一般相対論の数値計算手法

近畿大セミナー
2011/12/9-10

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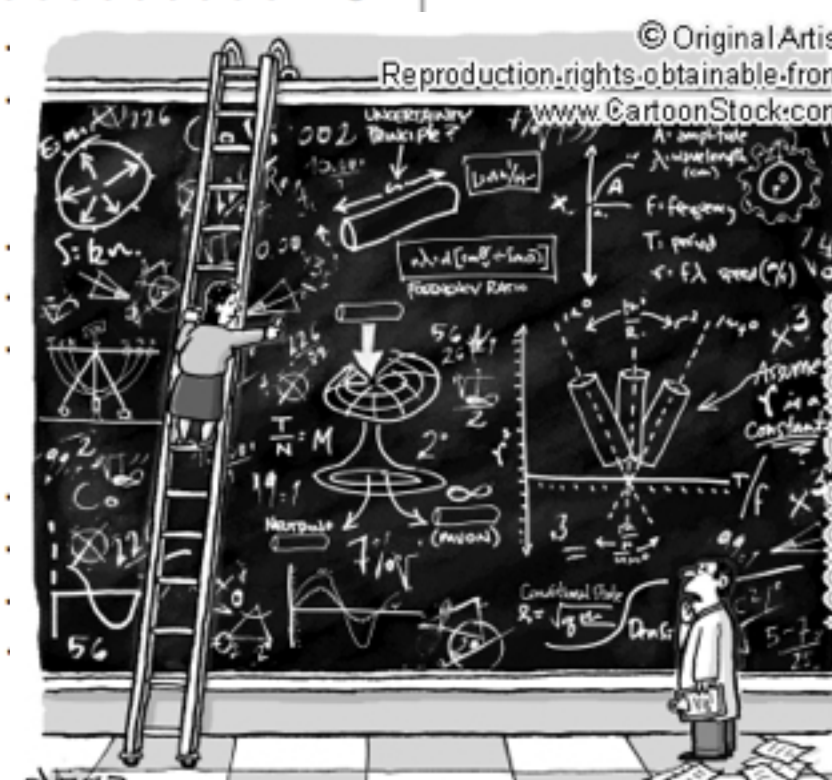
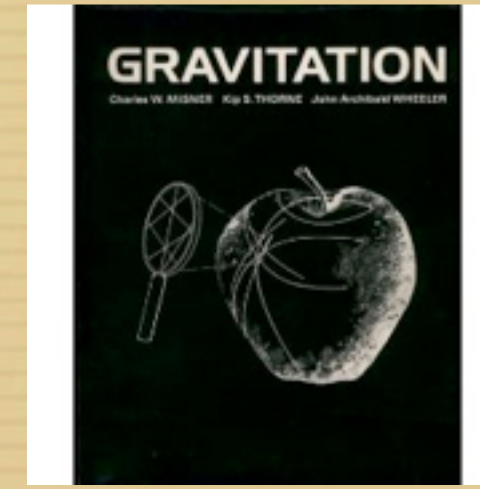
Contents

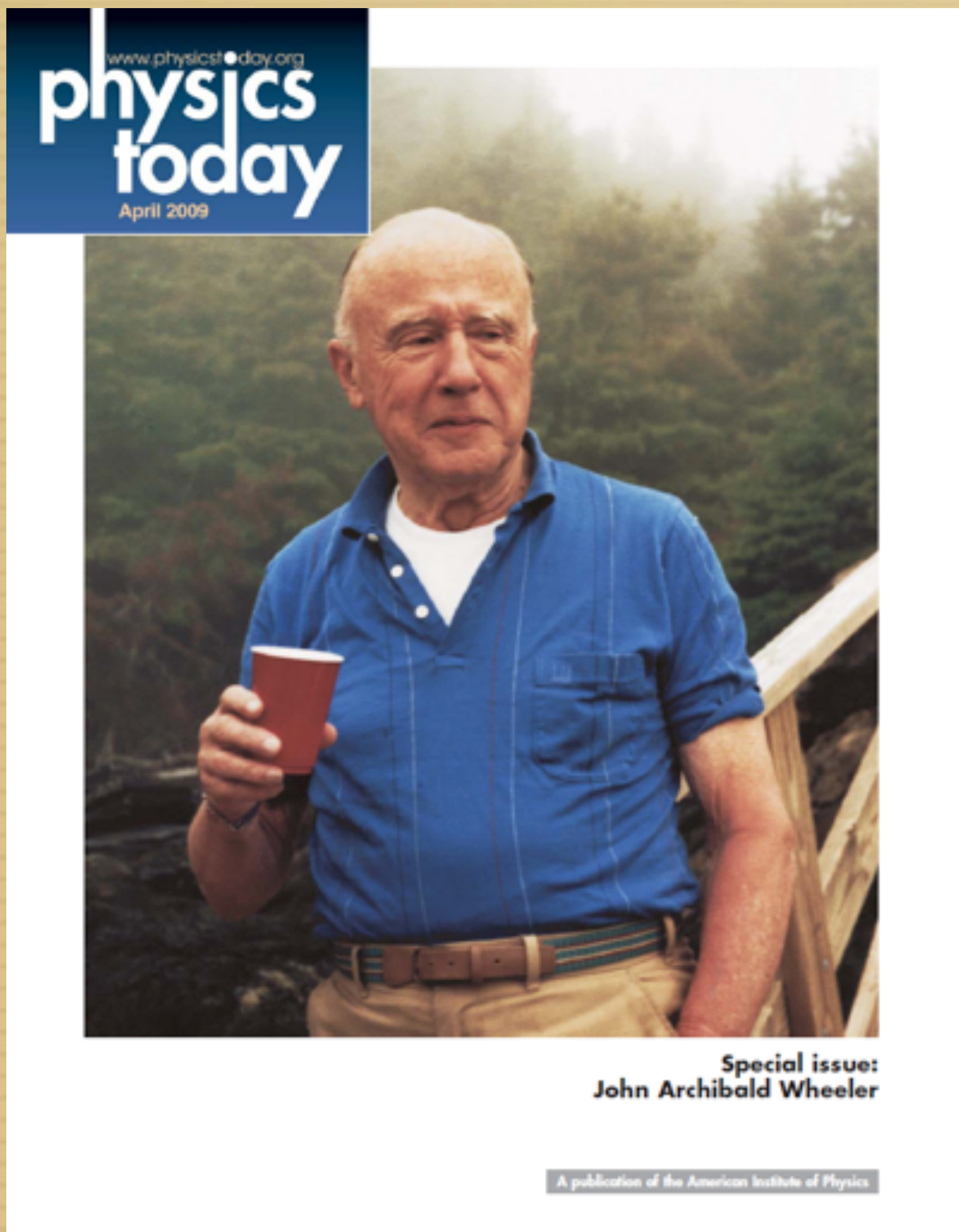


"If your calculations are correct, this completely revolutionizes everything we thought we knew about why chalk squeaks on a blackboard!"

