Summary of My Researches

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abstract

In this note, I describe briefly what I have studied. I hope that this note will help you to understand my approach to researches.

Contents

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1 Introduction

I am interested in physics under strong gravity, described by general relativity. The research targets are from black-holes to the early Universe, particularly with its nonlinear natures which are studied by numerical simulations. Most of my works are related to “numerical relativity”, including both actual simulations and fundamental formulations oriented to numerical applications.

My research interests directly link to the current experimental/observational efforts. Several earth-based interferometers designed to detect gravitational waves have been recently constructed. Detectors such as LIGO, VIRGO, GEO and TAMA are expected to begin operating within a few years (TAMA began operating 99). From the theoretical side, we are quite interested in predicting gravitational waveforms and extracting physics from these observations. In order to accomplish this, we need to solve the Einstein equations numerically. This is the reason why numerical relativity is one of the most active fields in general relativity.

My series of work have proposed how stably we can simulate space-time, together with alternative approaches to evolve space-time. I believe my viewpoint is original, and has certain impact to the community.

It is quite certain that researches of general relativity will couple with quantitative analysis in future which requires developments of numerical relativity. My research purpose is to reveal interesting nature of gravity along to this direction.

Research Interests in words

General Relativity and Cosmology

- **Numerical Relativity: Formulations and Simulations**
- **Gravitational Waves, Relativistic Objects**
- **Relativistic Cosmology, Theories of Gravitation**

2 PhD thesis

2.1 Title, etc

NUMERICAL ANALYSIS OF INHOMOGENEOUS SPACETIME STRUCTURE
March 1995, Waseda University, Tokyo, Japan

Supervisor:	Prof. Kei-ichi Maeda (Waseda Univ.)
Refereed by :	Prof. Katsuhiko Sato (Tokyo Univ.),
	Prof. Katsumi Yamada (Waseda Univ.),
	Prof. Tsuneaki Daishido (Waseda Univ.)
	and Prof. Ichiro Ohba (Waseda Univ.)

2.2 Abstract

One of the most important effects in general relativity is the non-linear nature of gravity. For highly dynamical phenomena, we expect that numerical simulations will provide the final answers. The research field of 'numerical relativity' is now actively developing because of the need for templates of gravitational wave forms by observations to be made in the near future. However, numerical relativists are still seeking the perfect formalisms for numerically integrating the Einstein equations and performing simulations under assumptions about the symmetries of spacetime.

In this dissertation, we show the results of our numerical investigations on the dynamics of inhomogeneous space-time. We treat inhomogeneities as one-dimensional problem, by assuming a plane symmetrical spacetime, i.e. all the physical quantities depend only on time and one spatial direction. Though the physical variables are constrained in this spacetime, we can study gravitational waves, which are absent in spherical symmetrical spacetime. Moreover, we can also examine the generality of the cosmic censorship conjecture, since a singularity formed in a plane symmetric spacetime is always naked. In the first half of the thesis, we presented our examination of the inflationary model; its generality and the detailed analysis of the 'topological inflationary model'. We see the competitive effects between the expansion of the background spacetime and the non-linearity of gravity. These calculations were performed using the Arnowitt-Deser-Misner formalism to integrate the Einstein equations. However, we think that Newman-Penrose formalism (characteristic formalism) is also attractive when studying the propagation of gravitational waves. In the latter half of the thesis, we proposed the transformation formula of the Weyl curvature between these two formalisms and applied them to the descriptions of principal null directions and Faraday rotation.

3 Research Interests

General Relativity and Cosmology [related published papers]

General Relativistic Effects
- Gravitational Radiation [7, 19]
- Gravitational Collapse [7]
- Relativistic Stars
  - Neutron Stars [11]
  - Boson Stars [9, 10]
  - Black Holes [18, 19]
  - Wormholes [23]

Cosmology
- Inflationary Universe [2, 3, 5]
- Higher dimensional models [15, 18, 26]

The Einstein equations
- Exact solutions [4]
- Characteristic formulations [4, 19]
- Cannonical formulations [4, 6, 20, 21]
- Hyperbolic formulations [12, 13, 14, 16, 17, 20]
- Asymptotically-constrained systems [17, 20, 21, 24, 26]
- Testbeds for numerical relativity [25]

Theory of gravitation:
- Connection formulation [6, 8, 12, 13, 14, 16, 17]
- Scalar-Tensor theory [7, 9, 10]
- Post-Newtonian approximation [11]

