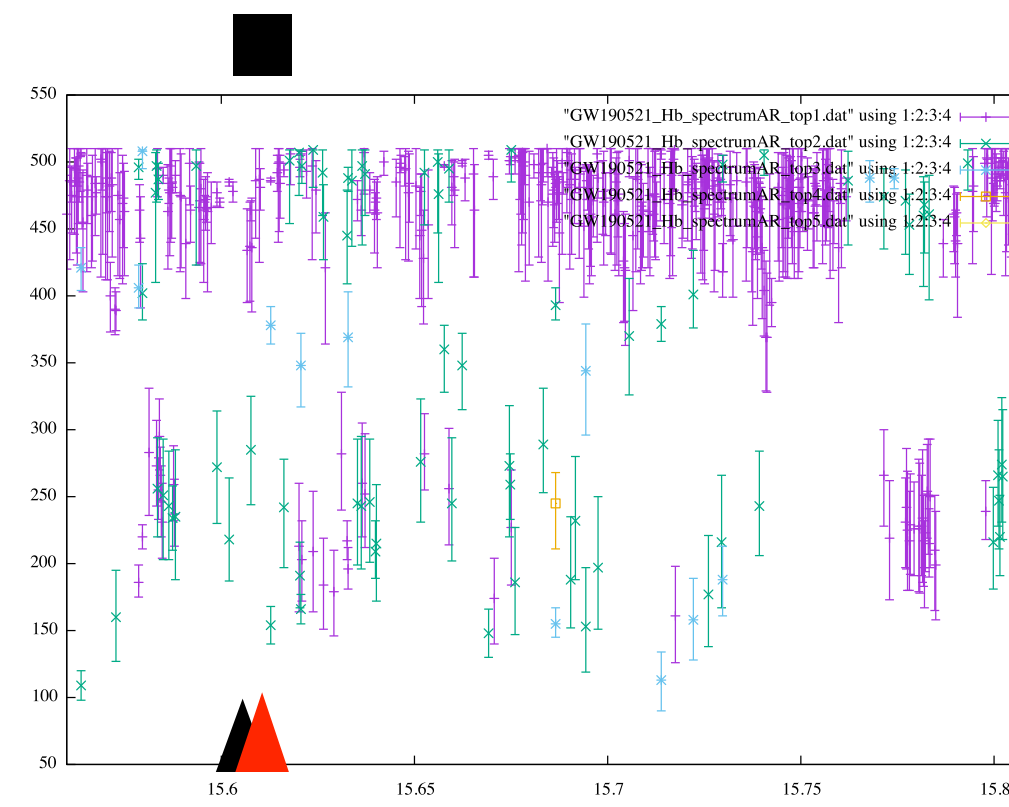
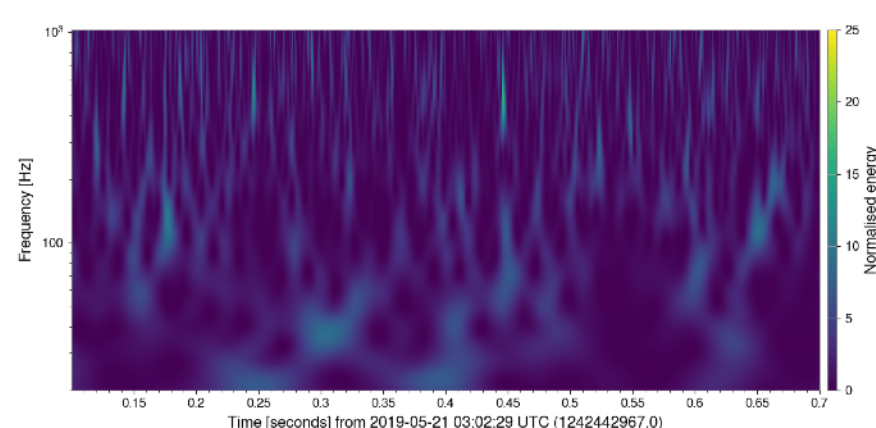
## Hanford



## Livingston



## Virgo



$f_{QNM}$   
@Earth ▶

$$f_{220} = 68.54 + i 10.00 \text{ Hz}, f_{221} = 67.28 + i 30.31 \text{ Hz}, f_{222} = 64.79 + i 51.07 \text{ Hz}$$

$$f_{210} = 91.92 + i 15.99 \text{ Hz}, f_{211} = 56.20 + i 30.67 \text{ Hz}, f_{200} = 61.42 + i 12.92 \text{ Hz}$$

$$f_{330} = 108.4 + i 10.23 \text{ Hz}, f_{331} = 107.7 + i 31.52 \text{ Hz}, f_{332} = 106.3 + i 56.17 \text{ Hz}$$

$$f_{320} = 96.67 + i 10.35 \text{ Hz}, f_{310} = 86.73 + i 14.37 \text{ Hz}, f_{300} = 78.55 + i 10.93 \text{ Hz}$$

低振動数はノイズまみれで無理だった

GW190521  
delay time (msec) from  $t_u$   
(+ delay, - advanced)

LHO = 2.6754  
LLO = 10.3767  
Virgo = -8.27122

$t_{merger} = 15.61 \text{ s}$   
 $t_{mergerH} = 15.613$   
 $t_{mergerL} = 15.620$   
 $t_{mergerV} = 15.602$

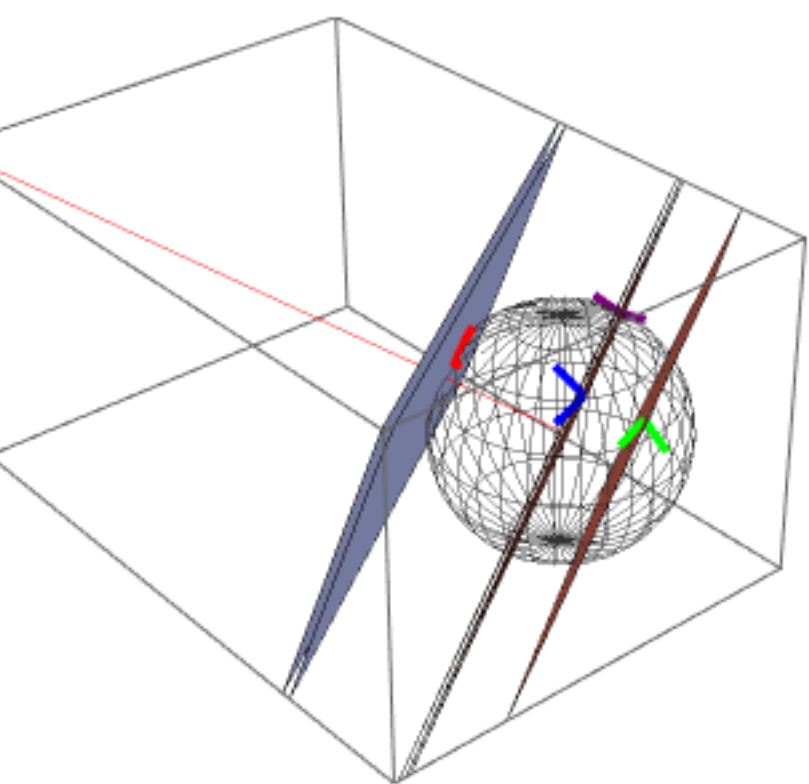
# GW190521\_074359

Network SNR=25.8

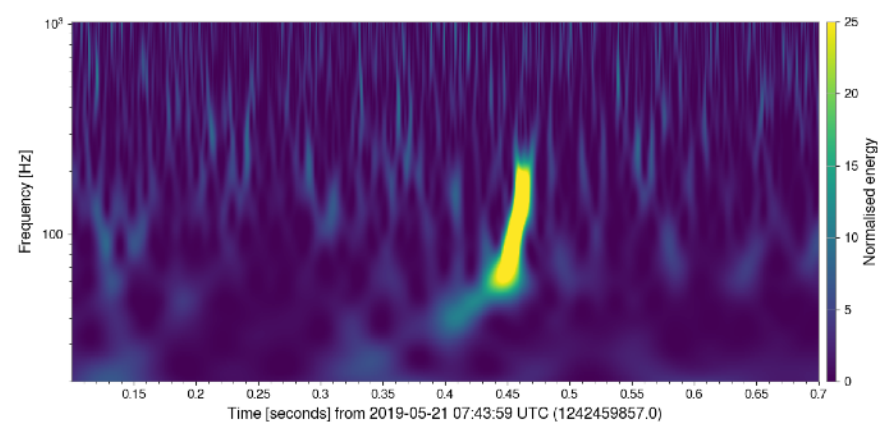
1 segment = 64 points  
= 1/64 sec = 15.625 ms

LV paper ▶

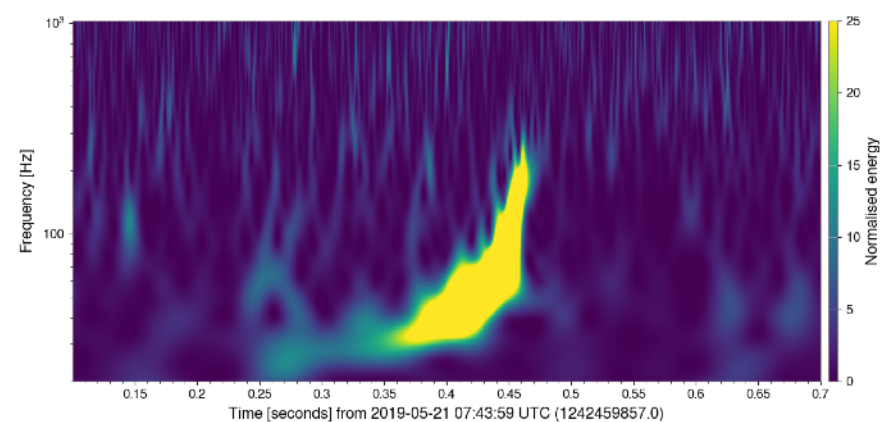
$$(M, a, z) = (70.7^{+6.4}_{-4.2}, 0.72^{+0.05}_{-0.07}, 0.25^{+0.06}_{-0.1})$$



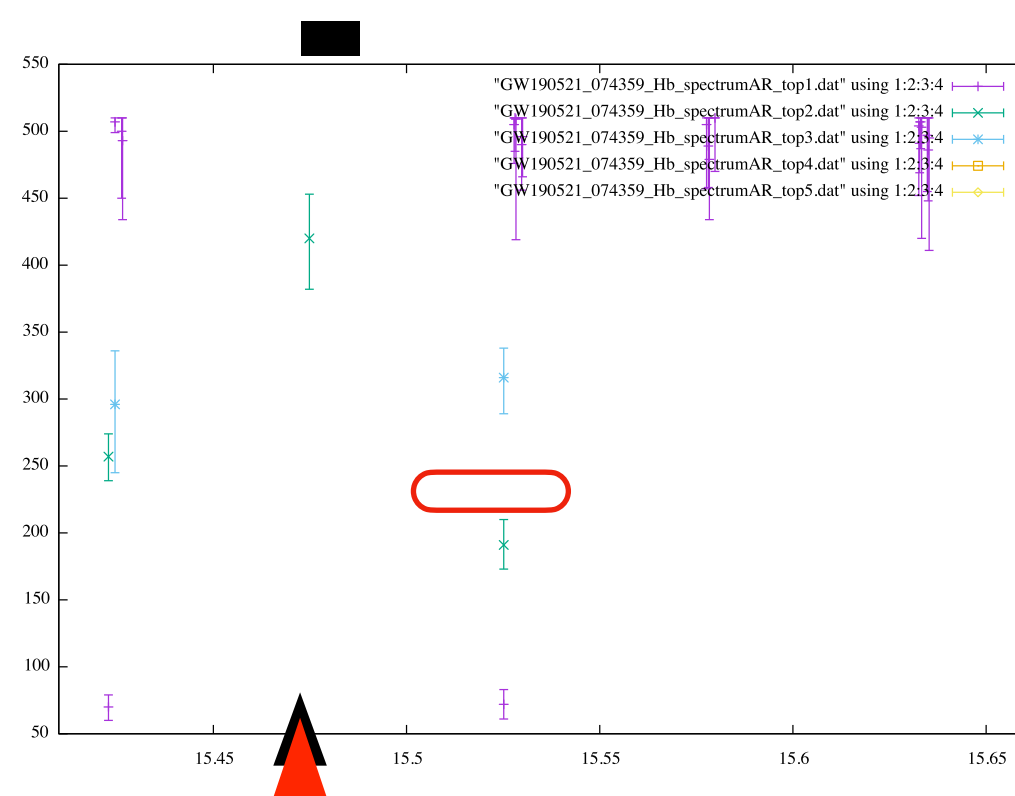
## Hanford



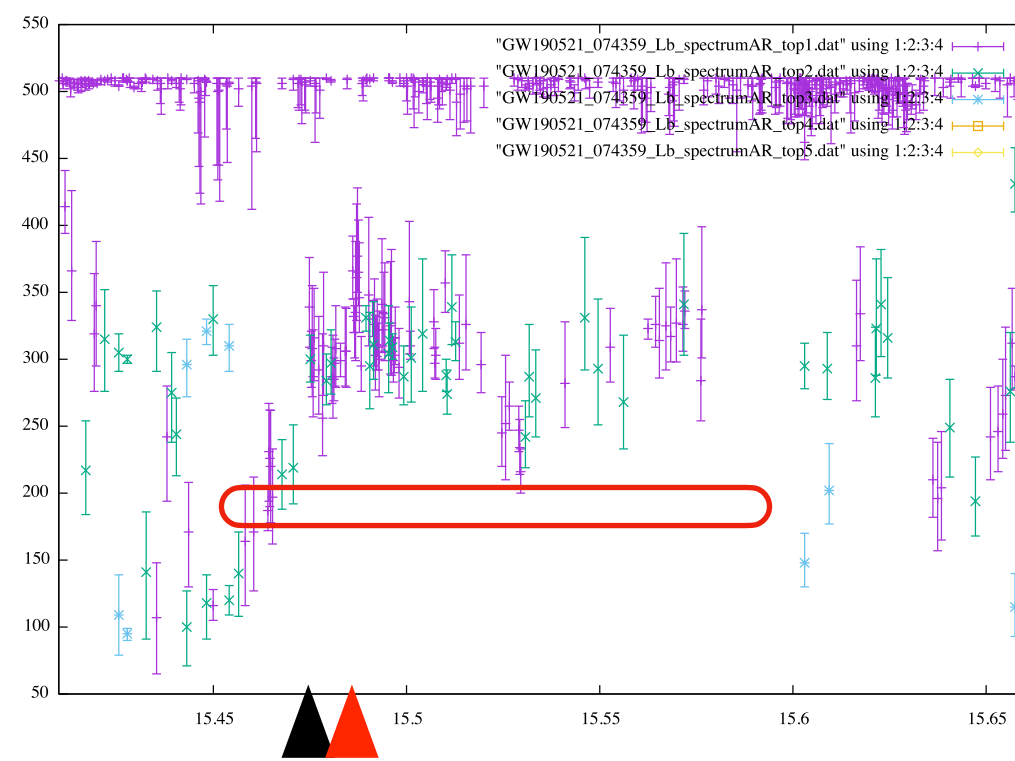
## Livingston



## Virgo X



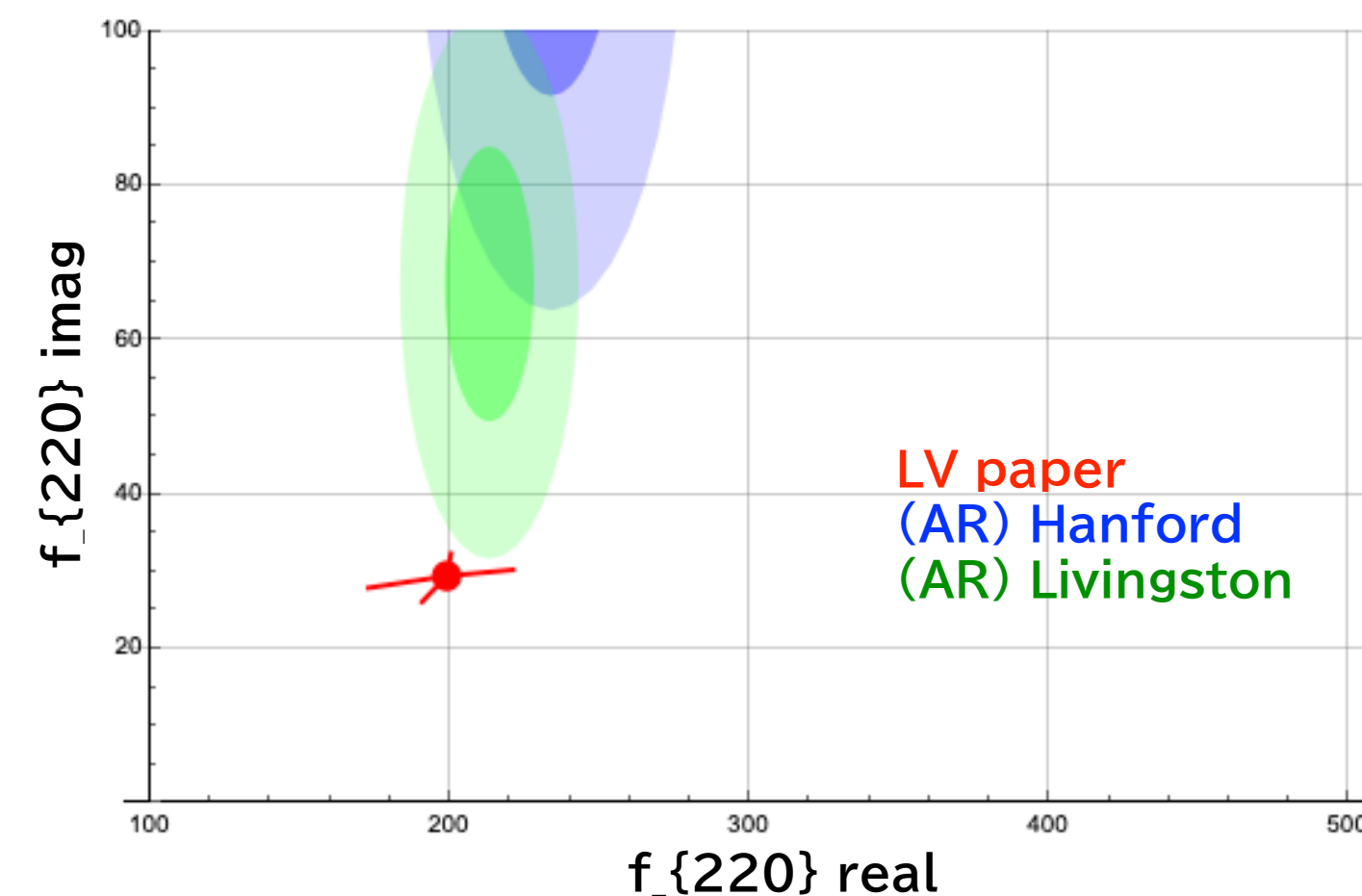
2 segments  
f = 233.66 ± 22.164 Hz  
t = 15.5256 -- 15.5266