John A. Wheeler

(July 9, 1911 – April 13, 2008)





Physics Today, 2009-4

Wheelerの育てた人材

Richard Feynman (PhD 1942)
Hugh Everett (PhD 1956)
Charles Misner (PhD 1957)
David Sharp (AB 1960)
Richard Lindquist (PhD 1962)
Kip Thorne (PhD 1965)
Robert Geroch (PhD 1967)
Yavuz Nutku (PhD 1969)
Wojciech Zurek (PhD 1979)
William Unruh (PhD 1971)
Demetrios Christodoulou (PhD 1971)
Robert Wald (PhD 1972)
Jacob Bekenstein (PhD 1972)
Warner A. Miller (PhD 1986)

....

Mentoring at Princeton University 1938–78*

Professor	PhD theses supervised	PhDs per year	Extra acknowledgments†	Senior theses supervised
John Wheeler	46	1.22	19	46
Thomas Carver	16	0.76	13	21
Robert Dicke	25	0.81	8	11
Val Fitch	15	0.71	5	5
Marvin Goldberger	19	0.95	10	4
Rubby Sherr	14	0.45	17	11
Sam Trieman	24	1.04	16	4
Arthur Wightman	24	0.93	14	11
Eugene Wigner	25	0.83	16	0

* Physics PhD and senior theses supervised during 1938–78 at Princeton by the nine professors who supervised the most doctoral theses during that period.
† Acknowledgments in PhD theses thanking a professor other than the adviser of record.

60-2 April 2009 Physics Today 55

Physics Today, 2009-4

Wheelerの創った言葉

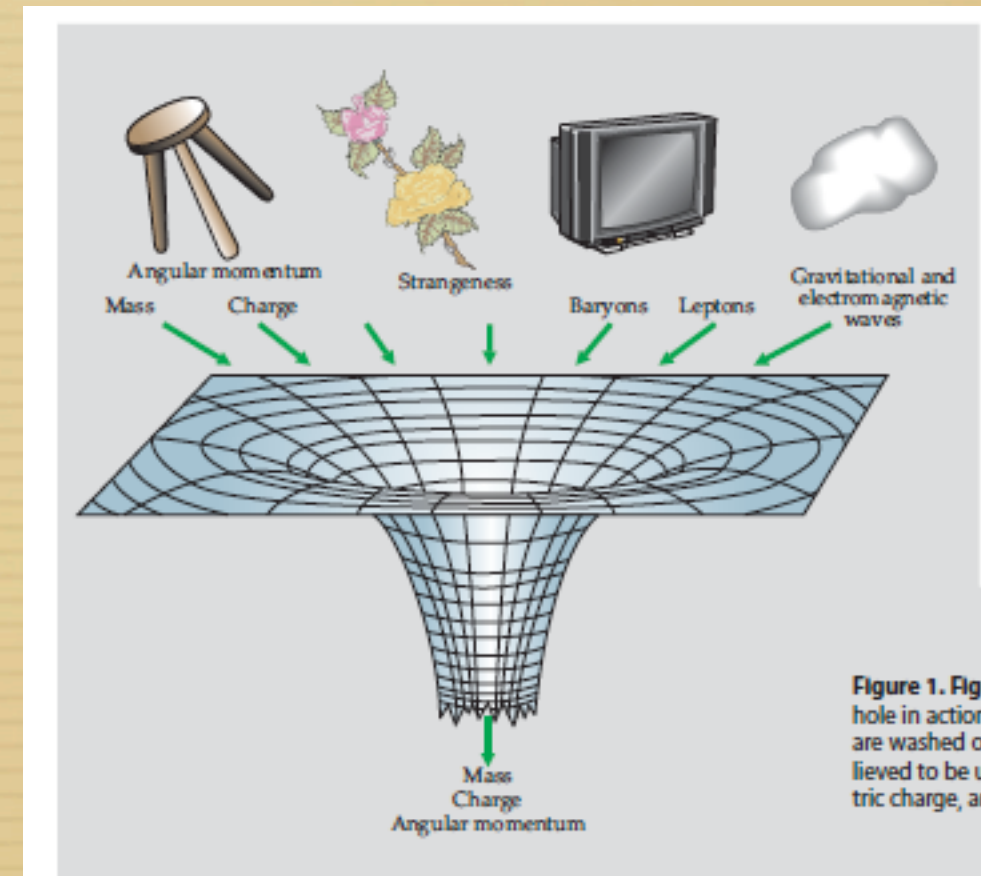
Box 1. Wheeler coinages

John Wheeler believed that the names given to concepts or to descriptions of an idea strongly influence how we think about concepts and ideas, even how we work on them and build on them. In short, the word inspires the deed. Accordingly, Wheeler spent many hours (often soaking in a warm bathtub) searching for the most apt terms. Here, in rough chronological order, are some of his coinages:

***S*-Matrix** the scattering operator in quantum mechanics
Sum over histories Richard Feynman's path-integral method
Moderator the material that slows neutrons in a nuclear reactor
Stellarator a plasma magnetic confinement device
Planck length, Planck time the scales at which quantum gravity dominates
Geon an object made from waves bound together by their energy's gravity
Mass without mass gravitating object containing no massive particles
Charge without charge wormholes as sources and sinks of electric field lines
Wormhole a topological "handle" in the geometry of curved space
Quantum foam quantum fluctuations in the geometry of spacetime
Black hole* the object formed by implosion of a sufficiently massive star
A black hole has no hair a classical black hole's properties are determined by only its mass, spin angular momentum, and charge
Space tells matter how to move and matter tells space how to curve the summarized content of general relativity
Law without law** emergence of law from random processes
It from bit** a physical world built of information units
Mutability** susceptibility of physical law to evolution and change
Observer-participancy** influence of the observer on reality
The universe as a self-excited circuit** shaping the past from the present
A single quantum cannot be cloned a theorem that puts a limit on quantum amplifiers

* The phrase "black hole" appears to have been used first, for the object formed by stellar implosion, by one or more non-physicists shortly after the 1963 discovery of quasars, but it did not stick. Wheeler recalls adopting it in 1968 after somebody at a lecture he was giving shouted it out as a suggestion, and in his hands it was quickly adopted worldwide.
**An influential, speculative idea due to Wheeler.

S-Matrix,
Sum over histories,
Planck length,
Planck time,
Wormhole,
Black Hole,
Geon,
Quantum foam,
A BH has no hair,
law without law,
it from bit, ...