4 Research Summary

4.1 Generality of the inflationary scenario in the early Universe

The Inflationary scenario solves many long-standing fundamental problems in the standard Big-bang model. However, whether inflation starts in anisotropic and/or inhomogeneous space-time and whether the space-time enters inflationary phase in general situation are not clear. By integrating the Einstein equations numerically in a plane-symmetric space-time, I show that even we set highly inhomogeneous scalar field (inflaton) or even if we assume locally strong gravitational wave packets, the space-time evolves into homogeneous de Sitter space-time. These results certify so-called "Cosmic No-hair Conjecture", which states all initially expanding universes with cosmological constants approach de Sitter space-time asymptotically [2, 3]. (The results of [2, 3] are reviewed in P. Anninos, Living Review of Relativity, 1998-2, http://www.livingreviews.org/Articles/Volume1/1998-2anninos, "Physical and Relativistic Numerical Cosmology", by being spent a subsection.)

In 1995, Linde and Vilenkin proposed that topological defects themselves expand rapidly and may become sources of inflation ("topological inflationary model"). I examine their models numerically for the case of planar domain wall and global monopole system and show the parameter ranges they work [5].

4.2 Transformations of variables between 3+1 and 2+2 formulations

The characteristic formulation of general relativity is attractive for treating gravitational waves. However, this formulation is not the best choice for numerical relativity, because it has focusing features. In order to extract and express some proper structure of gravitational field, I derive a transformation formula from the Weyl curvature $C_{\mu\nu\rho\sigma}$ to the Weyl scalar Ψ_i, using projections of Weyl curvature on 2-plane. Since I use a decomposition of the Weyl curvature into electric- and magnetic-parts, the formula has advantages for numerical relativists who work in ADM formulation [4].

One application is the proposal of a new method to present principal null directions pictorially. We can now easily see that the Petrov type of the space-time will damp to a simpler one according to the distance from a source (peeling-off theorem) even for a numerically generated (3+1) data. Another application is a new definition of a maximally polarized direction of propagating gravitational waves. Two modes of gravitational waves, +-mode and ×-mode, are often mixed by non-linear term in Einstein equations, and those effects are called a gravitational 'Faraday effect' from an analogy in the electromagnetism. I applied my definition to a numerical simulation and show some examples of such a rotation of a polarized plane. (Details are in the thesis §5.3).

4.3 Treatment of dynamics in the Ashtekar's connection approach

Motivated by a plan to apply the connection approach formulated by Ashtekar to classical numerical relativity, I show how to treat the constraints and reality conditions in the $SO(3)$-ADM (Ashtekar) version of general relativity. I clarify the difference between the reality conditions on the metric and on the triad from the point of the following the time developments of 3-hypersurfaces. Assuming the reality condition on the triad, we find a new variable, allowing us to solve the gauge constraint equations and the reality conditions simultaneously, which, I think, will play as an alternative variable together with those of Capovilla-Dell-Jacobson's [6].

I examined also whether Ashtekar's formulation of general relativity has an advantage or not when we apply it in numerical relativity, especially on a tractability of degenerate points in dynamics. Assuming that all dynamical variables are finite, we concluded that an essential trick for such a continuous evolution is in complexifying fundamental variables. In order to restrict the complex region locally, we proposed some 'reality recovering' conditions on space-time. We showed that this idea works in an actual dynamical problem [8].

This subject continues in the next section, "hyperbolic formulations and numerical relativity".

4.4 Hyperbolic formulations and Numerical relativity

Re-formulating the Einstein equations into hyperbolic form is extensively studied by several groups in these years. We started using Ashtekar's connection formulation, constructed three levels of hyperbolic forms, and compared their numerical differences systematically. We are trying to feed back these results into the standard ADM systems.

I checked the recent symmetric hyperbolic formulation in the Ashtekar's framework by Iriondo et al (1997), and found that their discussion was insufficient in the points that they did not mention characteristic speed and the consistency with the reality condition. I construct a symmetric hyperbolic system using the similar technique with theirs but use Hermite matrix to characterize the system symmetric [12]. We also construct three levels of hyperbolic systems (weakly, strongly and symmetric) and discuss each gauge requirements and reality constraints [13]. From our numerical comparisons of these hyperbolic systems, we observe the strongly and symmetric hyperbolic system show better stability properties, but not so much difference between the latter two. Rather, we find that the symmetric hyperbolic system is not always the best for controlling stability. [16].

In order to make the evolution equations more robust against the violations of the constraint equations and reality conditions, we proposed a new system based on Ashtekar's variables. By extending the space and introducing an artificial dissipative forces, the system has its attractor in the constraint and real-valued surfaces. The obtained systems may be useful for future numerical studies using Ashtekar's variables [14]. Actually we performed numerical simulations and showed that this system works as expected [17]. There we also proposed an alternative mechanism to obtain asymptotically constrained system, which will be explained in the next section.