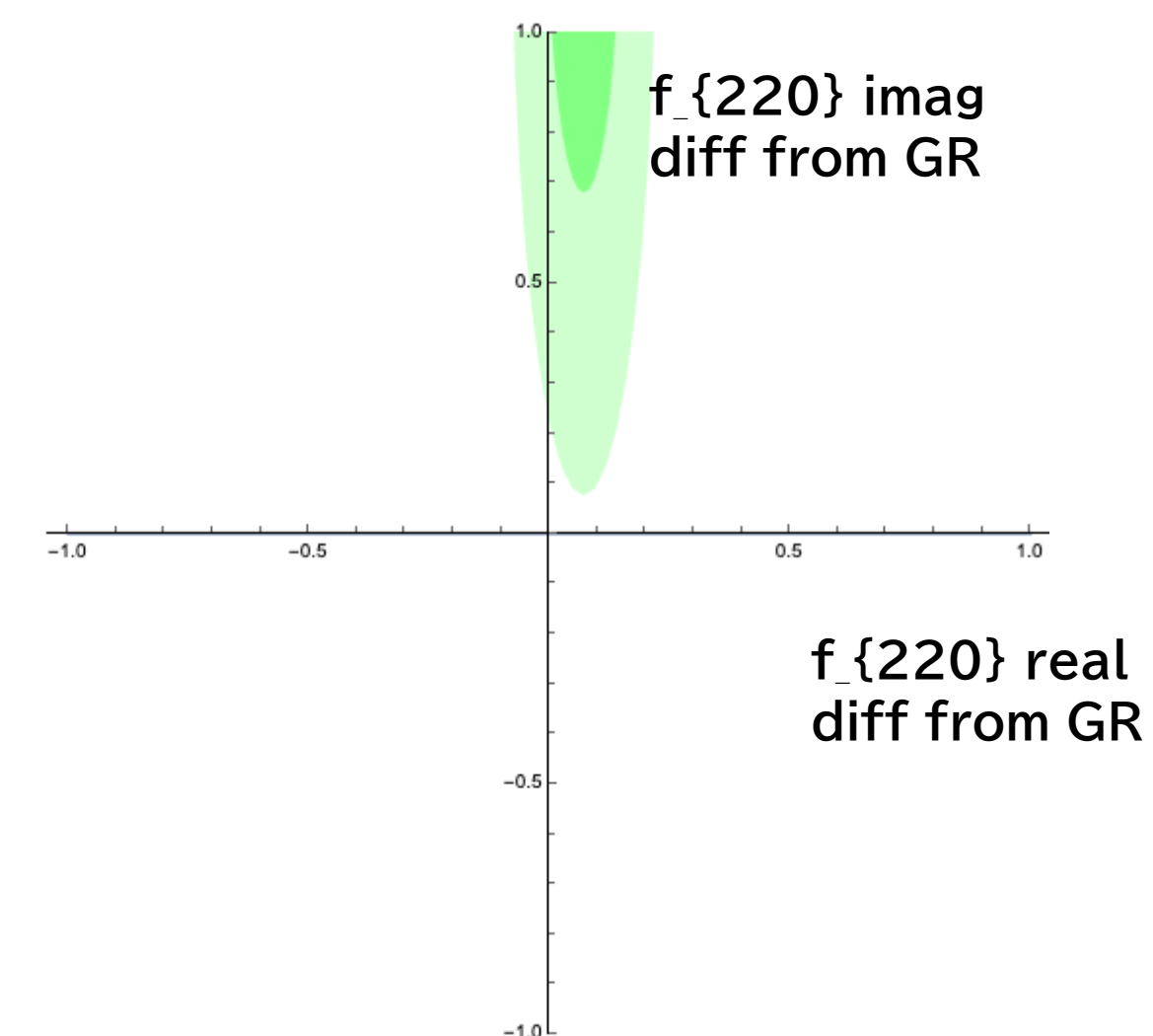
6 segments  
f = 213.03 ± 14.90 Hz  
t = 15.4639 - 15.5928

f<sub>QNM</sub>  
@Earth ▶

$f_{220} = 198.8 + i 29.43$  Hz,  $f_{221} = 195.0 + i 89.23$  Hz,  $f_{222} = 187.5 + i 150.5$  Hz  
 $f_{210} = 269.7 + i 47.34$  Hz,  $f_{211} = 163.6 + i 90.18$  Hz,  $f_{200} = 179.9 + i 37.98$  Hz  
 $f_{330} = 314.7 + i 30.14$  Hz,  $f_{331} = 312.5 + i 92.78$  Hz,  $f_{332} = 308.3 + i 165.7$  Hz  
 $f_{320} = 281.3 + i 30.46$  Hz,  $f_{310} = 252.9 + i 41.88$  Hz,  $f_{300} = 229.5 + i 32.02$  Hz



LV paper  
(AR) Hanford  
(AR) Livingston



このイベントは検出できなかった

GW190521\_074359  
delay time (msec) from t  
(+ delay, - advanced)

LHO = 1.11895  
LLO = 10.7412  
Virgo = 0.690039

t\_merger = 15.463 s  
t\_mergerH = 15.464  
t\_mergerL = 15.474  
t\_mergerV = 15.457



# GW200112\_155838

Network SNR=19.8

1 segment = 64 points  
= 1/64 sec = 15.625 ms

LV paper ▶

$$(M, a, z) = (60.8_{-4.3}^{+5.3}, 0.71_{-0.06}^{+0.06}, 0.24_{-0.08}^{+0.07})$$

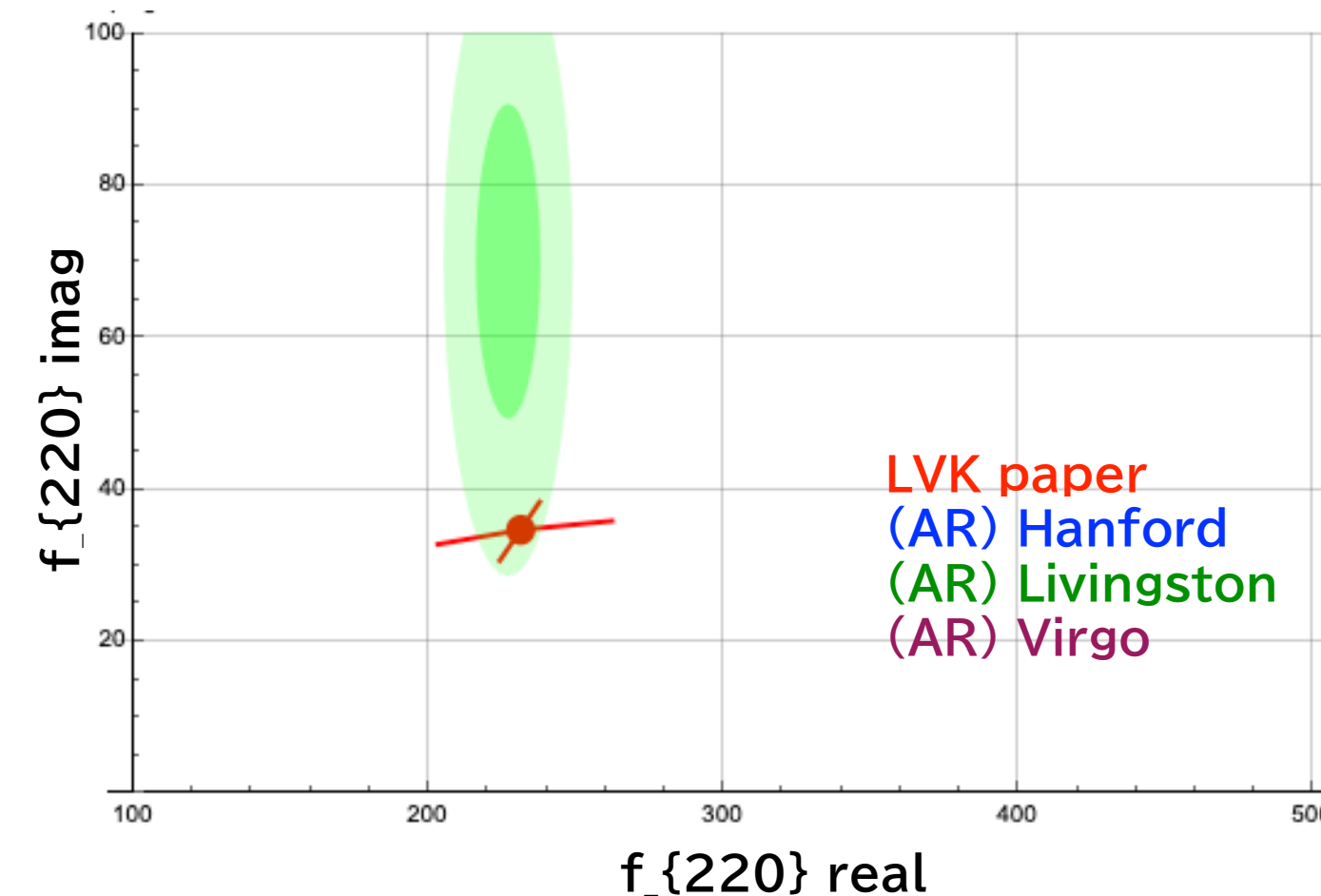
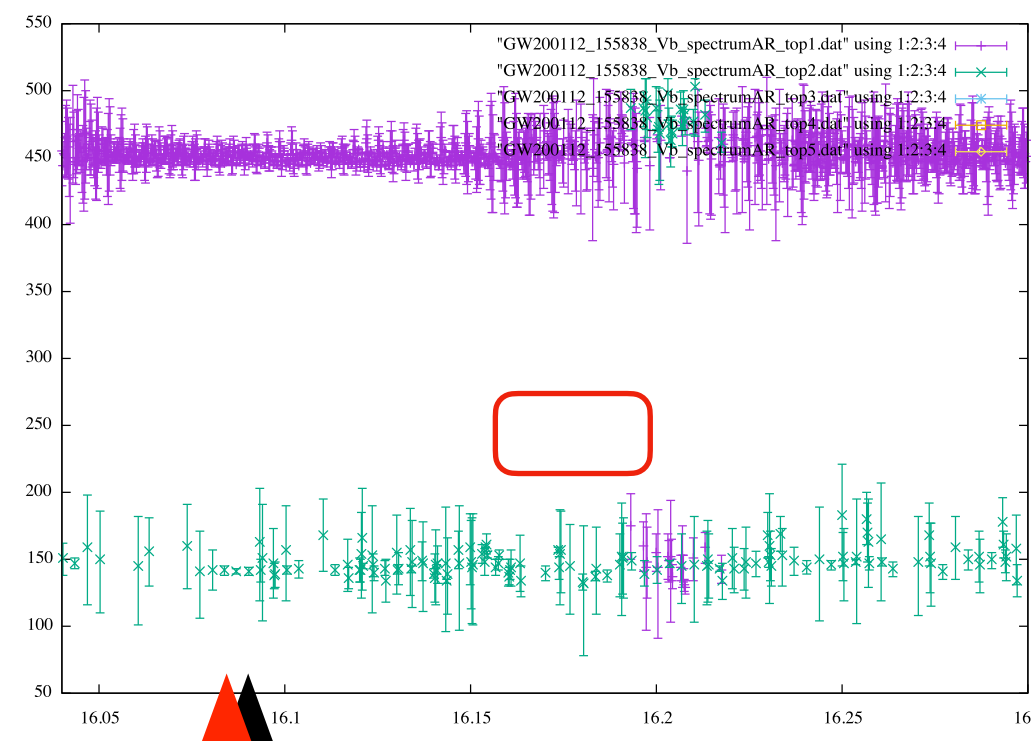
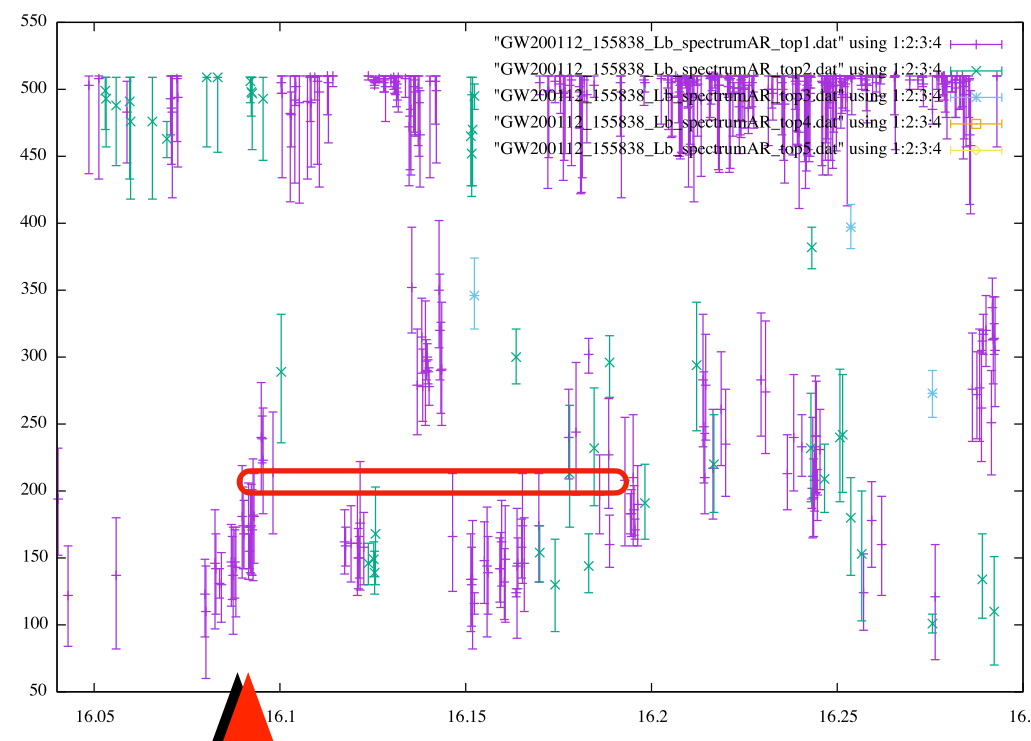
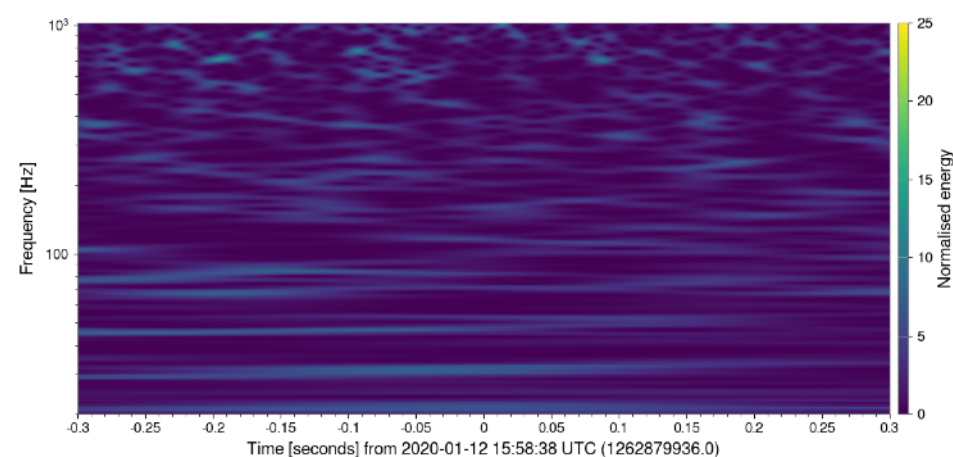
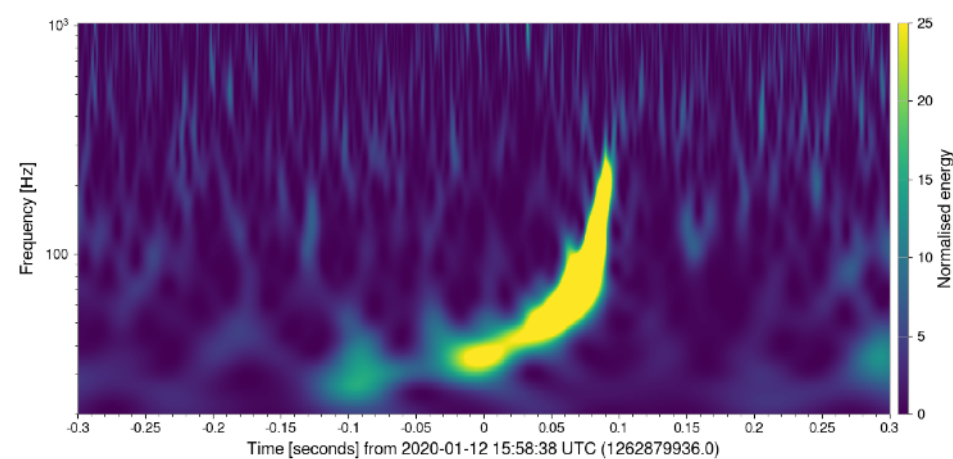
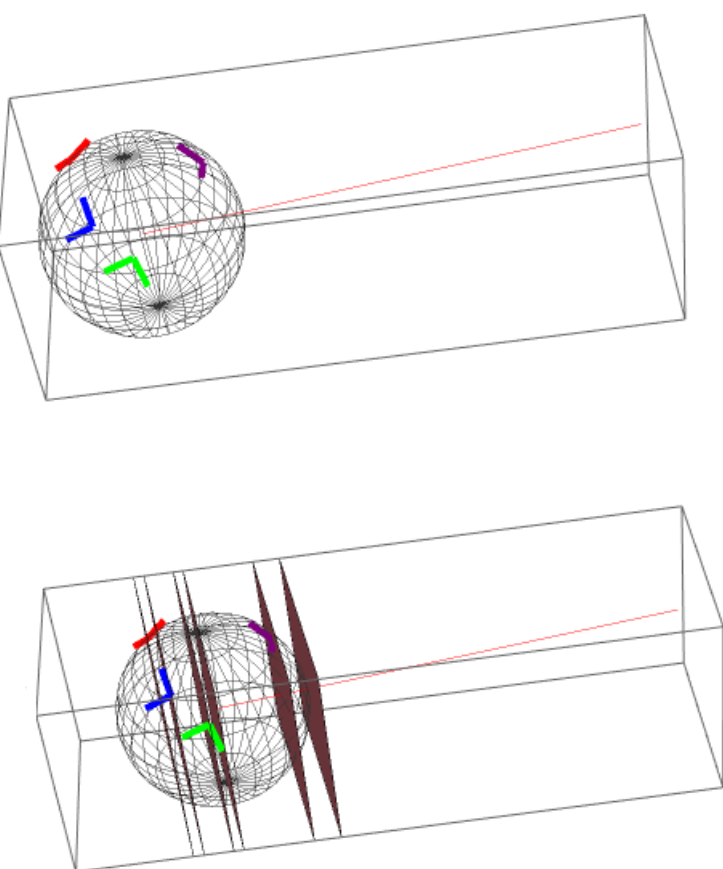
$f_{\text{QNM}}$   
@Earth ▶