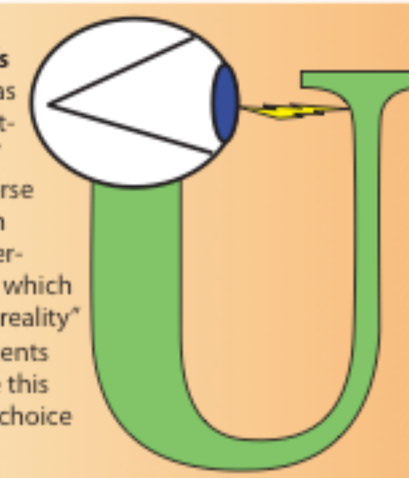


Figure 3. John Wheeler's diagram of the universe as a self-excited circuit: Starting small (thin part of "U" at upper right), the universe grows (loop of "U") and in time gives rise to observer-participancy (upper left), which in turn imparts "tangible reality" to even the earliest moments of the universe. Compare this notion with the delayed-choice experiment of figure 2.

Physics Today, 2009-4

５次元時空における特異点形成

真貝 寿明 （大阪工業大学）
work with 山田 祐太(D２)

Initial Data (Spheroid, Ring)
Yamada & HS, CQG 27 (2010) 045012
Evolution (Spheroid)
Yamada & HS, PRD 83 (2011) 064006
Evolution (Ring)
Yamada & HS, in preparation.

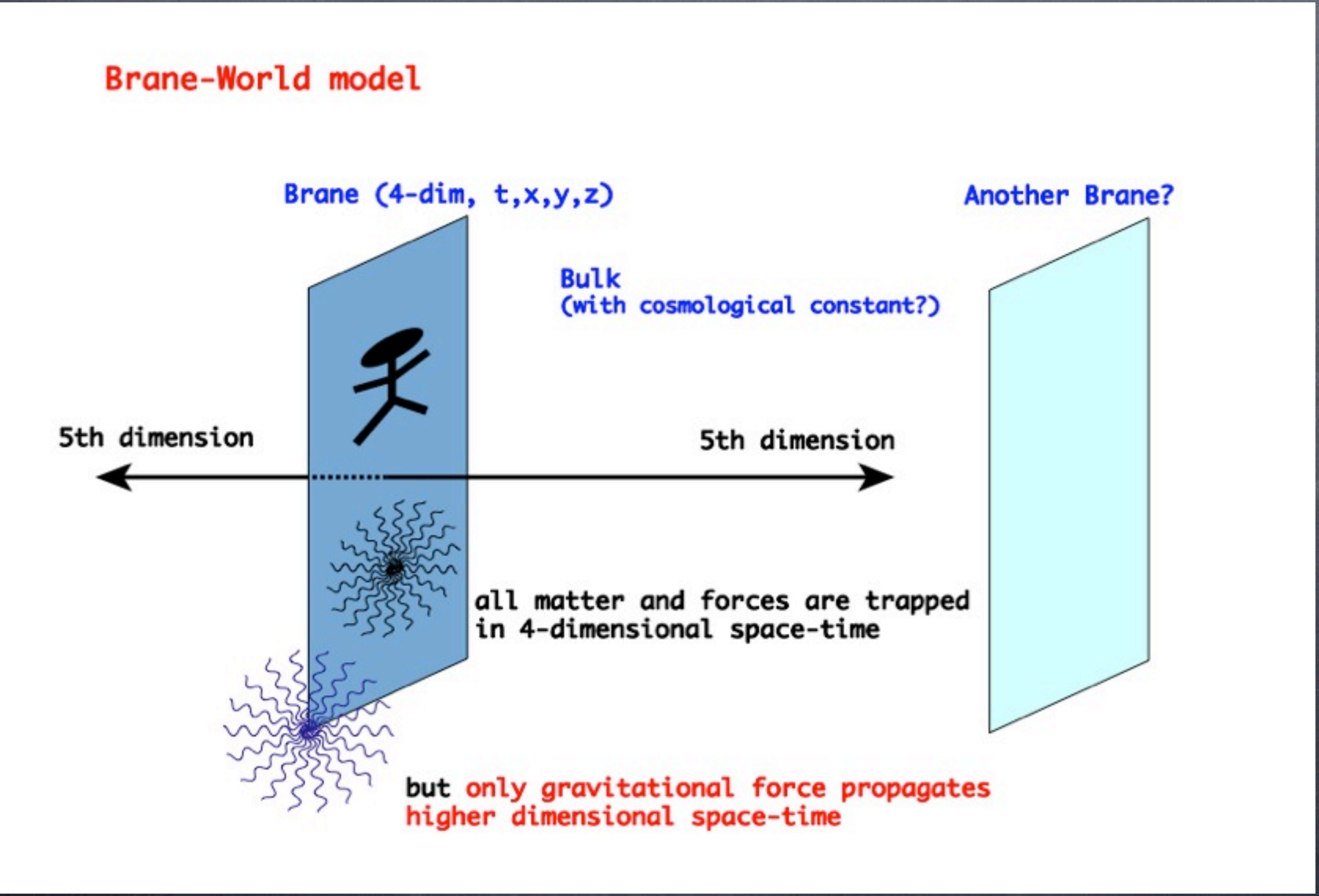
http://www.is.oit.ac.jp/~shinkai/

at 近畿大学セミナー　　2011/12/10

ケーススタディX

1999〜

ブレーンワールド



1. Motivation and Goal

Higher-Dim Black Holes have Rich Structures

Brane-World models give new viewpoints to gravity and cosmology

LHC experiments will (or will not) reveal Higher-Dim BHs in near future

4-dim BH :   horizon is S^2,

stable solutions

Schwarzschild --- Birkoff theorem (M)

Kerr --- uniqueness theorem (M, J)

1. Motivation and Goal

Higher-Dim Black Holes have Rich Structures

4-dim BHs		Higher-dim BHs :
Schwarzschild	→	Tangherlini
		--- unique & stable
Kerr	→	Myers-Perry
		--- maybe unstable in higher J
"Black Objects"		black string black ring (Emparan-Reall) black Saturn di-rings, orthogonal di-rings, ...