4.5 Asymptotically constrained systems and Numerical relativity

In order to perform long-term stable numerical integration of the Einstein equations, we studied the direction of hyperbolic formulation as I described above. Then we found that the mathematical notion of the hyperbolicity may not be enough to discuss the Einstein equations since it ignores the contribution of the non-principal parts.

We proceeded an idea of constructing an "asymptotically constrained" system, without a notion of hyperbolicity. By analyzing the procedure of adjusting constraints to the dynamical equations and their constraint propagation equations, we conjectured that eigenvalue analysis of the homogenized constraint propagation equations indicates asymptotically constrained feature. The idea certainly worked and numerically confirmed for the Maxwell system and the Ashtekar formulation [17]. We also show that this idea is applicable to the ADM evolution equations by adjusting them with constraints [20]. We predicted several possibilities based on the ADM formulation for the Schwarzschild space-time [21]. The generalization for higher dimensional space-time is also presented [26].

We further applied the same idea to the so-called BSSN (Baumgarte-Shapiro-Shibata-Nakamura) version of ADM formulation, and succeeded to show what part of the modifications contributes to give us stable evolution system, and proposed similar systems which are expected to be better than the current set of equations [22].

The series of above analysis provided us to analyze the stability of reformulations of the constraint dynamics systematically. However, the conditions for stability should be supported by mathematical proves. In [24], we classified asymptotical behaviors of constraint-violation into three (asymptotically constrained, asymptotically bounded, and diverge), and gave their necessary and sufficient conditions. We find that degeneracy of eigenvalues sometimes leads constraint evolution to diverge (even if its real-part is not positive), and conclude that it is quite useful to check the diagonalizability of constraint propagation matrix.

4.6 Gravitational waves in Brans-Dicke theory of gravity

A direct observation of gravitational waves can be used as a tool of testing theory of gravity. We studied gravitational radiation in Brans-Dicke theory of gravity, which is an alternative theory to general relativity, and compared with those in general relativity. Analyzing test particles falling into a Kerr black hole, we estimated the waveform and emitted energy of both scalar and tensor modes of gravitational radiation. By applying these results in non-spherical dust collapse, we found that the scalar modes dominate only for highly spherical collapse, and clarified in which parameter ranges we can expect to observe the scalar modes of gravitational waves [7].

4.7 Boson stars in scalar-tensor theories

In order to show some interesting features of scalar-tensor theory, I began studying a structure of boson stars. Boson stars are formed by complex scalar field. If we construct them in scalar-tensor theory of gravity in conformally transformed Einstein frame, then we just need to add one more scalar field in energy-momentum tensor.

I show a sequence of equilibrium configuration of boson star in both Brans-Dicke theory and Damour-Nordtvedt's quadratic coupling model. I found that if we pose an observational constraint for the theories, then the solutions are identical with those of general relativity. Applying a catastrophe theory, I also discuss their stability. What is interesting, I found that there are no stable sequence of boson stars in the earlier cosmological era if we work in quadratic coupling model [9].

By integrating the dynamical equations of this system numerically, I checked the above stability discussion in Brans-Dicke case. I also compared the scalar emissions from this system with GR [10].

The debates starting from [9] are summarized in A.W.Whinnett, Phys. Rev. D. 61 (2000) 124014, concluding that the results in [9] are confirmed.

4.8 Post-Newtonian treatment in Numerical Relativity

One of the most important problem in simulating coalescing binary neutron star is to prepare physically satisfactory initial data. Towards this problem in fully general relativistic (GR) hydrodynamical simulations, I am working to apply the post-Newtonian (PN) approach to construct initial data.

As a preliminary step, I showed that the discontinuous matching surface of the PN and GR in the vacuum region will be smoothed out in fully relativistic evolution in a particular slicing condition.

I also tested a post-Newtonian approach by constructing a single neutron star model. I show how the PN approximation converges together with mass-radius relations for several polytropic equation of state. By solving the Hamiltonian constraint equation with these density profiles as trial functions, I examine the differences in the final metric. We conclude the second 'post-Newtonian' approximation is close enough to describe general relativistic single star[11].

4.9 Higher dimensional cosmology

According to the recent development of string theories and related studies, our 4-dimensional space-time was reduced from higher dimensional space-time via a sort of compactifications. Such dimensional reductions are not yet well understood, and we do not have fundamental consensus in hand. However, we studied the following two topics based on the latest plausible scenario.