$f_{220} = 231.1 + i 34.69$  Hz,  $f_{221} = 226.5 + i 105.2$  Hz,  $f_{222} = 217.5 + i 177.6$  Hz  
 $f_{210} = 317.0 + i 56.13$  Hz,  $f_{211} = 190.8 + i 106.2$  Hz,  $f_{200} = 211.1 + i 44.72$  Hz  
 $f_{330} = 366.1 + i 35.55$  Hz,  $f_{331} = 363.3 + i 109.4$  Hz,  $f_{332} = 358.3 + i 195.8$  Hz  
 $f_{320} = 328.0 + i 35.89$  Hz,  $f_{310} = 295.6 + i 48.9$  Hz,  $f_{300} = 268.7 + i 37.61$  Hz

Hanford X

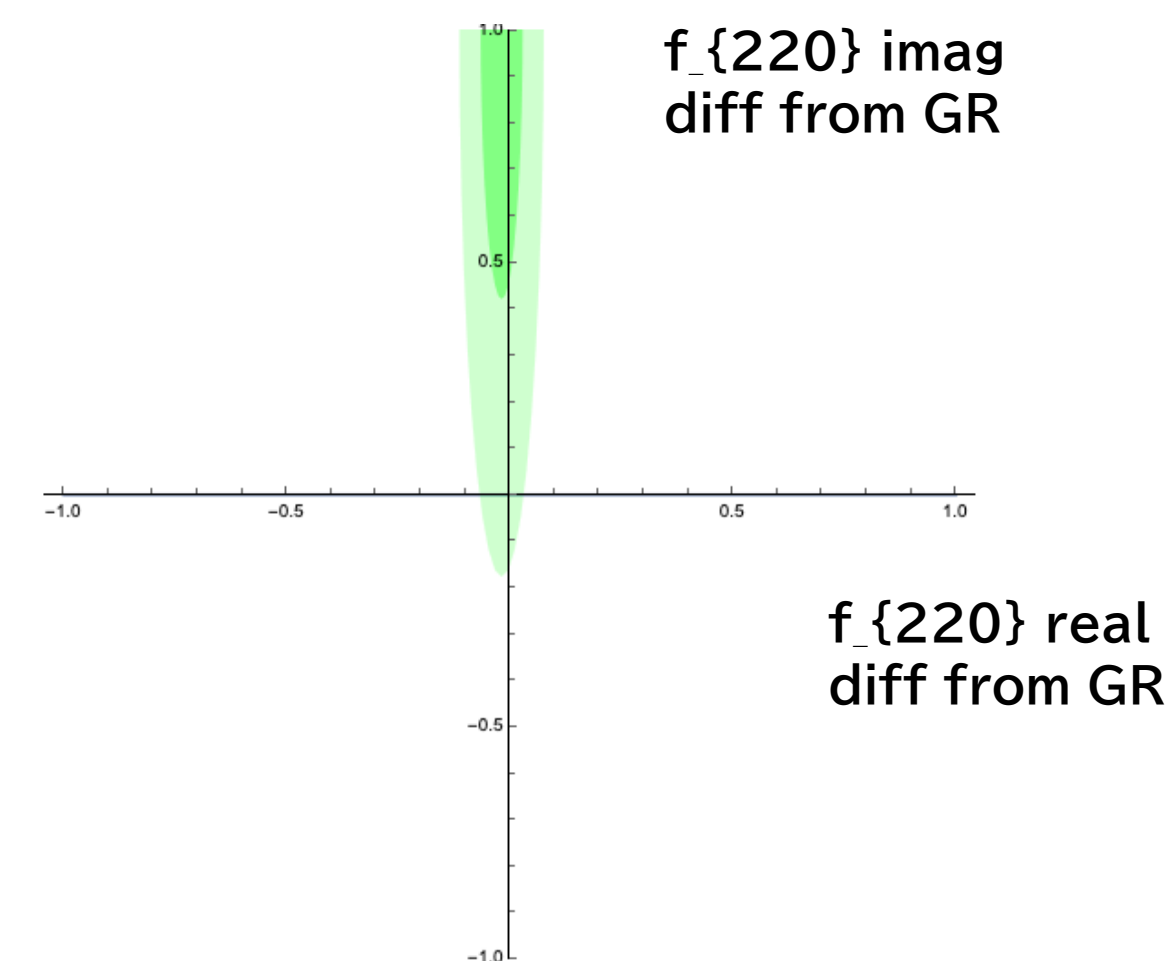
Livingston

Virgo



31 segments  
 $f = 226.88 \pm 10.95$  Hz  
 $t = 16.0950 \text{ -- } 16.1992$

2 segments  
 $f = 276.78 \pm 76.15$  Hz  
 $t = 16.1528 \text{ -- } 16.1992$



GW200112  
 delay time (msec) from  $t_0$   
 (+ delay, - advanced)

LHO = 8.634  
 LLO = 2.1455  
 Virgo = -15.3773

$t_{\text{merger}} = 16.094$  s  
 $t_{\text{mergerH}} = 16.103$   
 $t_{\text{mergerL}} = 16.096$   
 $t_{\text{mergerV}} = 16.080$

# GW200129\_065458

Network SNR=26.8

1 segment = 256 points  
= 1/16 sec = 62.5 ms

LV paper ▶

$$(M, a, z) = (60.3_{-3.3}^{+4.0}, 0.73_{-0.05}^{+0.06}, 0.18_{-0.07}^{+0.05})$$

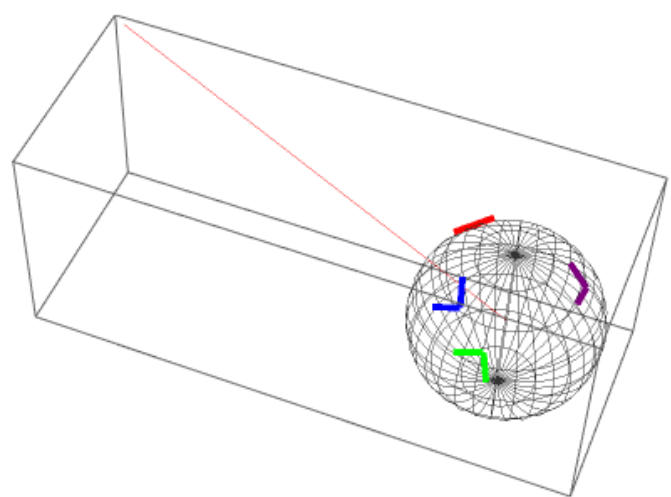
$f_{QNM}$   
@Earth ▶

$$f_{220} = 249.0 + i 36.34 \text{ Hz}, f_{221} = 244.4 + i 110.1 \text{ Hz}, f_{222} = 235.4 + i 185.5 \text{ Hz},$$

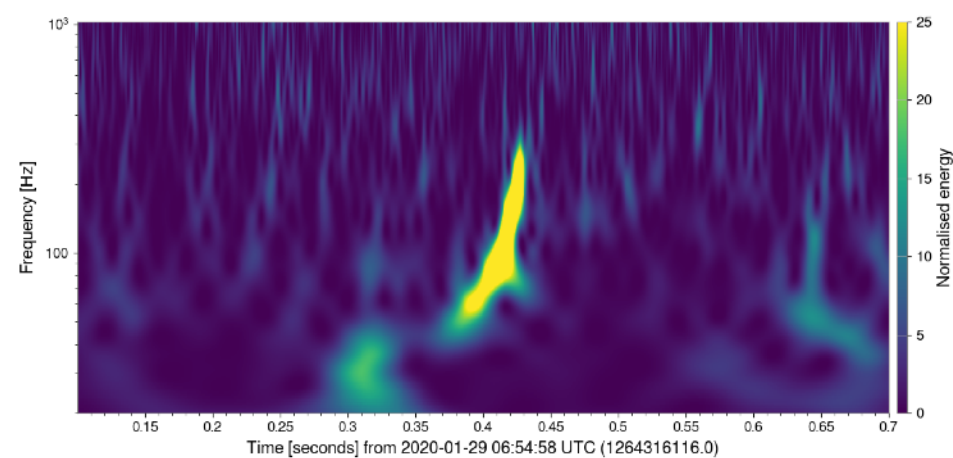
$$f_{210} = 334.0 + i 58.09 \text{ Hz}, f_{211} = 204.2 + i 111.4 \text{ Hz}, f_{200} = 223.1 + i 46.94 \text{ Hz},$$

$$f_{330} = 394.0 + i 37.18 \text{ Hz}, f_{331} = 391.3 + i 114.5 \text{ Hz}, f_{332} = 386.3 + i 204.1 \text{ Hz},$$

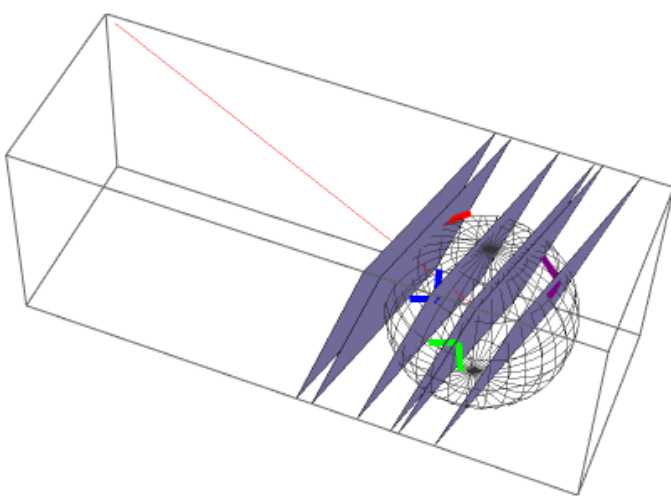
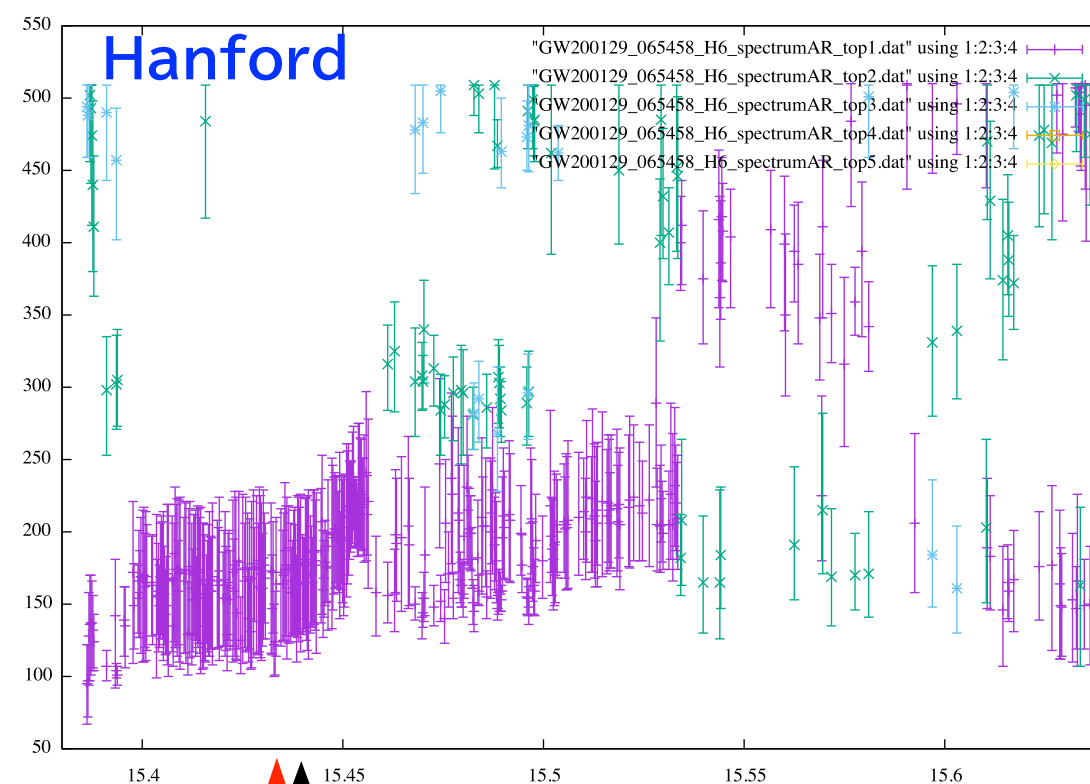
$$f_{320} = 351.2 + i 37.62 \text{ Hz}, f_{310} = 315.1 + i 52.23 \text{ Hz}, f_{300} = 285.4 + i 39.7 \text{ Hz},$$