1. Motivation and Goal

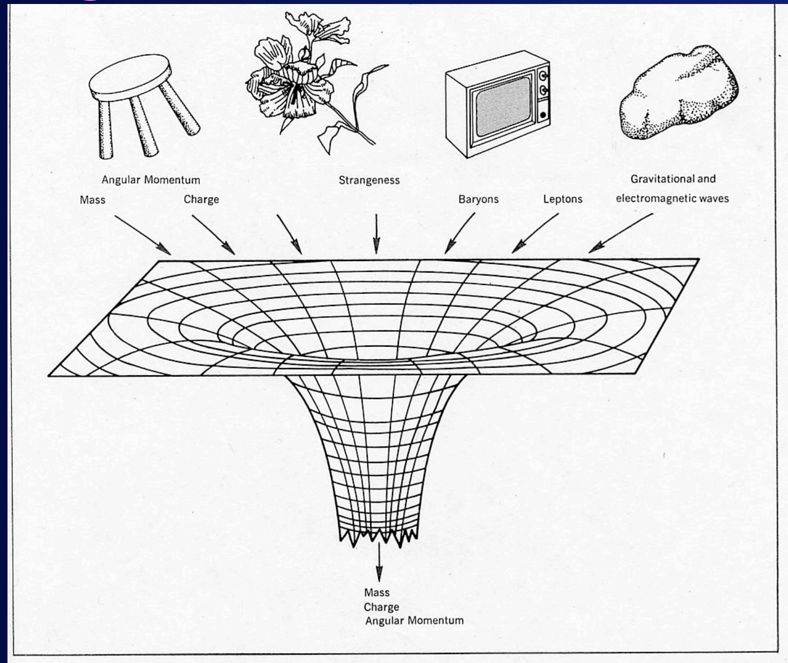
Higher-Dim Black Holes have Rich Structures

"Black Objects"

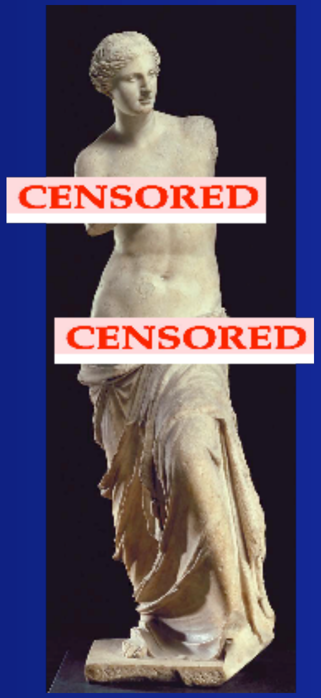
black hole
black string
black ring
black Saturn
di-rings, orthogonal di-rings ...

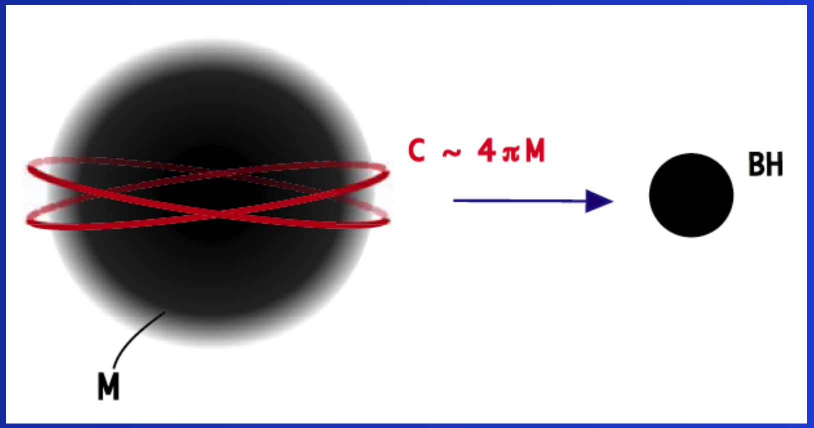
Uniqueness (only in spherical sym.)
Stability?
Formation Process?
Dynamical Features? ...

No Hair Conjecture?
Cosmic Censorship?
Hoop Conjecture?



Figurative representation of a black hole in action. All details of the infalling matter are washed out. The final configuration is believed to be uniquely determined by mass, electric charge, and angular momentum. Figure 1





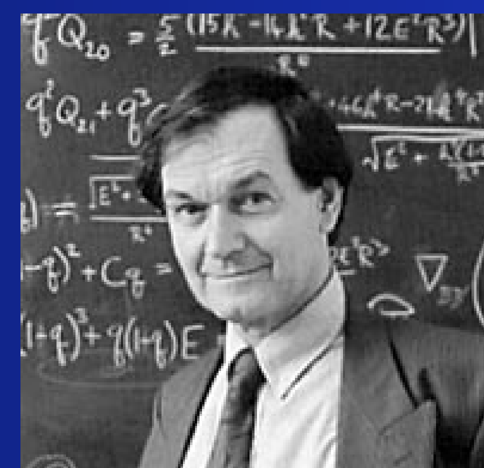
裸の特異点 と 宇宙検閲官仮説

naked singularity vs cosmic censorship conjecture

弱い宇宙検閲仮説　R. Penrose (1969)

「漸近的に平坦な時空で，物理的に適当な初期条件から出発し，物理的に適当な物質および輻射の重力崩壊によって発生するすべての特異点は，ブラックホールの中に隠され，遠方の観測者はそれを見ることができない．」

「裸の特異点は，見えてはならない」



強い宇宙検閲仮説　R. Penrose (1979)

「物理的にもっともらしいすべての時空には，初期特異点以外に観測可能な特異点は存在しない．」

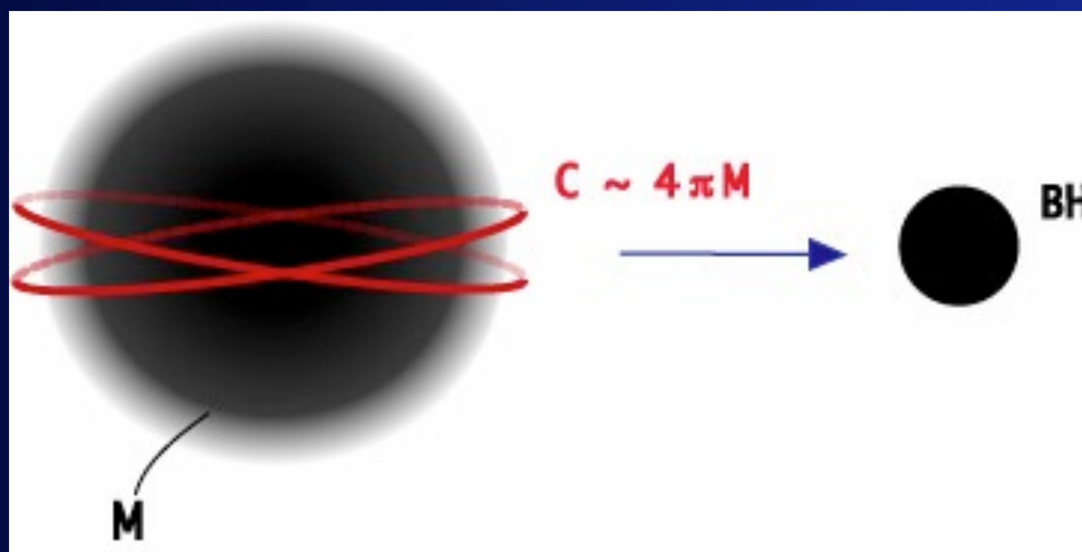
「特異点は，遠方の観測者のみならず，ブラックホールに落ちた観測者からも，見えてはならない」