- Stability of 5-dimensional Kaluza-Klein bubble space-time [15].
  The stability of 4-dimensional space-time can be judged by the positiveness of its total energy. However, Brill-Horowitz's solution, which was found as a momentarily static initial data in 5-dimensional Kaluza-Klein space-time, allows arbitrary negative total mass, and that negative mass solution is conjectured as a stable (expanding) solution by Corley-Jacobson. We show numerically that such a negative energy bubble will collapse and form a naked singularity, while the evolution start expanding.

  In our analysis, we simply use 5-dimensional Einstein equations. If we apply the recent 'brane-world' scenario, the fundamental equations are not the conventional Einstein's ones therefore our results might not directly proceed understanding of the higher dimensional cosmology. We, however, are quite interested in the fact that the stability of space-time can be judged simply by the positiveness of its total energy even in the higher dimensional space-time.

- The 5th dimensional structure of the black holes on the brane [18].
  We study charged brane-world black holes in the model of Randall and Sundrum in which our universe is viewed as a domain wall in asymptotically anti-de Sitter space. Such black holes can carry two types of "charge", one arising from the bulk Weyl tensor and one from a gauge field trapped on the wall. We use a combination of analytical and numerical techniques to study how these black holes behave in the bulk. It has been shown that a Reissner-Nordstrom geometry is induced on the wall when only Weyl charge is present. However, we show that such solutions exhibit pathological features in the bulk. For more general charged black holes find that the extent of the horizon in the fifth dimension is less than for an uncharged black hole that has the same horizon radius on the wall.

4.10 Dual-null formulation for describing space-time geometry

Evolving space-time with dual-null $(2+2)$ formulation has many advantages than the traditional space and time decomposition $(3+1$ formulation), but still a challenging project.

- Proposal of the new approximation of space-time: Quasi-spherical approach [19]
  We proposed a new approximation scheme in a dual-null decomposition of space-time, and report its numerical test on its validity. This proposal is with the aim of providing a computationally inexpensive estimate of the gravitational waveforms produced by a black-hole or neutron-star collision, given a full numerical simulation up to (or close to) coalescence, or an analytical model thereof. The scheme truncates the Einstein equations by removing second-order terms which would vanish in a spherically symmetric space-time. We numerically implemented this scheme, testing it against angular momentum by applying it to Kerr black holes. As error measures, we take the conformal strain and specific energy due to spurious gravitational radiation. The strain is found to be monotonic rather than wavelike. The specific energy is found to be at least an order of magnitude smaller than the 1% level expected from typical black-hole collisions, for angular momentum up to at least 70% of the maximum, for an initial surface as close as $r = 3m$.

- Dynamical properties of traversible wormholes [23]
  Wormholes are known as a kind of solution of the Einstein equations, and have become a popular research topic, raising theoretical possibilities of rapid interstellar travel, time machines and warp drives. These topics sound like science fiction, but after an influential study of traversible wormholes by Morris & Thorne (1988), it became widely accepted as a scientific topic.
  We study numerically the stability of Morris & Thorne's traversible wormhole. We observe that the wormhole throat is unstable against Gaussian pulses in either exotic or normal massless Klein-Gordon fields. The wormhole throat suffers a bifurcation of horizons and either explodes to form an inflationary universe or collapses to a black hole, if the total input energy is respectively negative or positive. For normal matter, such as a traveller traversing the wormhole, collapse to a black hole always results. However, additional ghost radiation can maintain the wormhole for a limited time. The black-hole formation from a traversible wormhole confirms the recently proposed duality between them.

4.11 Standard testbeds for numerical relativity

In recent years, many different numerical evolution schemes for Einstein's equations have been proposed to address stability and accuracy problems that have plagued the numerical relativity community for decades. Some of these approaches have been tested on different spacetimes, and conclusions have been drawn based on these tests. We propose to build up a suite of standardized testbeds for comparing approaches to the numerical evolution of Einstein's equations that are designed to both probe their strengths and weaknesses and to separate out different effects, and their causes, seen in the results [25].

4.12 Simulation of Ising models using a special-purpose computer

This study was done when I visited the Delft University in the Netherlands as an exchanging student of IAESTE (International Association of Exchanging Student for Technical Experiences) one summer.

We investigate the phenomenon of magnetization bistability in a two- dimensional Ising model with a non-Hamiltonian Glauber dynamics by means of Monte Carlo simulations. This effect has previously been observed in the Toom model, which supports two stable phases with different magnetization, even in the presence of a nonzero field. We find that such bistability is also present in an Ising model in which the transition probabilities are expressed in terms of Boltzmann factors depending only on the nearest-neighbor spins and the associated bond strengths. The strength on each bond assumes different values with respect to the spins at either of its ends, introducing an asymmetry like that of the Toom model [1].

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