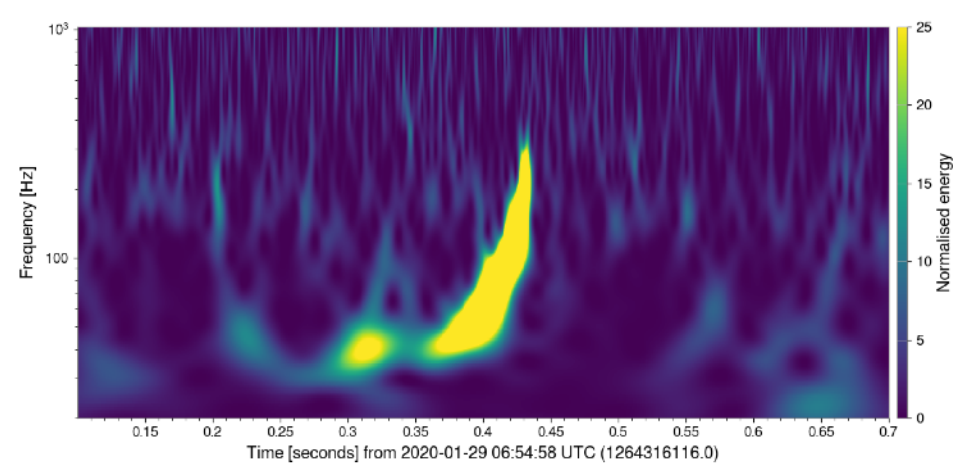
## Hanford



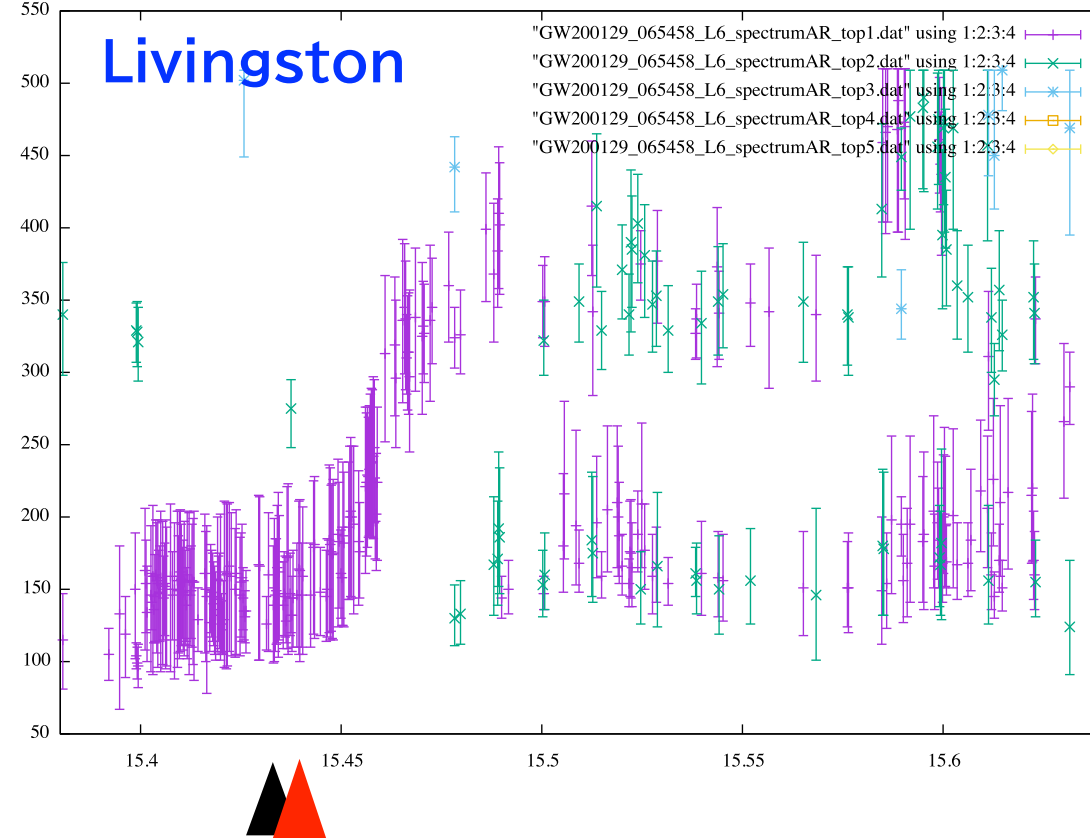
## Hanford



## Livingston



## Livingston



5 segments

$$f = 296.12 \pm 8.29 \text{ Hz}$$

$$t = 15.4736 - 15.5012$$

10 segments

$$f = 258.19 \pm 3.90 \text{ Hz}$$

$$t = 15.4607 - 15.5081$$

73 segments

$$f = 246.45 \pm 10.08 \text{ Hz}$$

$$t = 15.4556 - 15.5493$$

29 segments

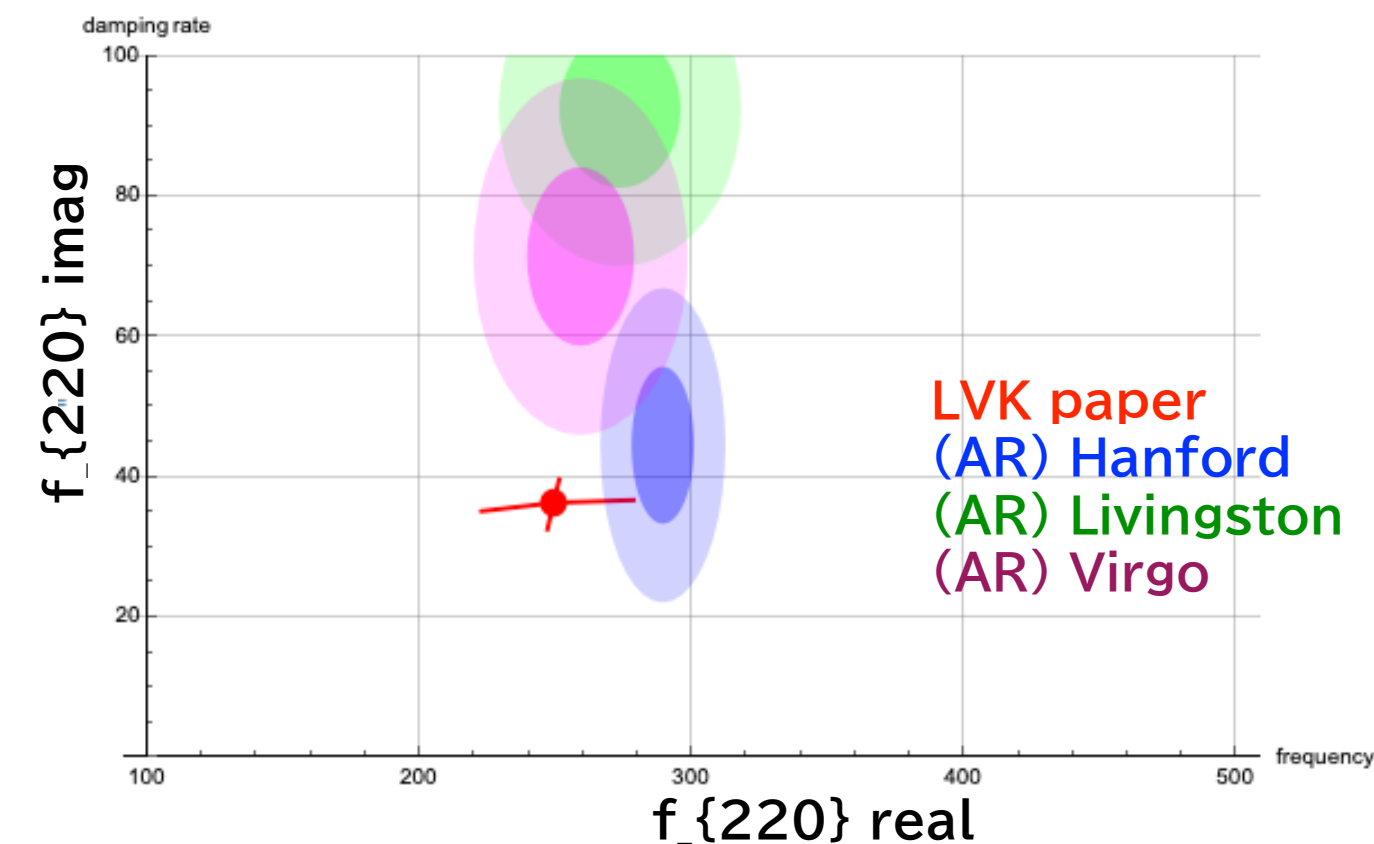
$$f = 316.65 \pm 10.77 \text{ Hz}$$

$$t = 15.4609 - 15.4993$$

39 segments

$$f = 259.97 \pm 11.16 \text{ Hz}$$

$$t = 15.4553 - 15.5461$$



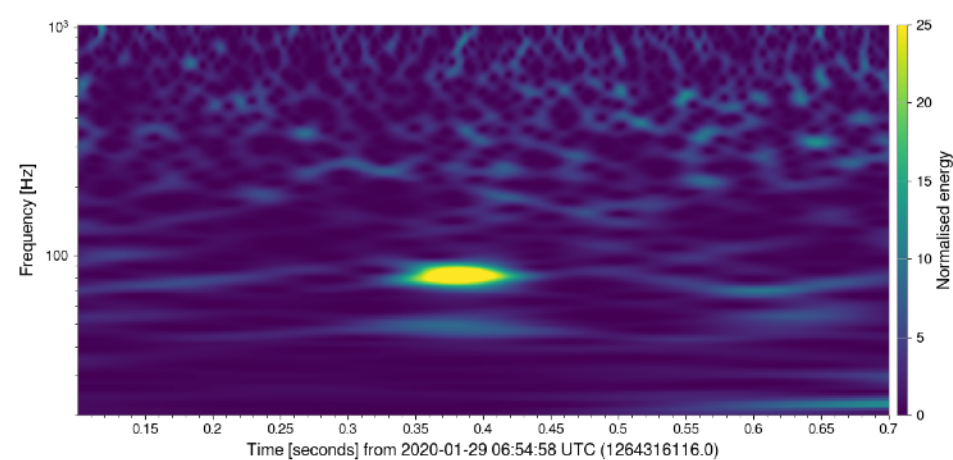
LVK paper  
(AR) Hanford  
(AR) Livingston  
(AR) Virgo

GW200129  
delay time (msec) from  $t_0$   
(+ delay, - advanced)

LHO = - 7.5579  
LLO = 1.71251  
Virgo = 9.73468

$t_{\text{merger}} = 15.435 \text{ s}$   
 $t_{\text{mergerH}} = 15.427$   
 $t_{\text{mergerL}} = 15.437$   
 $t_{\text{mergerV}} = 15.445$

## Virgo





# GW200129\_065458

Network SNR=26.8

1 segment = 64 points  
= 1/64 sec = 15.625 ms

LV paper ▶

$$(M, a, z) = (60.3_{-3.3}^{+4.0}, 0.73_{-0.05}^{+0.06}, 0.18_{-0.07}^{+0.05})$$

$f_{QNM}$   
@Earth ▶

$$f_{220} = 249.0 + i 36.34 \text{ Hz}, f_{221} = 244.4 + i 110.1 \text{ Hz}, f_{222} = 235.4 + i 185.5 \text{ Hz},$$

$$f_{210} = 334.0 + i 58.09 \text{ Hz}, f_{211} = 204.2 + i 111.4 \text{ Hz}, f_{200} = 223.1 + i 46.94 \text{ Hz},$$

$$f_{330} = 394.0 + i 37.18 \text{ Hz}, f_{331} = 391.3 + i 114.5 \text{ Hz}, f_{332} = 386.3 + i 204.1 \text{ Hz},$$

$$f_{320} = 351.2 + i 37.62 \text{ Hz}, f_{310} = 315.1 + i 52.23 \text{ Hz}, f_{300} = 285.4 + i 39.7 \text{ Hz},$$

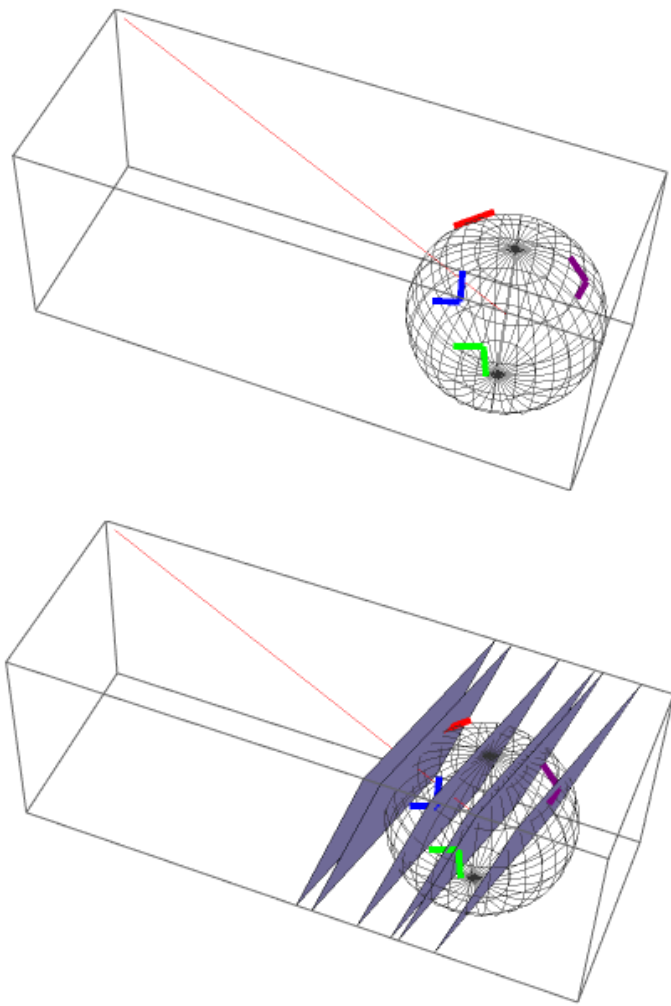
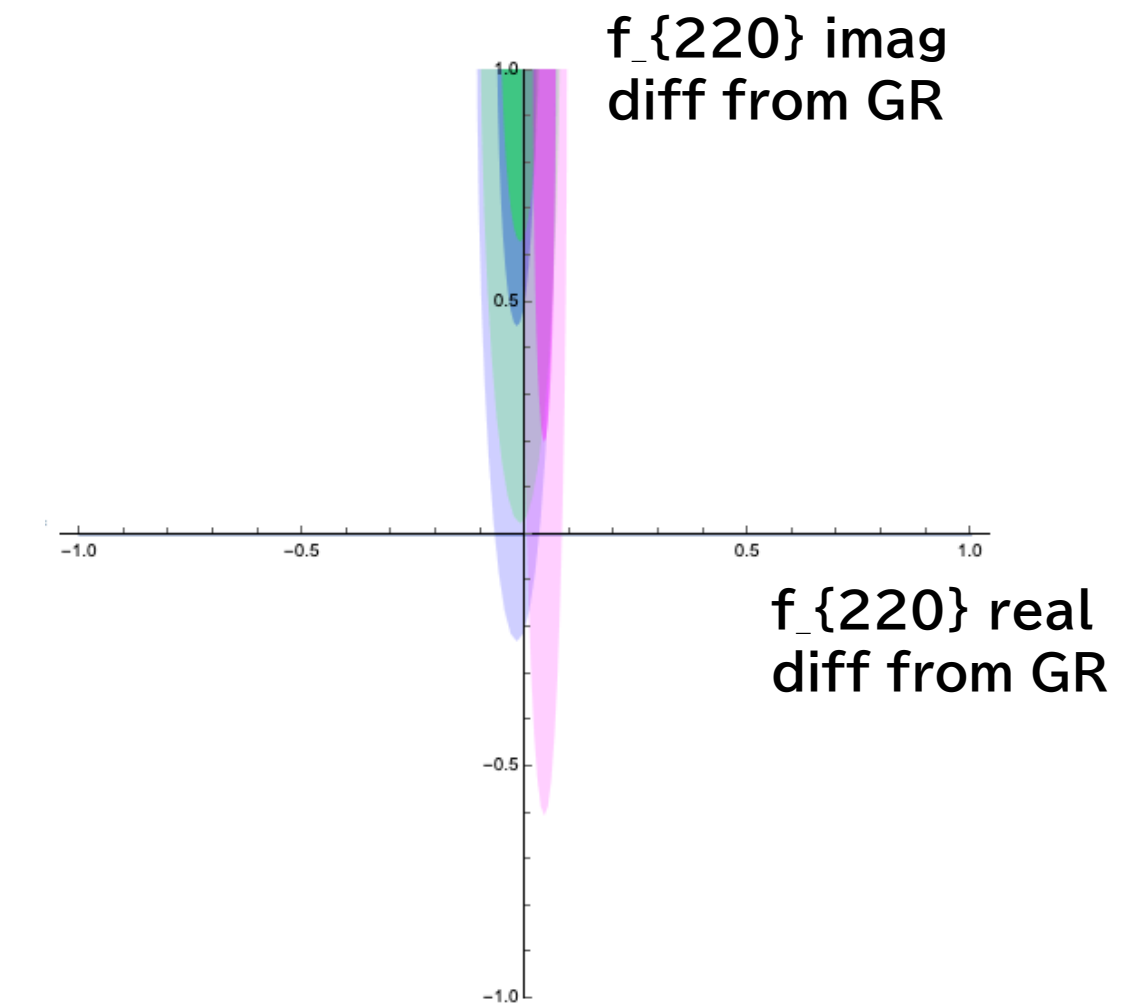
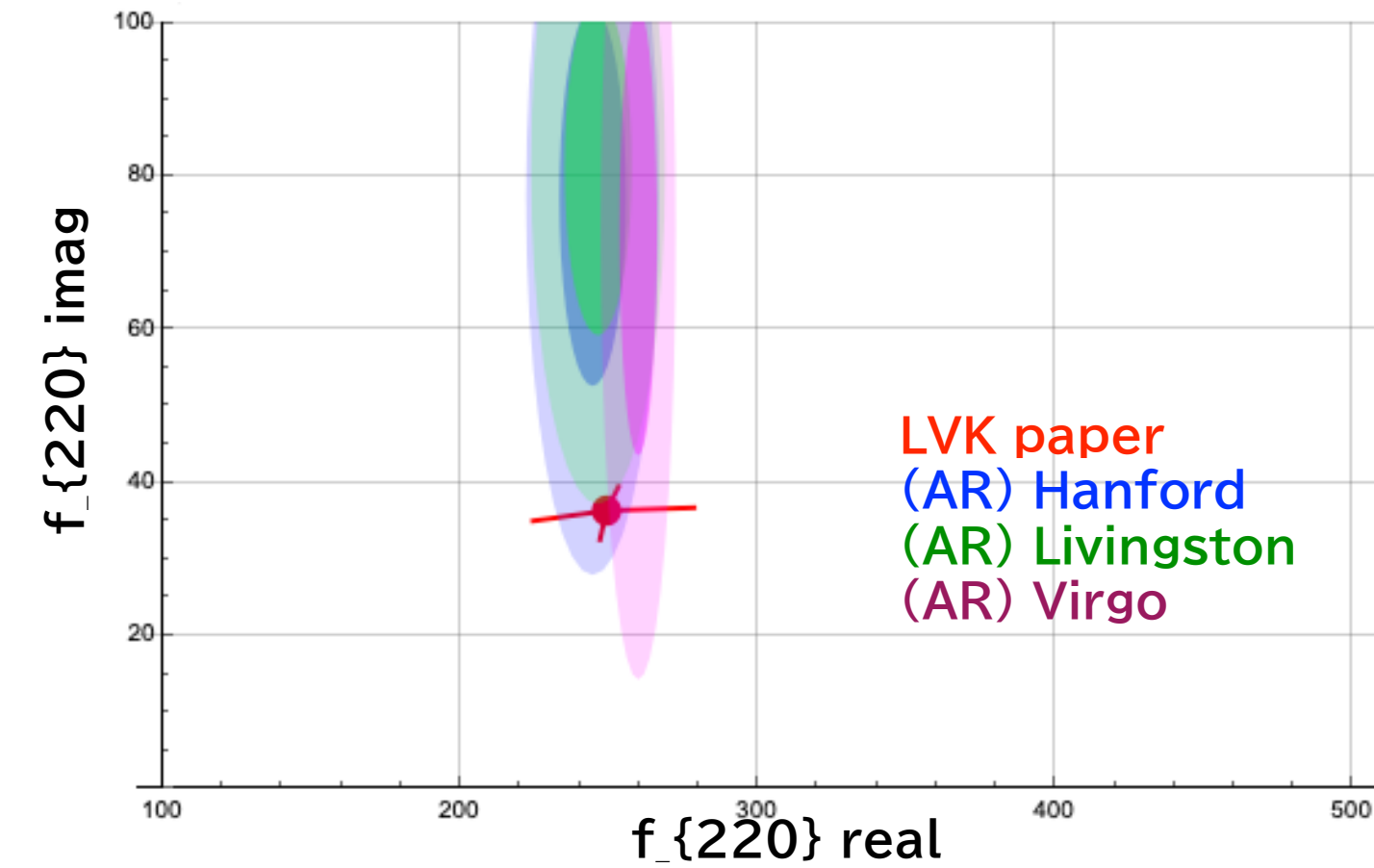
45 segments  
 $f = 244.68 \pm 10.90 \text{ Hz}$   
 $t = 15.4399 \text{ -- } 15.5796$