「裸の特異点は，存在しない」

フープ仮説

hoop conjecture

K. Thorne (1972)



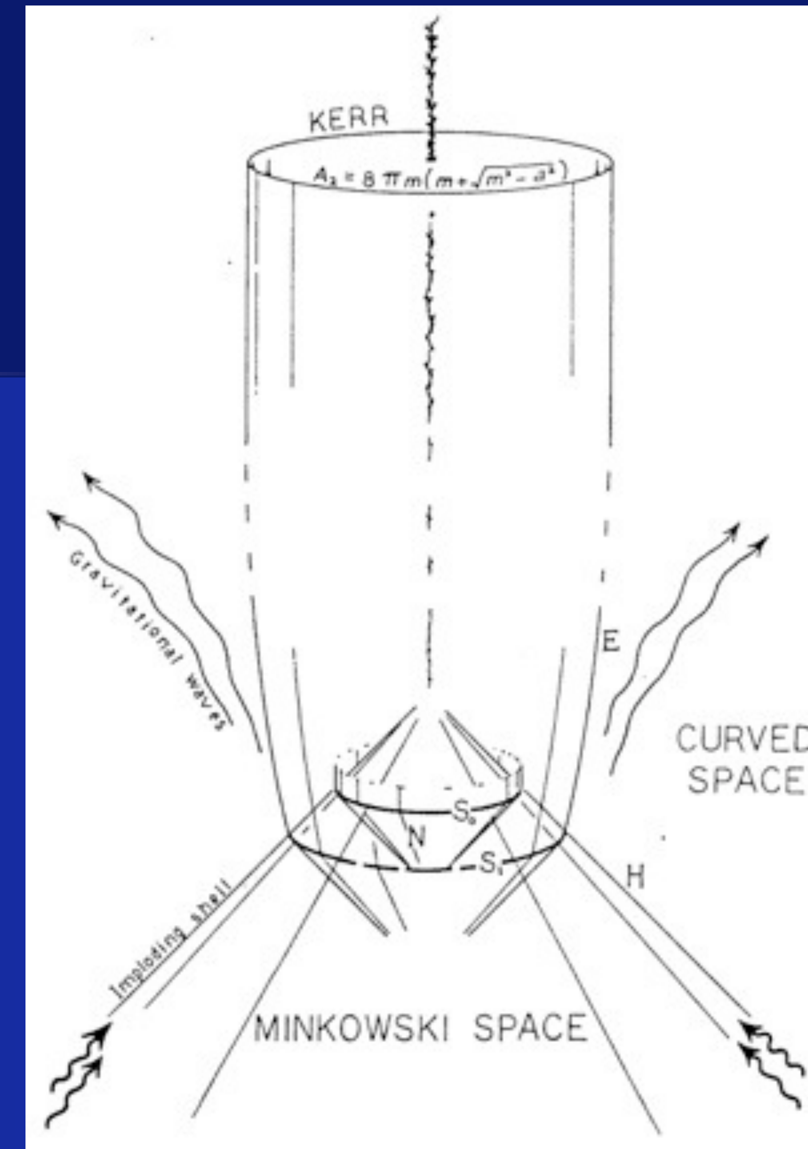


FIGURE 3. An attempt to derive a contradiction with the "establishment" viewpoint. A shell of photons collapses into a flat space-time. The assumption that a K- black hole results implies the existence of a certain inequality on the initial geor

R. Penrose (1969)

Penrose inequality

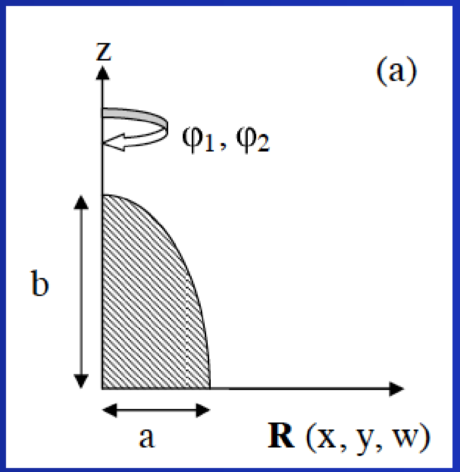
$$A \leq 16\pi m^2$$

Plan of the talk

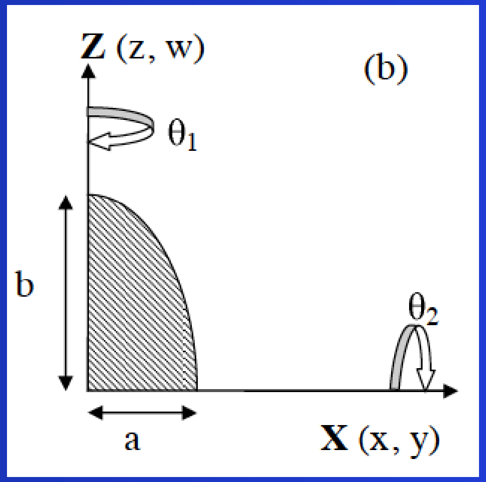
2. Numerical method

3. Spheroidal matter collapse
   Initial data analysis
   Evolutions

S^3 horizon



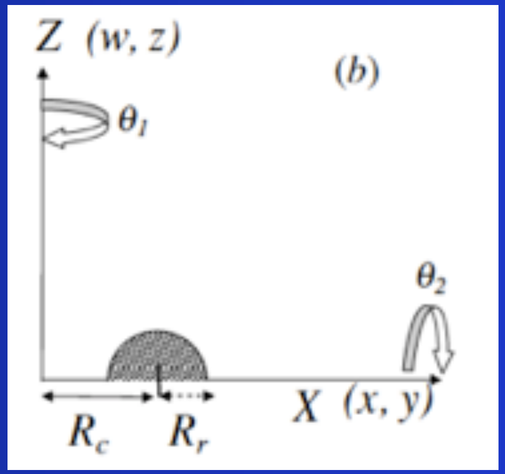
$SO(3)$



$U(1) \times U(1)$

4. Ring matter collapse
   Initial data analysis
   Evolutions

$S^2 \times S^1$ horizon



5. Hoop Conjecture?

2. Numerical method A. ADM

The 3+1 decomposition of space-time: The ADM formulation

[1] R. Arnowitt, S. Deser and C.W. Misner, in *Gravitation: An Introduction to Current Research*, ed. by L.Witten, (Wiley, New York, 1962).

