2012 年度前期　確率・統計（2 年）期末試験　　2012 年 8 月 2 日実施　真貝

【Important】Write your answers to the answer-sheet. Write not only the solutions but also the process how you obtain solutions. The answering order is free.

Use the table of the standard normal distribution when you need.

Use the equation $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$, if needed.

For passing the test with "C", solve 1, 2, 3, and 5.

The judgement "A" or "B" is by all the problems including 4.

1. Probability (20 点)

(1) Mr A and B toss two coins, and decide the winner who gets "Head-Head" or "Tail-Tail" combination first. The turn starts from Mr A, and rounds until whoever wins. Show the winning probabilities P_A and P_B for Mr A and B.

(2) Mr C received a report of his pre-test for the entrance exams of universities, in which 5 universities are listed with passing probability 20%. If he actually tried these 5 universities' entrance exams, what is the possibility for him to pass at least one exam. (Consider as a problem of probabilities.)

2. There is one student who came from the ancient era using a time-machine in a campus of 1000 students. We have an "ancient-man detector", but its judgement has always 5% error. That is, a present-aged man can be judged as an ancient-man in 5%, likewise an ancient-aged man can be judged as a present-man in 5%. Show the probabilities below. (15 点)

(1) Probability of the judgement of "ancient-man", when we check a person in the campus.

(2) When the detector judges she is from the ancient, probability that the answer is true.

3. Regarding the normal distribution (ND) and the standard normal distribution (SND). (30 点)

(1) Let x the random variable of ND, μ the average (expectation value) and σ^2 the variance. Show the probability distribution function of ND, $f(x)$.

(2) Show the graph $y = f(x)$, and explain the meaning of σ using the graph.

(3) Let z the random variable of SND. Show the transformation from x to z.

(4) Show the average of x is μ by integrating $f(x)$.

(5) Results of a test was expressed by a ND with the average 60, and the standard deviation 12. A professor judges "A/B/C" by dividing number of students one-third simply. Fill in the blanks.

judges	test points	standard deviation	number of students
A	more than [ア]	more than [A]	33%
B	more than [イ], less than [ア]	more than [B], less than [A]	33%
C	less than [イ]	less than [B]	33%

4. Answer two of the below three problems. (20 点)

(1) When you throw a dice 500 times, what is the probability for obtaining ⚄ more than 50 and less than 100 times?

(2) In a country of population 100 million, a newspaper company collects interviews from 1000 people whether they support the government. The result is 40%. What is the supporting rate in population? Answer with the confidence level 99%.

(3) Mr A and B try Japanese chess 50 times, and A won 40. Can we say A is stronger than B? Tests with significance level 5%.

5. Write your original problem for solving a conditional probability and calculating an expected value with your answer. (15 点)