$f_{\{220\}}$  starts 5ms  
after  $t_{\text{merger}}$

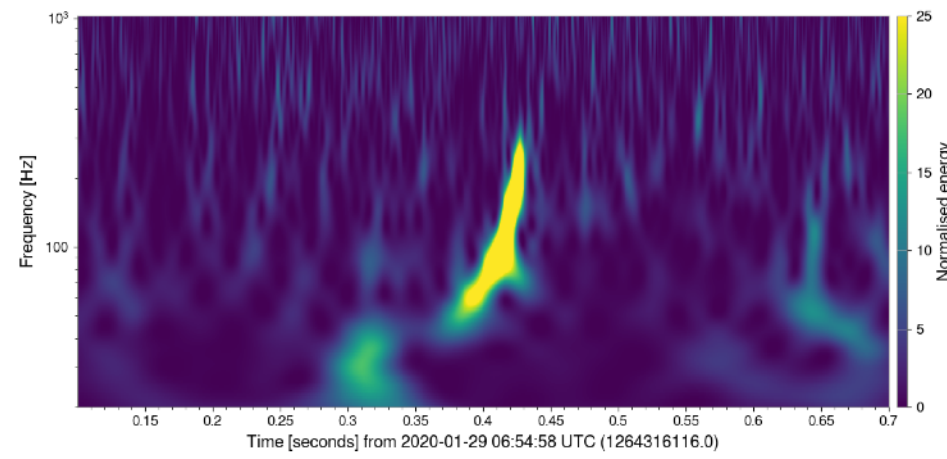
31 segments  
 $f = 246.20 \pm 11.28 \text{ Hz}$   
 $t = 15.4370 \text{ -- } 15.5840$

33 segments  
 $f = 259.69 \pm 6.31 \text{ Hz}$   
 $t = 15.4548 \text{ -- } 15.6497$

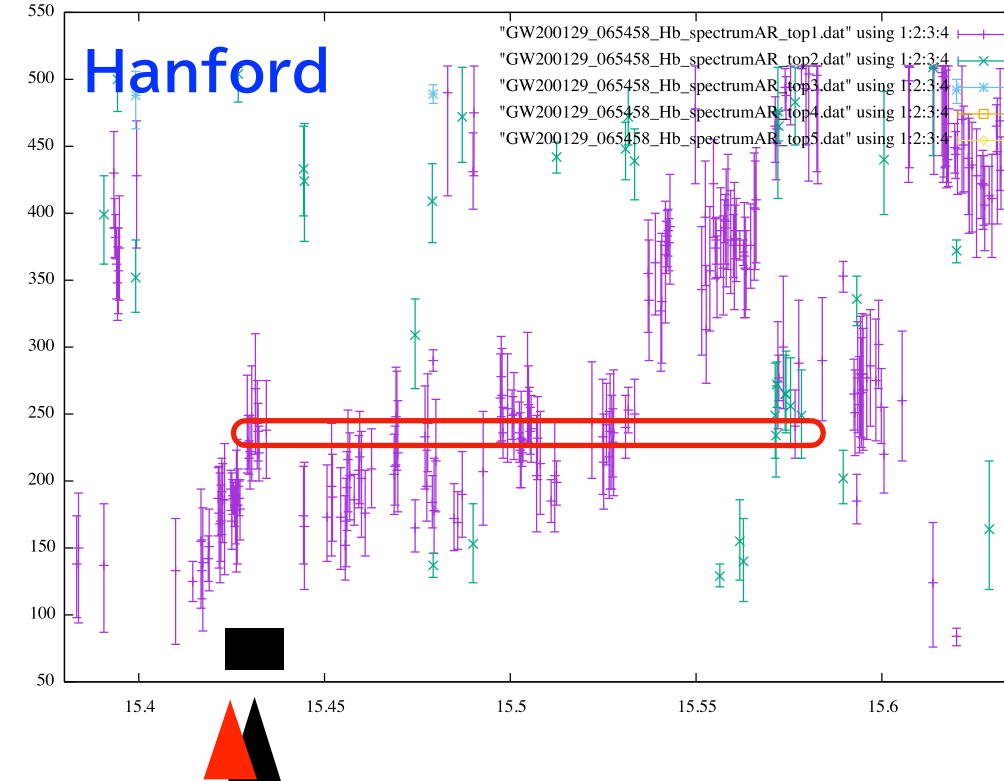
$f_{\{220\}}$  starts 3ms  
after  $t_{\text{merger}}$



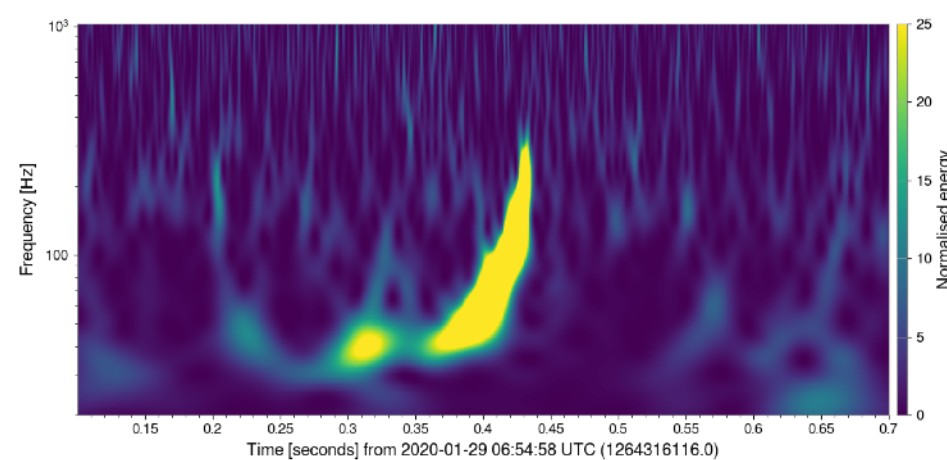
Hanford



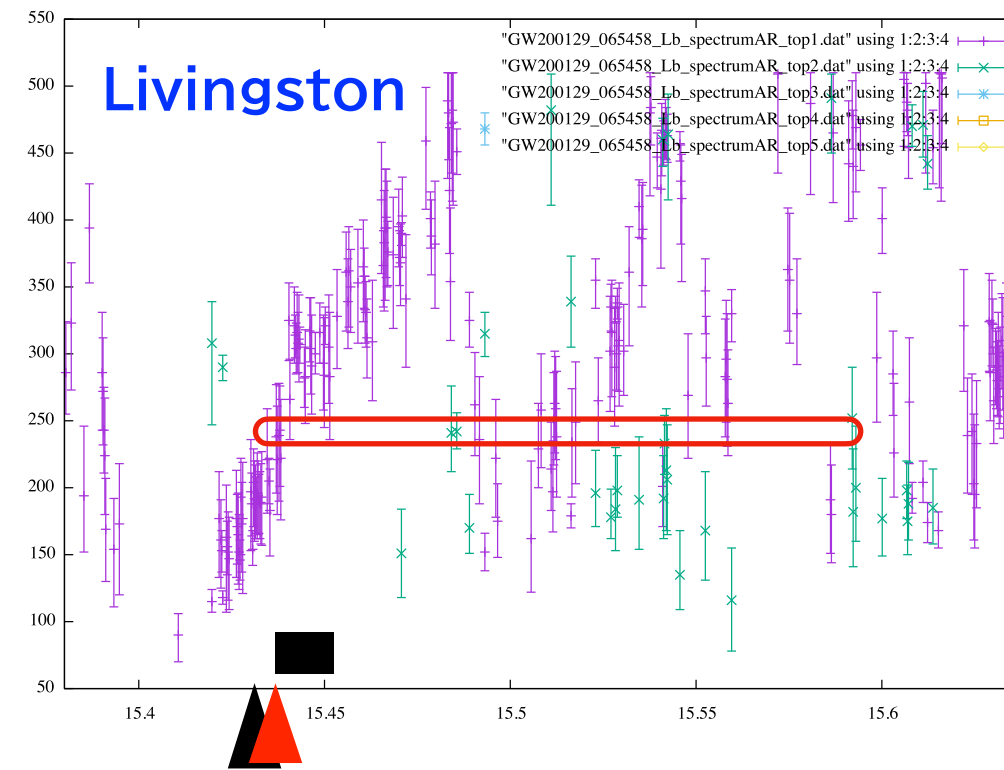
Hanford



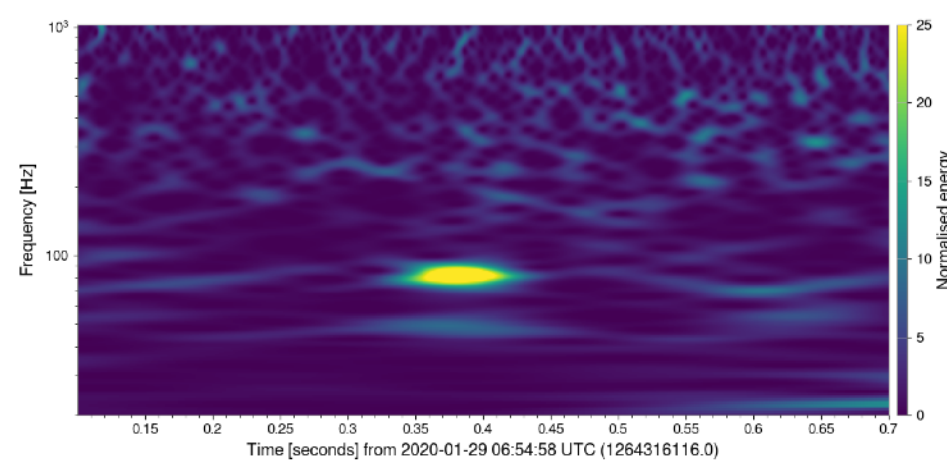
Livingston



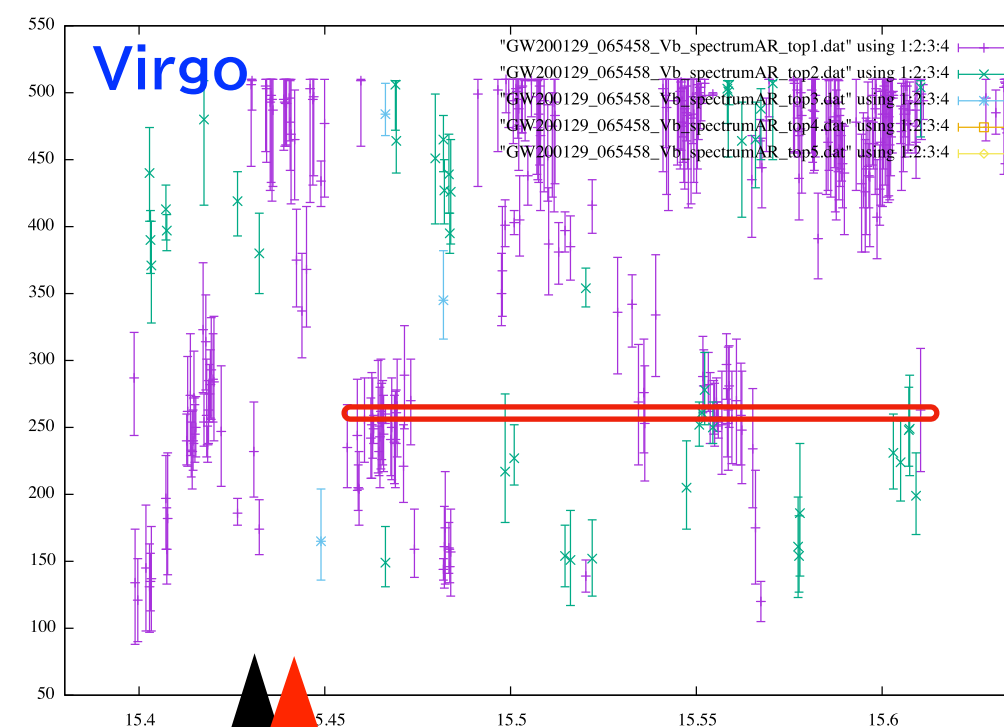
Livingston



Virgo



Virgo



GW200129  
delay time (msec) from  $t_0$   
(+ delay, - advanced)

LHO = - 7.5579  
LLO = 1.71251  
Virgo = 9.73468

$t_{\text{merger}} = 15.435 \text{ s}$   
 $t_{\text{mergerH}} = 15.427$   
 $t_{\text{mergerL}} = 15.437$   
 $t_{\text{mergerV}} = 15.445$

+ 7.8125s

$t_{\text{mergerH}} = 15.435$   
 $t_{\text{mergerL}} = 15.445$   
 $t_{\text{mergerV}} = 15.452$



# GW200224\_222234

Network SNR=20.0

1 segment = 64 points  
= 1/64 sec = 15.625 ms

LV paper ▶

$$(M, a, z) = (68.6_{-4.7}^{+6.6}, 0.73_{-0.07}^{+0.07}, 0.32_{-0.11}^{+0.08})$$

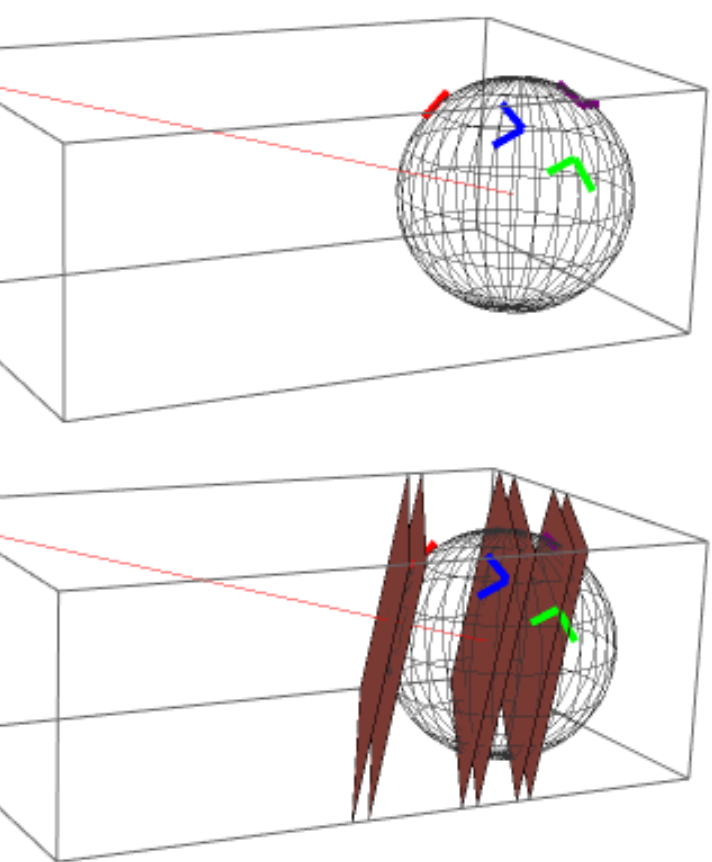
$f_{\text{QNM}}$   
@Earth ▶

$$f_{220} = 195.7 + i 28.55 \text{ Hz}, f_{221} = 192.1 + i 86.53 \text{ Hz}, f_{222} = 185.0 + i 145.8 \text{ Hz}$$

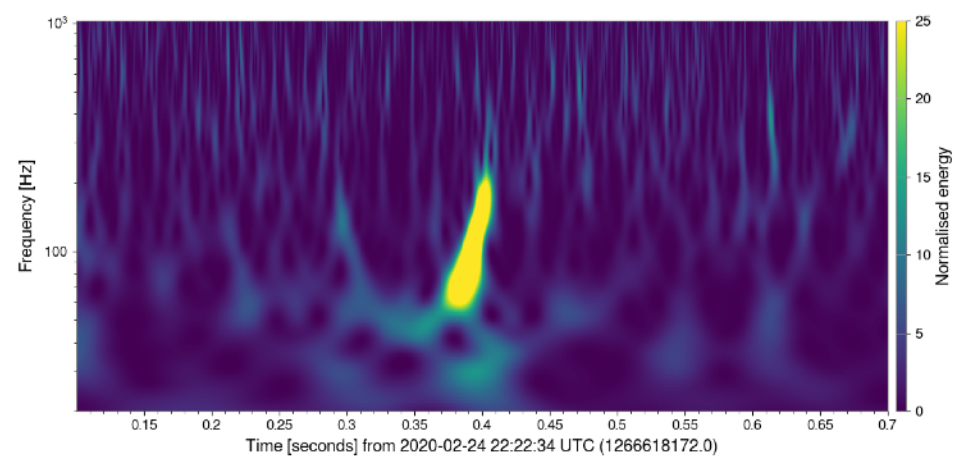
$$f_{210} = 262.4 + i 45.65 \text{ Hz}, f_{211} = 160.4 + i 87.55 \text{ Hz}, f_{200} = 175.3 + i 36.89 \text{ Hz}$$

$$f_{330} = 309.6 + i 29.22 \text{ Hz}, f_{331} = 307.4 + i 89.99 \text{ Hz}, f_{332} = 303.5 + i 160.3 \text{ Hz}$$

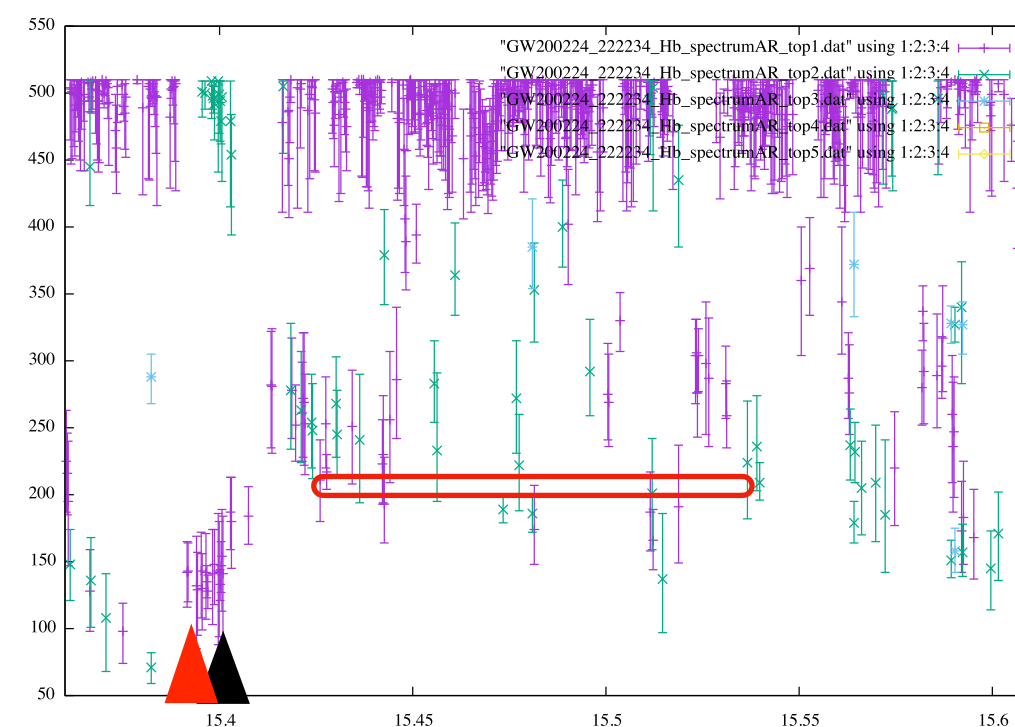
$$f_{320} = 276.0 + i 29.56 \text{ Hz}, f_{310} = 247.6 + i 41.04 \text{ Hz}, f_{300} = 224.3 + i 31.2 \text{ Hz}$$