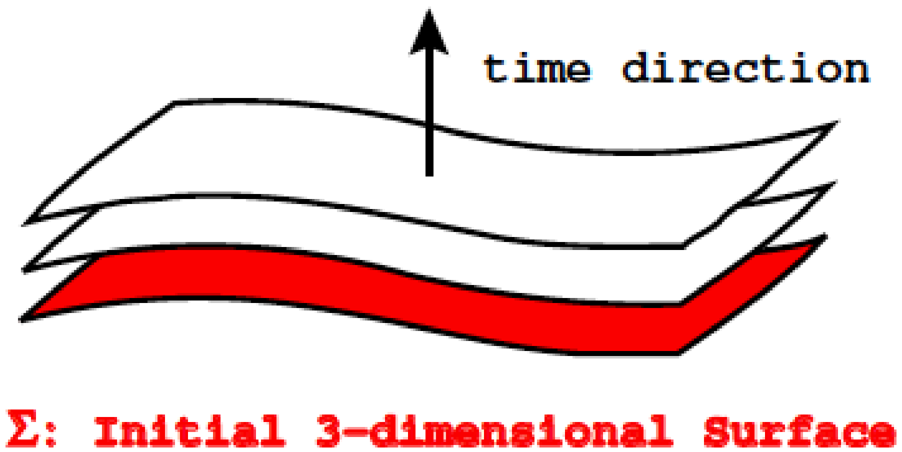
[2] J.W. York, Jr. in *Sources of Gravitational Radiation*, (Cambridge, 1979)

Dynamics of Space-time = Foliation of Hypersurface

- Evolution of t =const. hypersurface $\Sigma(t)$.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (\mu, \nu = 0, 1, 2, 3)$$

on $\Sigma(t)$...
$$d\ell^2 = \gamma_{ij} dx^i dx^j, \quad (i, j = 1, 2, 3)$$



- The unit normal vector of the slices, n^μ.

$$n_\mu = (-\alpha, 0, 0, 0)$$
$$n^\mu = g^{\mu\nu} n_\nu = (1/\alpha, -\beta^i/\alpha)$$

- The lapse function, α. The shift vector, β^i.

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

The decomposed metric:

$$\begin{aligned} ds^2 &= -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \\ &= (-\alpha^2 + \beta_l \beta^l) dt^2 + 2\beta_i dt dx^i + \gamma_{ij} dx^i dx^j \end{aligned}$$

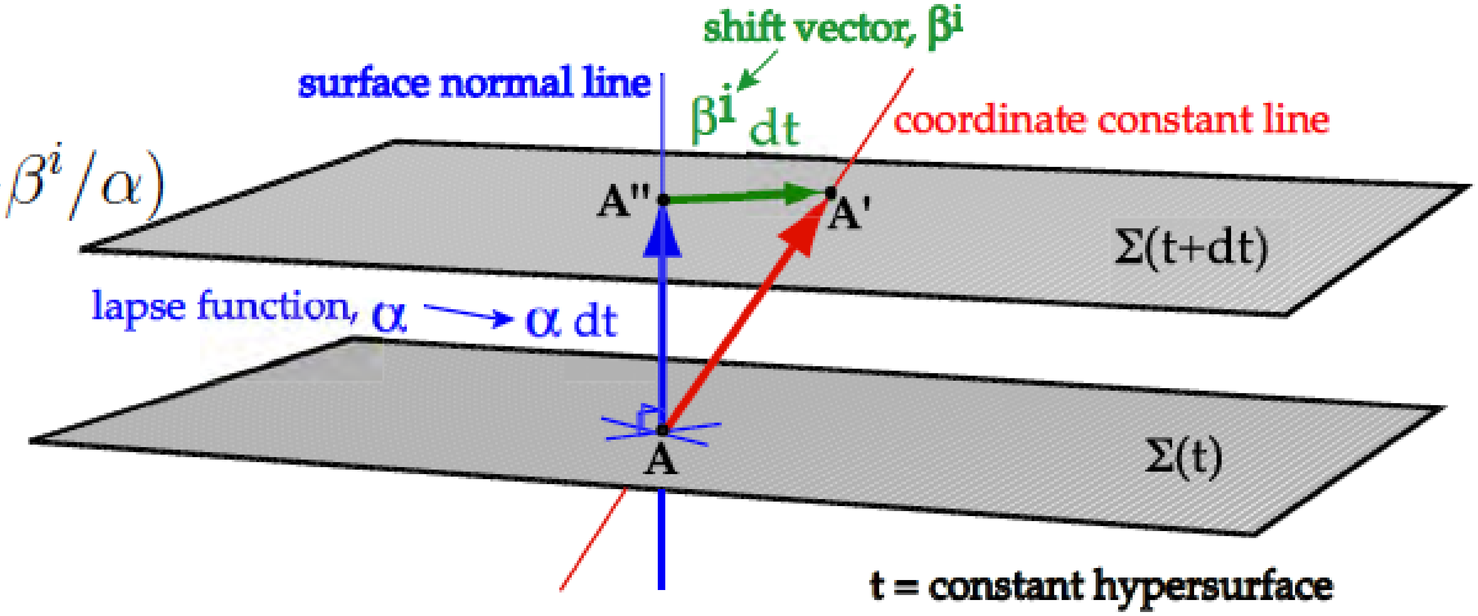
$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_l \beta^l & \beta_j \\ \beta_i & \gamma_{ij} \end{pmatrix}, \qquad g^{\mu\nu} = \begin{pmatrix} -1/\alpha^2 & \beta^j/\alpha^2 \\ \beta^i/\alpha^2 & \gamma^{ij} - \beta^i\beta^j/\alpha^2 \end{pmatrix}$$

where α and β_j are defined as $\alpha \equiv 1/\sqrt{-g^{00}}$, $\quad \beta_j \equiv g_{0j}$.

- The unit normal vector of the slices, n^μ.

$$\begin{aligned} n_\mu &= (-\alpha, 0, 0, 0) \\ n^\mu &= g^{\mu\nu} n_\nu = (1/\alpha, -\beta^i/\alpha) \end{aligned}$$

- The lapse function, α.
- The shift vector, β^i.



2. Numerical method A. ADM (4-dim)

The Standard ADM formulation (aka York 1978):

J.W. York, Jr. in *Sources of Gravitational Radiation*, (Cambridge, 1979)

The fundamental dynamical variables are (γ_{ij}, K_{ij}), the three-metric and extrinsic curvature. The three-hypersurface Σ is foliated with gauge functions, (α, β^i), the lapse and shift vector.

- The evolution equations:

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i, \qquad (1)$$

$$\partial_t K_{ij} = \alpha\, {}^{(3)}R_{ij} + \alpha K K_{ij} - 2\alpha K_{ik}K^k{}_j - D_iD_j\alpha + (D_i\beta^k)K_{kj} + (D_j\beta^k)K_{ki} + \beta^k D_k K_{ij} - 8\pi G\alpha\{S_{ij} + (1/2)\gamma_{ij}(\rho_H - \mathrm{tr}S)\}, \qquad (2)$$

  where $K = K^i{}_i$, and ${}^{(3)}R_{ij}$ and D_i denote three-dimensional Ricci curvature, and a covariant derivative on the three-surface, respectively.

- Constraint equations:

Hamiltonian constr. $\qquad \mathcal{H}^{ADM} := {}^{(3)}R + K^2 - K_{ij}K^{ij} - 2\Lambda - 2\kappa\rho \approx 0,$

momentum constr. $\qquad \mathcal{M}_i^{ADM} := D_jK^j{}_i - D_iK - \kappa J^i \approx 0,$

where ${}^{(3)}R = {}^{(3)}R^i{}_i$.

2. Numerical method A. ADM (N-dim)

The Standard ADM formulation in $N+1$-dim.

cf. H. Shinkai and G. Yoneda, Gen. Rel. Grav. **36**, 1931 (2004)

The fundamental dynamical variables are (γ_{ij}, K_{ij}), the three-metric and extrinsic curvature. The three-hypersurface Σ is foliated with gauge functions, (α, β^i), the lapse and shift vector.

- The evolution equations:

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_j\beta_i + D_i\beta_j, \tag{1}$$

$$\begin{aligned}\partial_t K_{ij} = {} & \alpha\, {}^{(N)}R_{ij} + \alpha K K_{ij} - 2\alpha K^{\ell}{}_j K_{i\ell} - D_i D_j \alpha \\ & + \beta^k (D_k K_{ij}) + (D_j \beta^k) K_{ik} + (D_i \beta^k) K_{kj} - \kappa\alpha \left(S_{ij} - \frac{1}{N-1}\gamma_{ij} T \right) - \frac{2\alpha}{N-1}\gamma_{ij}\Lambda, \end{aligned} \tag{2}$$

where $K = K^i{}_i$, and ${}^{(N)}R_{ij}$ and D_i denote N-dimensional Ricci curvature, and a covariant derivative on the three-surface, respectively.

- Constraint equations:

Hamiltonian constr. $\quad \mathcal{H}^{ADM} := {}^{(N)}R + K^2 - K_{ij}K^{ij} - 2\Lambda - 2\kappa\rho \approx 0,$

momentum constr. $\quad \mathcal{M}_i^{ADM} := D_j K^j{}_i - D_i K - \kappa J^i \approx 0,$

where ${}^{(N)}R = {}^{(N)}R^i{}_i$.

2. Numerical method B. procedures

Procedure of the Standard Numerical Relativity

■ 3+1 (ADM) formulation

■ Preparation of the Initial Data

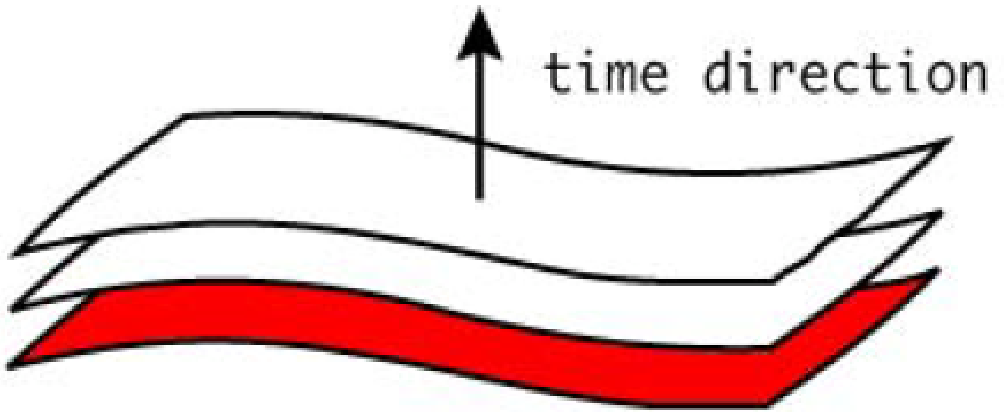
- ◆ Assume the background metric
- ◆ Solve the constraint equations

■ Time Evolution

do time=1, time_end

- ◆ Specify the slicing condition
- ◆ Evolve the variables
- ◆ Check the accuracy
- ◆ Extract physical quantities

end do



Σ: Initial 3-dimensional Surface

2. Numerical method B. procedures

Procedure of the Standard Numerical Relativity

■ 3+1 (ADM) formulation

■ Preparation of the Initial Data

- ◆ Assume the background metric
- ◆ Solve the constraint equations

Need to solve elliptic PDEs
-- Conformal approach
-- Thin-Sandwich approach

■ Time Evolution

do time=1, time_end

- ◆ Specify the slicing conditions
- ◆ Evolve the variables
- ◆ Check the accuracy
- ◆ Extract physical quantities

end do

singularity avoidance,
simplify the system,
GW extraction, ...

Robust formulation ?
-- modified ADM
-- hyperbolization
-- asymptotically constrained

2. Numerical method C. initial data (3-dim)

Conformal approach for solving constraints (York-ÓMurchadha, 1974)

N.ÓMurchadha and J.W.York Jr., Phys. Rev. **D 10**, 428 (1974)

One way to set up $(\gamma_{ij}, K_{ij}, \rho, J^i)$ so as to satisfy the constraints:

1. Specify metric components $\hat{\gamma}_{ij}$, tr$K$, $\hat{A}^{TT}_{ij}$, and matter distribution $\hat{\rho}$, $\hat{J}$ in the conformal frame.
2. Solve the next equations for $(\psi,\ W^i)$

$$8\hat{\Delta}\psi = \hat{R}\psi - (\hat{A}_{ij}\hat{A}^{ij})\psi^{-7} + [(2/3)(\mathrm{tr}K)^2 - 2\Lambda]\psi^5 - 16\pi G\hat{\rho}\psi^{5-n} \quad (1)$$

$$\hat{\Delta}W^i + (1/3)\hat{D}^i\hat{D}_kW^k + \hat{R}^i{}_kW^k = (2/3)\psi^6\hat{D}^i\mathrm{tr}K + 8\pi G\hat{J}^i \quad (2)$$

   where $\hat{A}^{ij} = \hat{A}^{ij}_{TT} + \hat{D}^iW^j + \hat{D}^jW^i - (2/3)\hat{\gamma}^{ij}\hat{D}_kW^k$.
3. Apply the inverse conformal transformation and get the metric and matter components $\gamma_{ij},\ K_{ij},\ \rho,\ J^i$ in the physical frame:

$$\gamma_{ij} = \psi^4\hat{\gamma}_{ij},$$
$$K_{ij} = \psi^{-2}[\hat{A}^{TT}_{ij} + (\hat{\mathbb{l}}W)_{ij}] + (1/3)\psi^4\hat{\gamma}_{ij}\mathrm{tr}K,$$
$$\rho = \psi^{-n}\hat{\rho},$$
$$J^i = \psi^{-10}\hat{J}^i$$

2. Numerical method C. initial data (N-dim)

Conformal approach for solving constraints in $(N+1)$-dim.