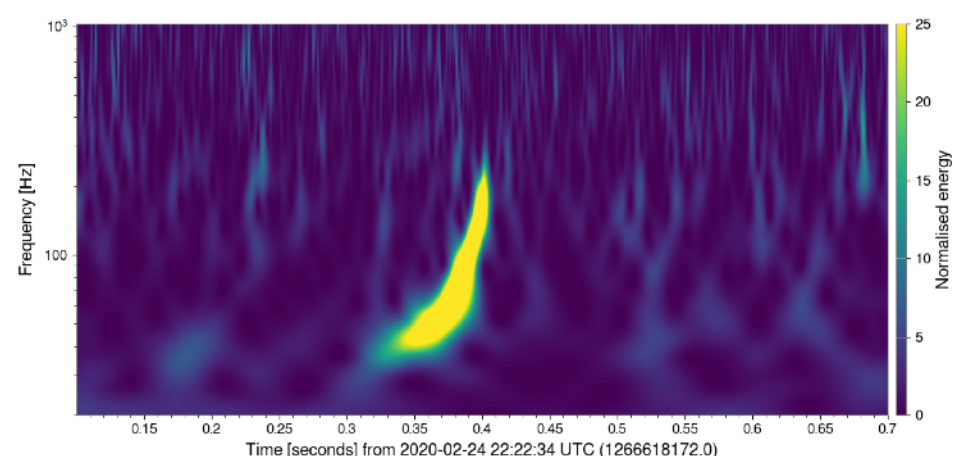
## Hanford



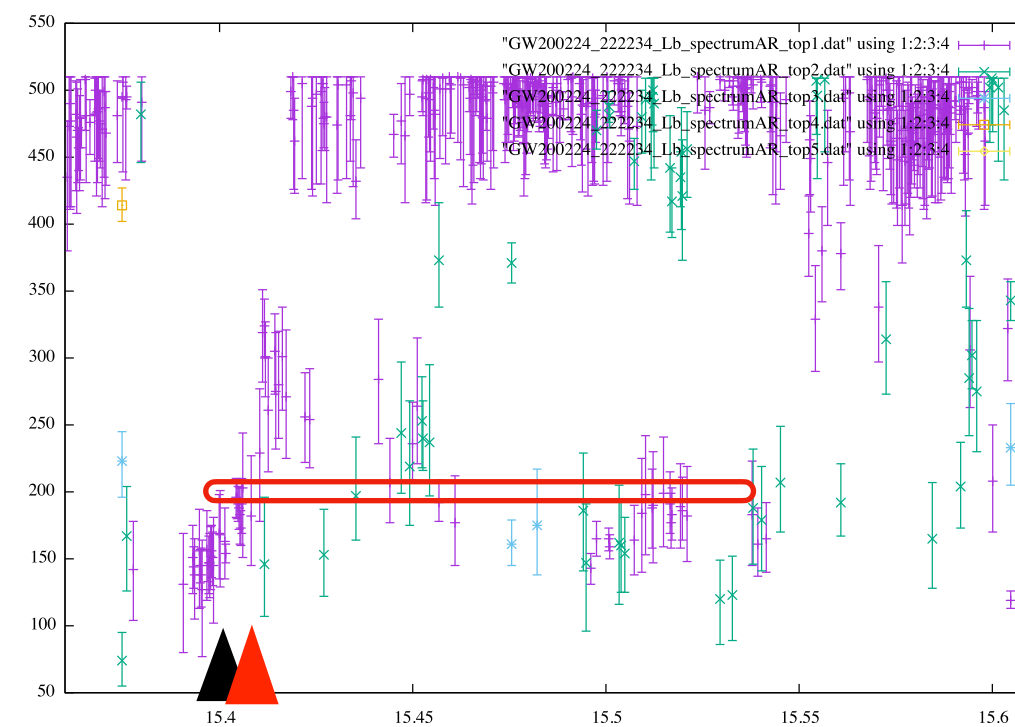
## Hanford



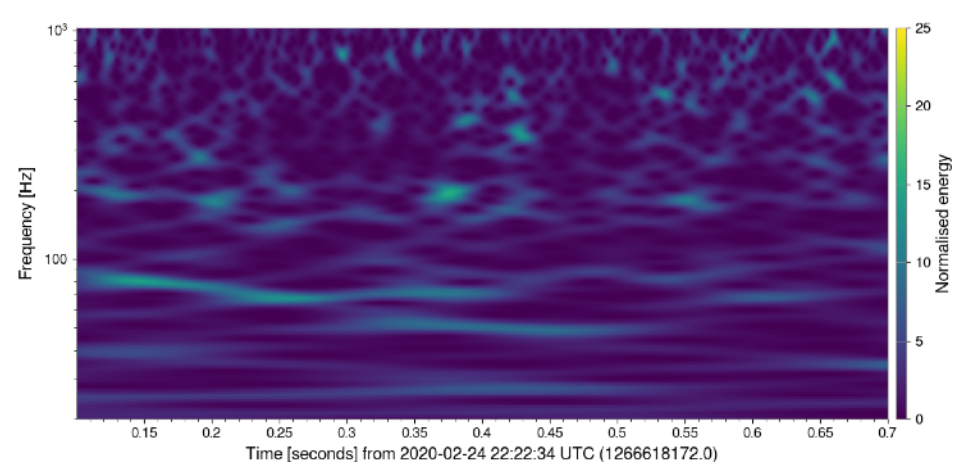
## Livingston



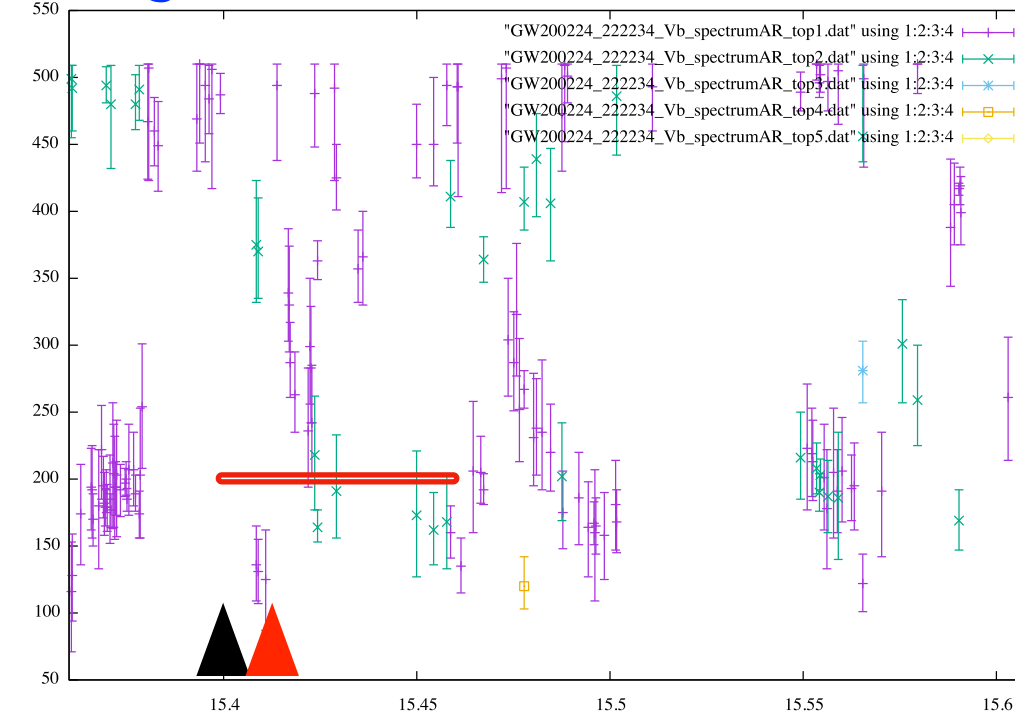
## Livingston



## Virgo



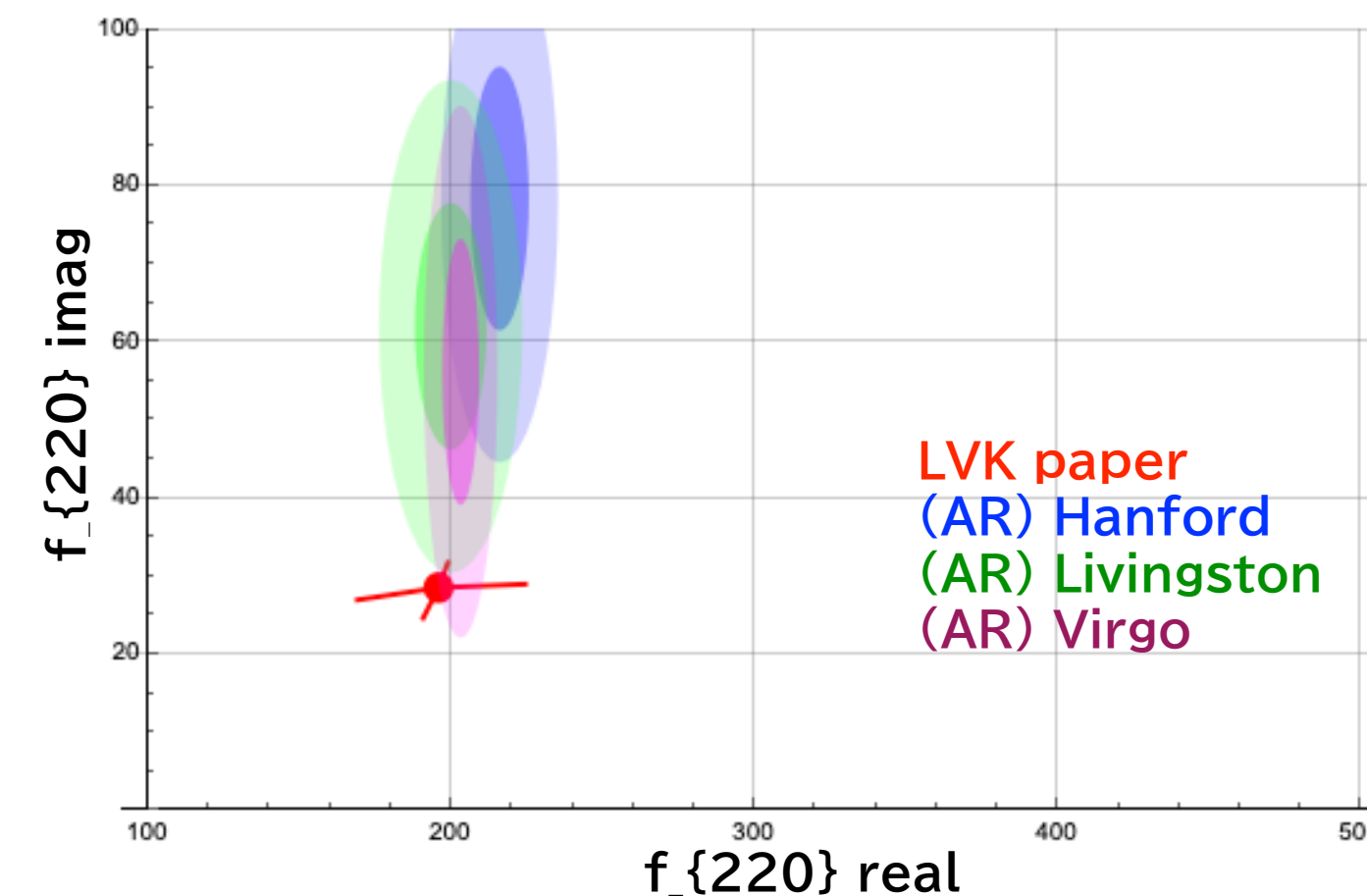
## Virgo



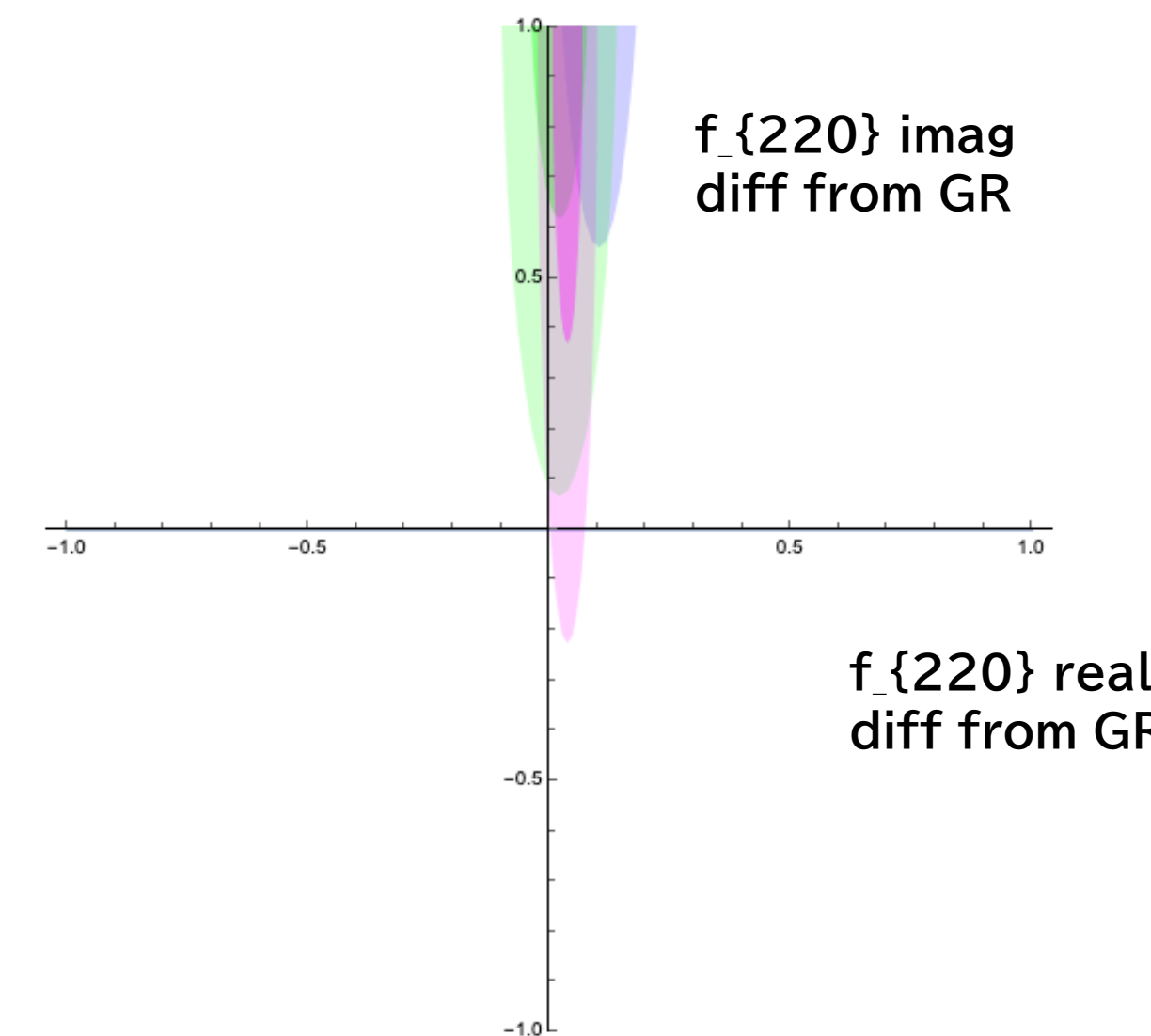
9 segments  
f = 215.82 ± 9.65 Hz  
t = 15.4395 -- 15.5396

28 segments  
f = 199.68 ± 11.82 Hz  
t = 15.4060 -- 15.5452

8 segments  
f = 202.96 ± 6.07 Hz  
t = 15.4060 -- 15.5452



LVK paper  
(AR) Hanford  
(AR) Livingston  
(AR) Virgo



f\_{220} imag  
diff from GR

f\_{220} real  
diff from GR

GW200224  
delay time (msec) from t0  
(+ delay, - advanced)

LHO = - 2.70101  
LLO = 6.94325  
Virgo = 9.23521  
KAGRA = -18.5399

t\_merger = 15.402 s  
t\_mergerH = 15.399  
t\_mergerL = 15.409  
t\_mergerV = 15.411

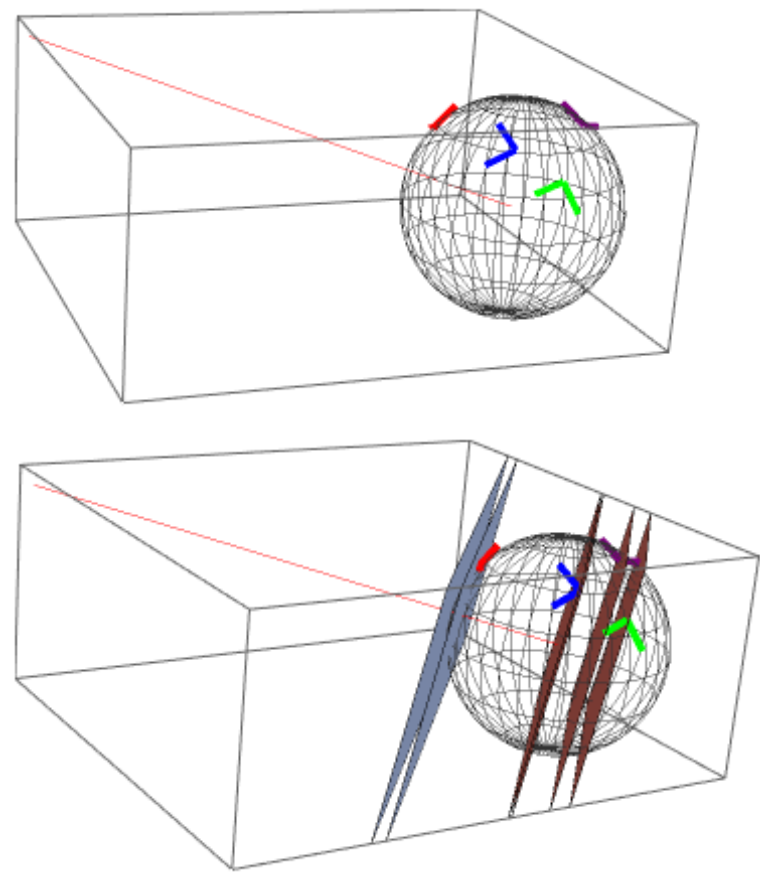


# GW200311\_115853

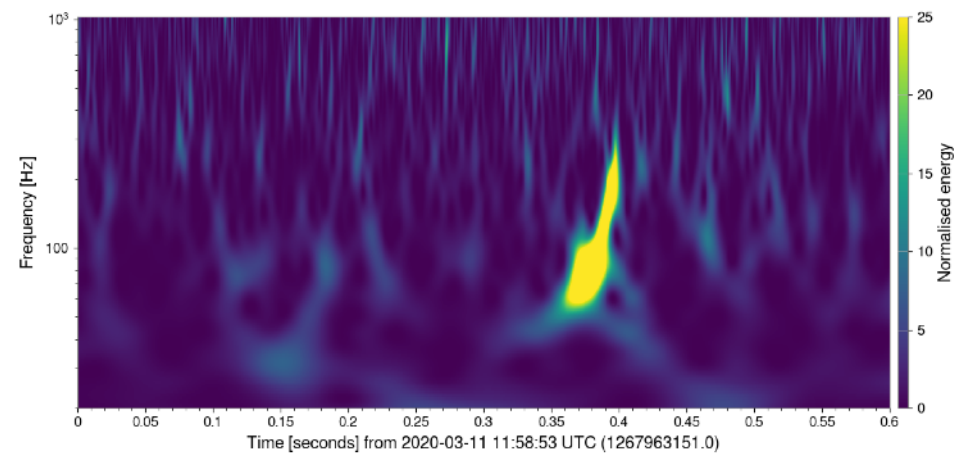
Network SNR=17.8

1 segment = 64 points  
= 1/64 sec = 15.625 ms

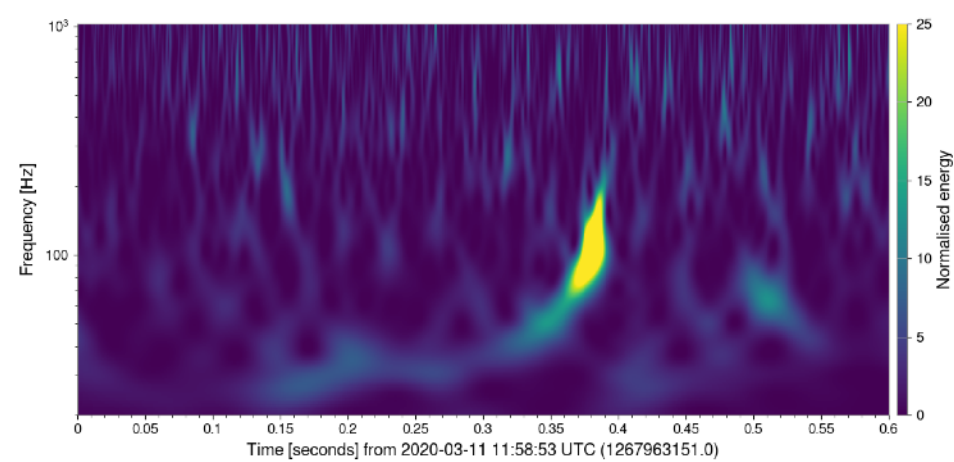
LV paper ▶  $(M, a, z) = (59.0^{+4.8}_{-3.9}, 0.69^{+0.07}_{-0.08}, 0.23^{+0.05}_{-0.07})$



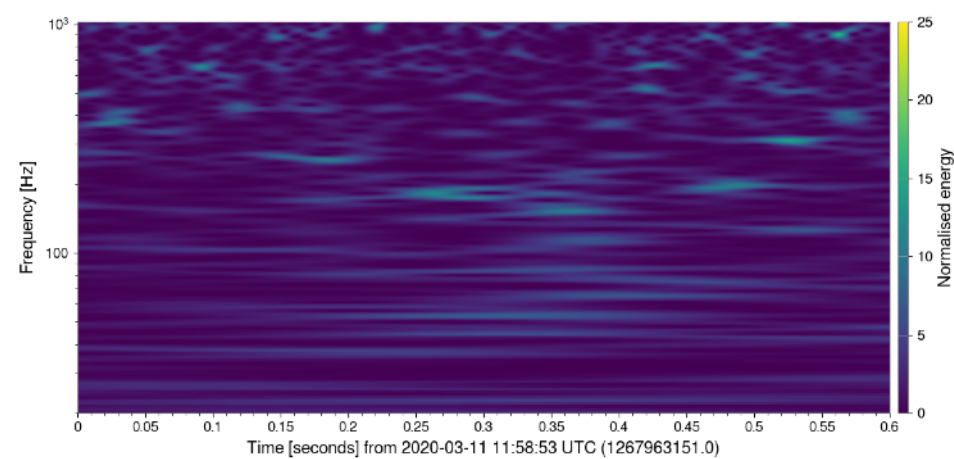
Hanford



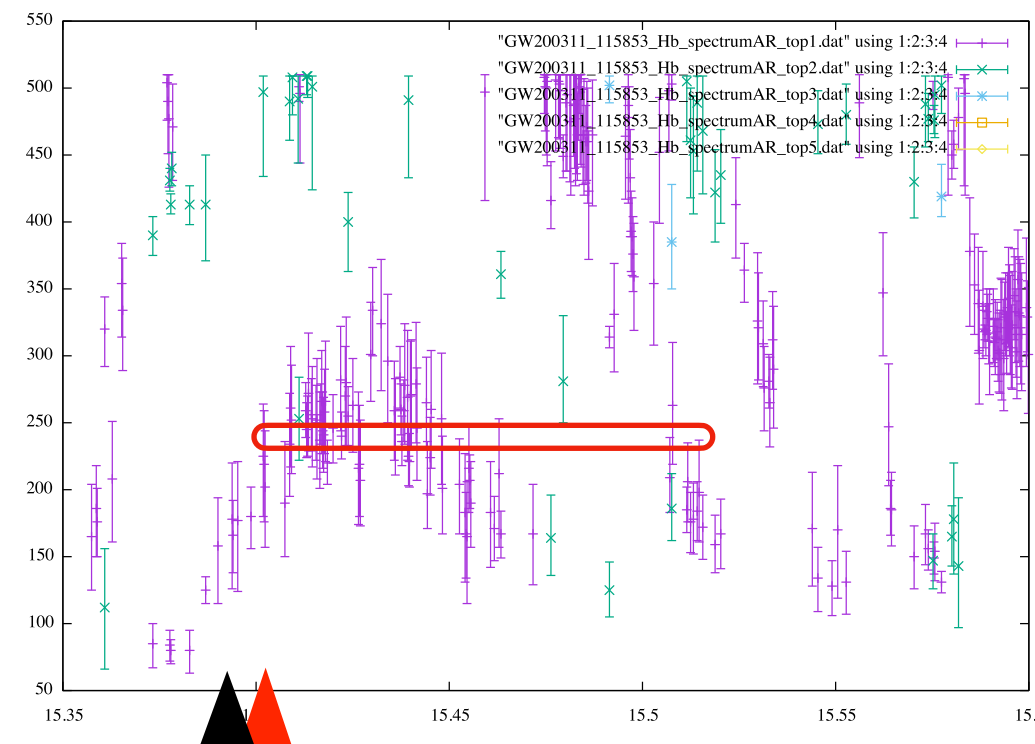
Livingston



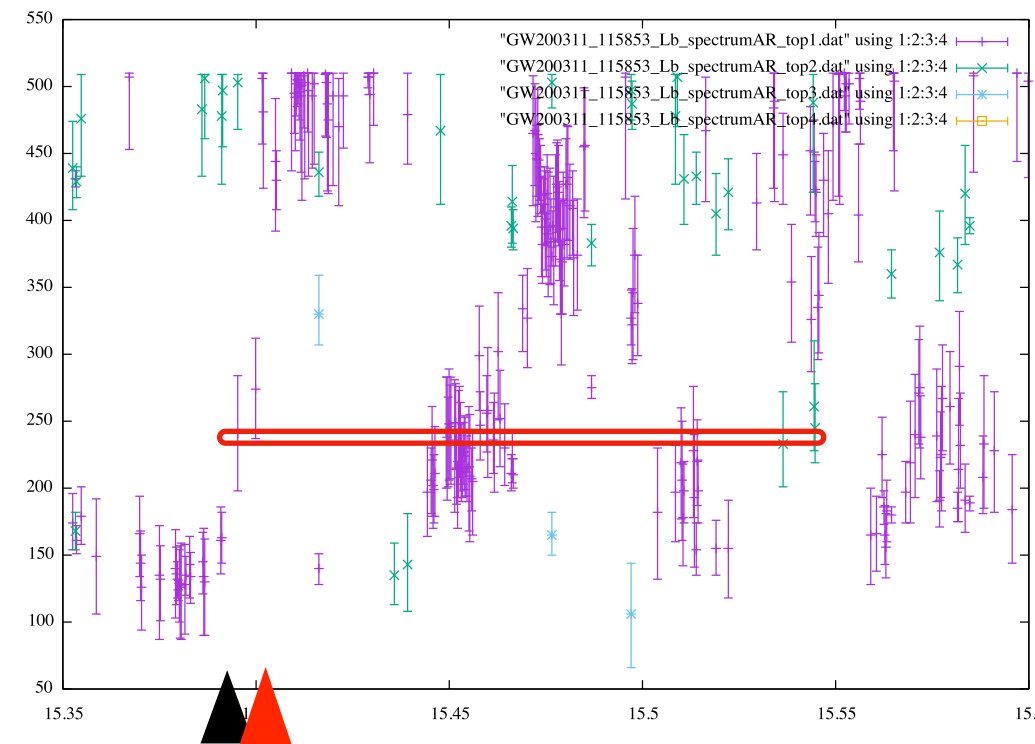
Virgo



Hanford



Livingston

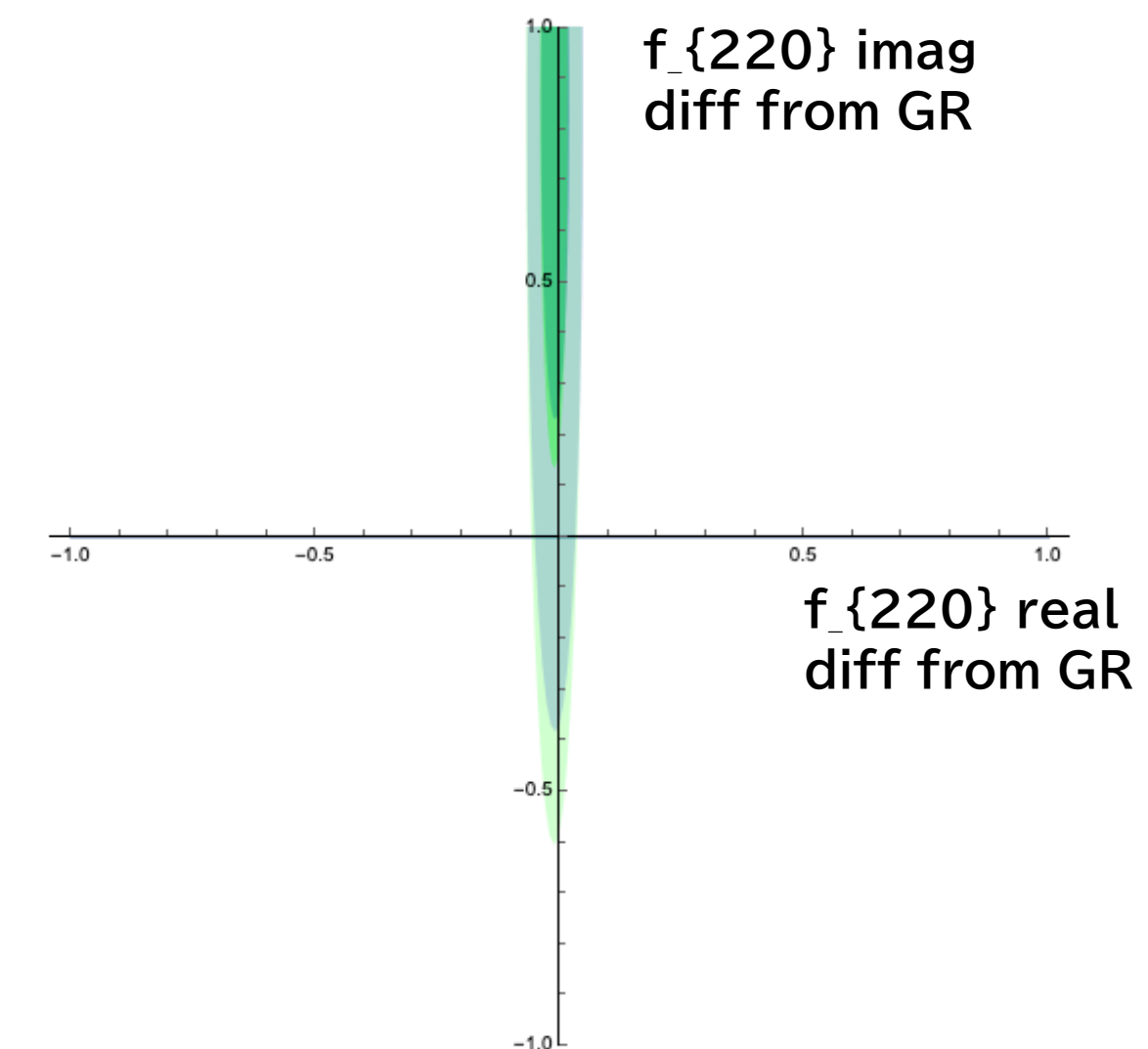
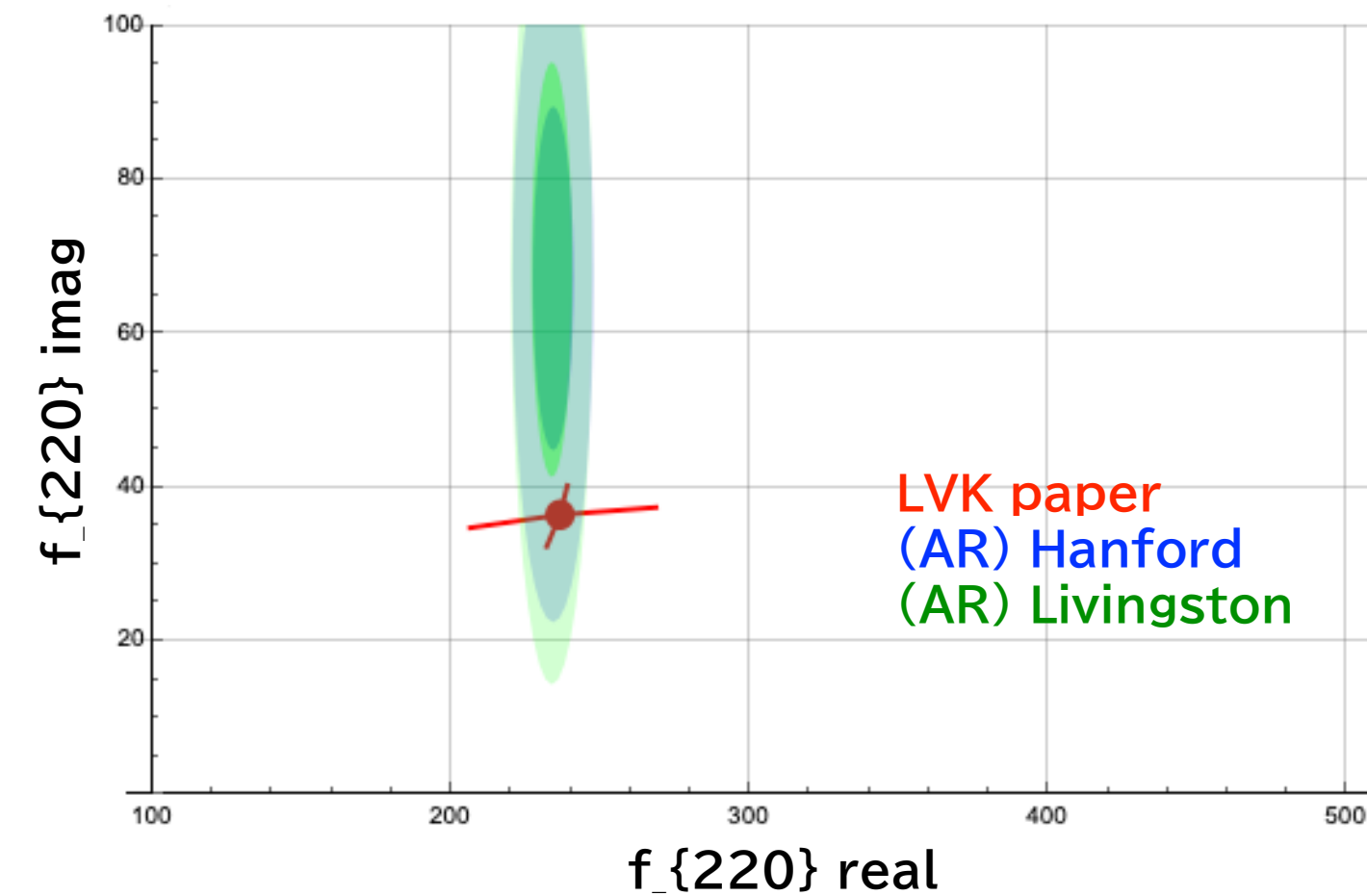


$f_{QNM}$   
@Earth ▶

$f_{220} = 236.3 + i 36.41$  Hz,  $f_{221} = 231.3 + i 110.5$  Hz,  $f_{222} = 221.5 + i 186.9$  Hz  
 $f_{210} = 331.0 + i 59.62$  Hz,  $f_{211} = 196.3 + i 111.4$  Hz,  $f_{200} = 219.8 + i 46.87$  Hz  
 $f_{330} = 374.7 + i 37.35$  Hz,  $f_{331} = 371.6 + i 114.8$  Hz,  $f_{332} = 366.1 + i 206.5$  Hz  
 $f_{320} = 337.3 + i 37.65$  Hz,  $f_{310} = 305.3 + i 50.42$  Hz,  $f_{300} = 278.5 + i 39.2$  Hz

22 segments  
 $f = 234.01 \pm 6.81$  Hz  
 $t = 15.4019 \text{ -- } 15.5437$

28 segments  
 $f = 233.61 \pm 6.82$  Hz  
 $t = 15.3982 \text{ -- } 15.5449$



GW200311  
 delay time (msec) from  $t_0$   
 (+ delay, - advanced)

LHO = 0.48509  
 LLO = 10.1988  
 Virgo = 6.83823  
 KAGRA = -19.049

$t_{\text{merger}} = 15.398$  s  
 $t_{\text{mergerH}} = 15.404$   
 $t_{\text{mergerL}} = 15.408$   
 $t_{\text{mergerV}} = 15.405$

## まとめと展望

### 自己回帰モデル (AR法)

$$\begin{aligned}
 x_n &= a_1 x_{n-1} + a_2 x_{n-2} + \dots + a_M x_{n-M} + \varepsilon \\
 &= \sum_{j=1}^M a_j x_{n-j} + \varepsilon
 \end{aligned}$$

ノイズを含む短時間実データ (64 pts~15ms) に対しても, 周波数と減衰率を抽出できるデータのみを用いて, テンプレートは不要(理論に独立). 複数のモードも抽出可能. 検出器ごとのデータ解析. 波形抽出の時刻も明確にできる.

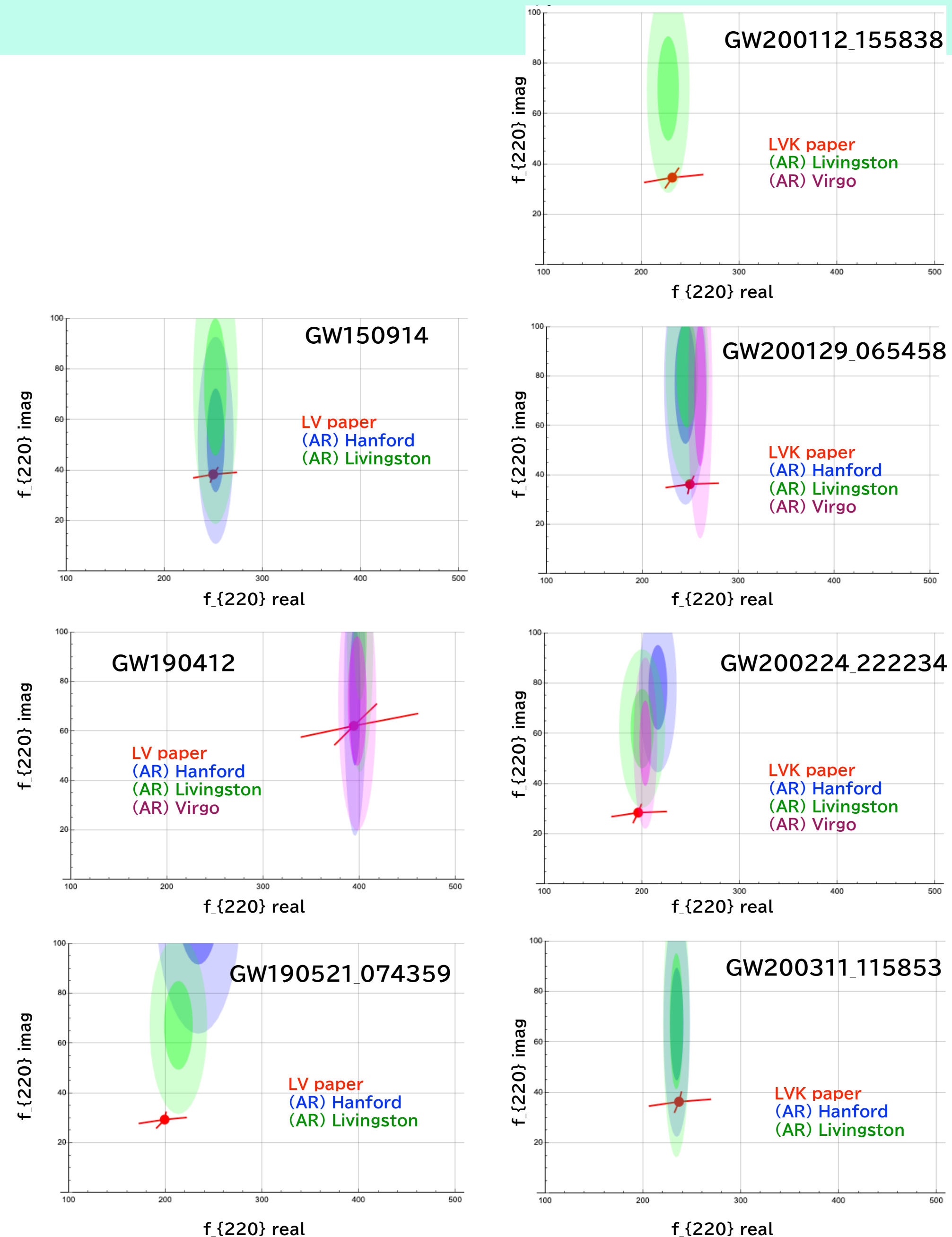
### LV O1/O2/O3a & LVK O3b: リングダウン波形の抽出を試みた.

S/N >= 15 (インスパイラル部含む)のイベントに対しては, リングダウン波形は取り出せる. (時間幅 [merger time + 200 ms], Band Filterings [20-600Hz] など parameterはイベントごとに設定せず共通)

- ➡ AR法はリングダウン波を抽出しているか **Yes**
- ➡ 3干渉計(Hanford, Livingston, Virgo) で合致した値になるか **Yes**
- ➡ LVK catalogと合致した値になるか **Almost Yes**
- ➡ f\_imag (減衰率)の精度は悪い. 大きく見積もる傾向がある.

- ➡ リングダウン波の開始時刻はいつか **3ms-5ms**
- ➡ 高次モード, 高調波モードはみつかるか **Not Yet**
- ➡ GRと矛盾しないか **今のところ consistent といえる**

- ➡ 他の方法との整合性check, パラメータ調整
- ➡ S/N の良いイベント待ち





おまけ

合体後エコーは見えるか？



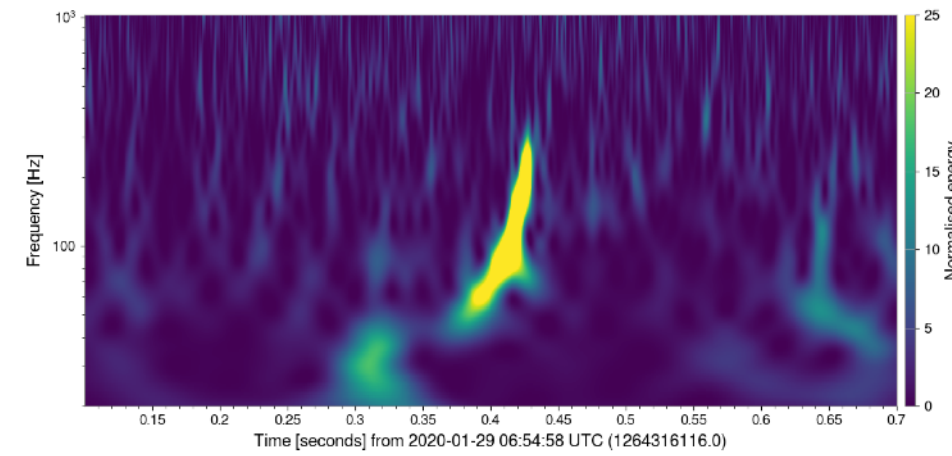
# GW200129\_065458

Network SNR=26.8

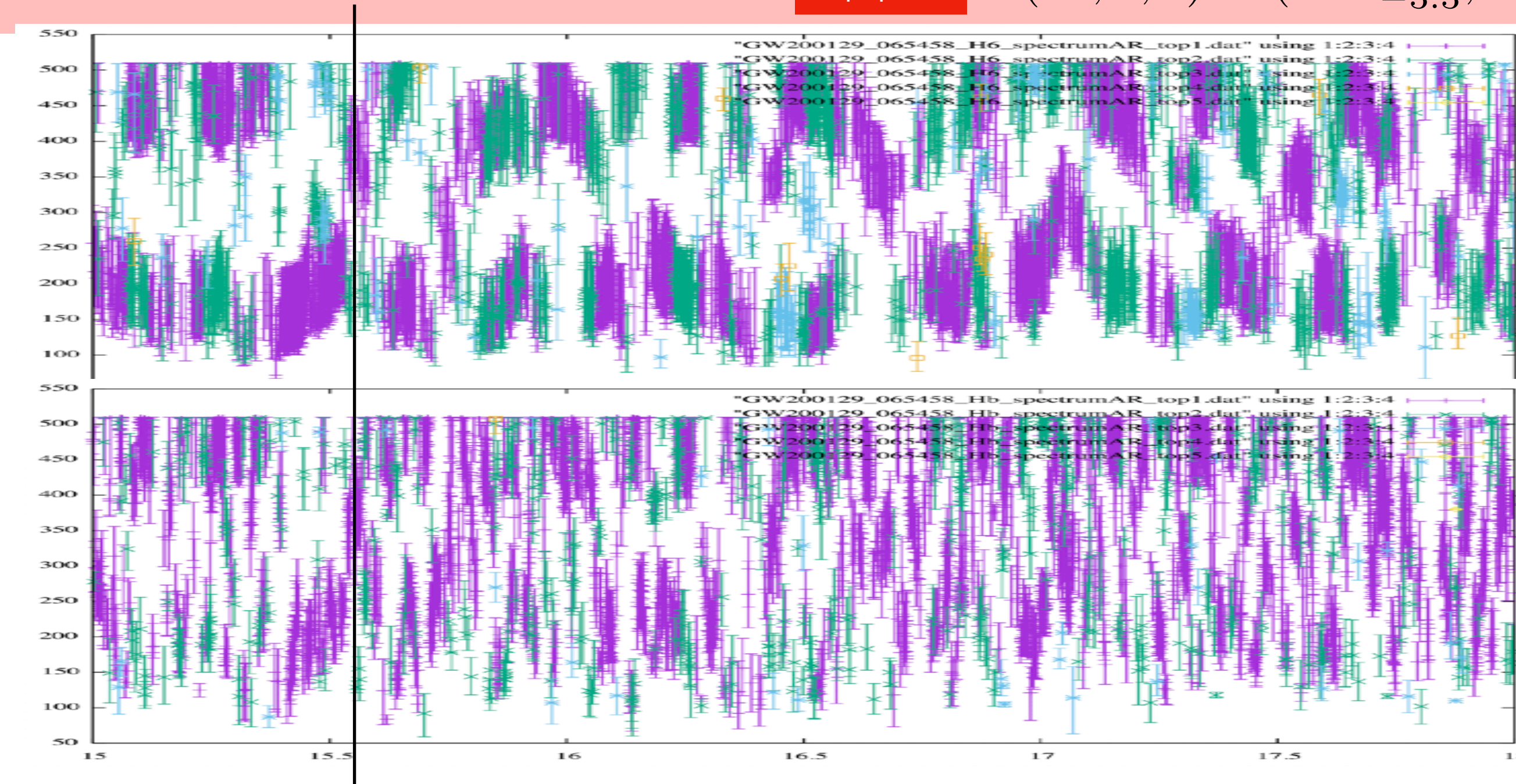
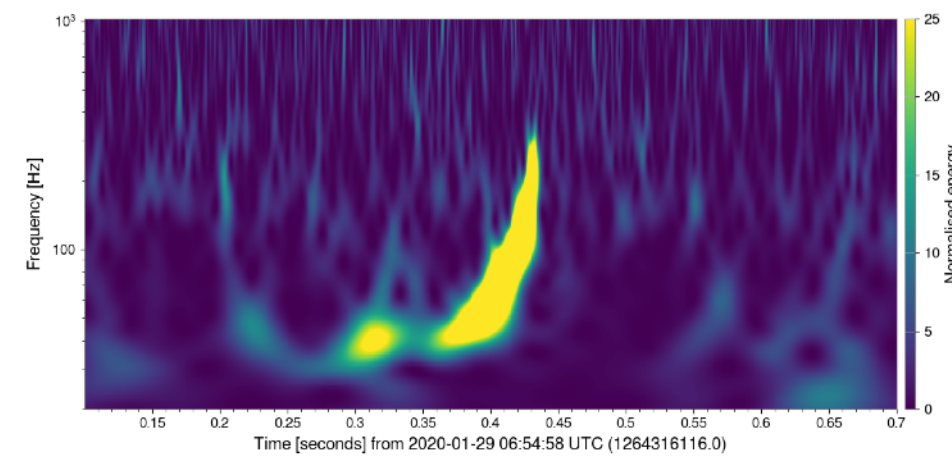
LV paper ▶

$$(M, a, z) = (60.3_{-3.3}^{+4.0}, 0.73_{-0.05}^{+0.06}, 0.18_{-0.07}^{+0.05})$$

Hanford

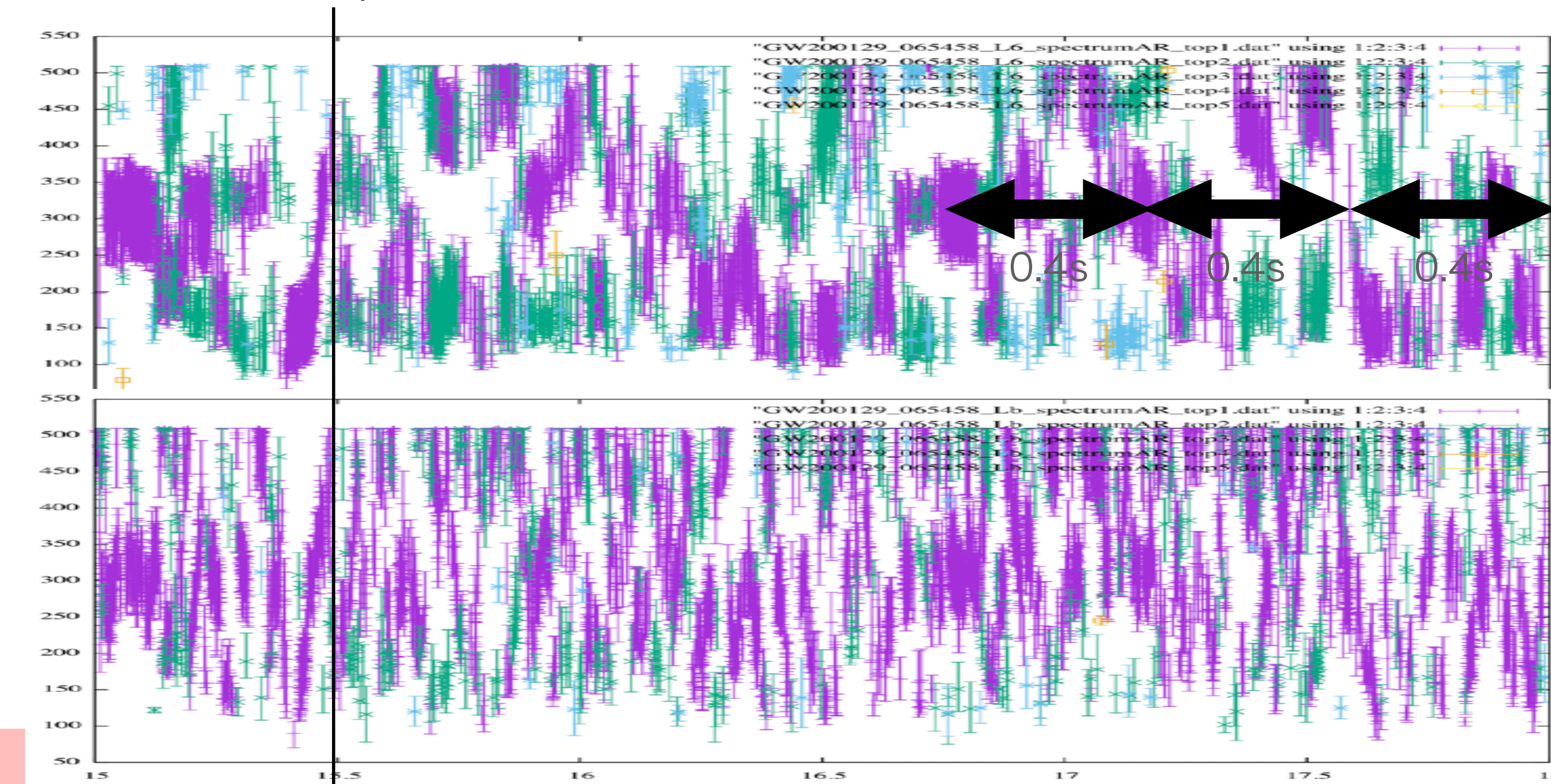


Livingston



1 segment = 256 points  
= 1/16 sec = 62.5 ms

1 segment = 64 points  
= 1/64 sec = 15.625 ms



1 segment = 256 points  
= 1/16 sec = 62.5 ms

1 segment = 64 points  
= 1/64 sec = 15.625 ms