T. Torii and H. Shinkai, Phys. Rev. **D 78**, 084037 (2008)

One way to set up $(\gamma_{ij}, K_{ij}, \rho, J^i)$ so as to satisfy the constraints:

1. Specify metric components $\hat{\gamma}_{ij}$, tr$K$, $\hat{A}^{TT}_{ij}$, and matter distribution $\hat{\rho}$, $\hat{J}$ in the conformal frame.
2. Solve the next equations for (ψ, W^i)

$$\frac{4(N-1)}{N-2}\hat{\Delta}\psi = \hat{R}\psi - \varepsilon\psi^{2\ell+1-4/(N-2)}(\hat{K}^2 - \hat{K}_{ab}\hat{K}^{ab}) + 2\varepsilon\kappa^2\hat{\rho}\psi^{-p} - 2\hat{\Lambda} \qquad (1)$$

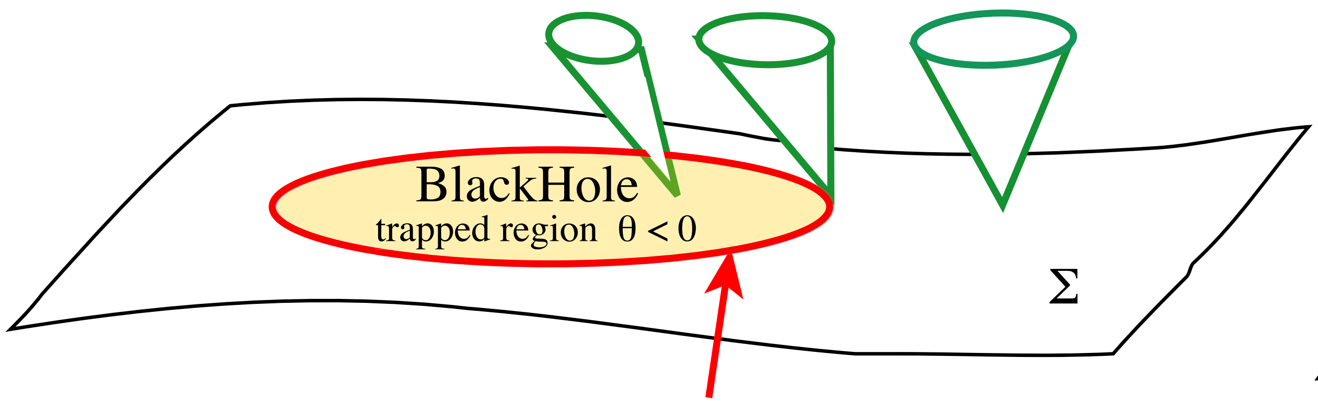
$$\hat{\Delta}W_i + \frac{N-2}{N}\hat{D}_i\hat{D}_kW^k + \hat{R}_{ik}W^k + \psi^{-1}(\ell+2)(\hat{D}^aW^b + \hat{D}^bW^a - \frac{2}{N}\hat{\gamma}^{ab}\hat{D}_kW^k)\hat{\gamma}_{bi}\hat{D}_a\psi$$
$$-\frac{N-1}{N}\left[(\ell - \frac{4}{N-2})(\hat{D}_i\psi)\hat{K} + \hat{D}_i\hat{K}\right] = \kappa^2\psi^{8/(N-2)-\ell-q}\hat{J}_i \qquad (2)$$

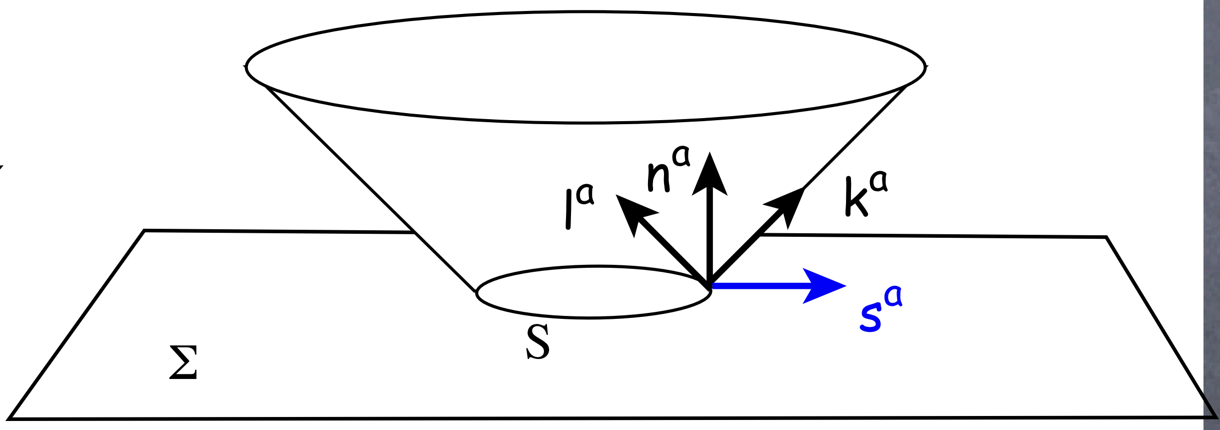
3. Apply the inverse conformal transformation and get the metric and matter components γ_{ij}, K_{ij}, ρ, J^i in the physical frame:

$$\gamma_{ij} = \psi^{4/(N-2)}\hat{\gamma}_{ij},$$
$$K_{ij} = \psi^{\ell}[\hat{A}^{TT}_{ij} + (\hat{\mathbb{l}}W)_{ij}] + \frac{1}{N}\psi^{\ell-4/(N-2)}\hat{\gamma}_{ij}\mathrm{tr}K,$$
$$\rho = \psi^{-p}\hat{\rho},$$
$$J^i = \psi^{-q}\hat{J}^i$$

2. Numerical method D. Horizon Finder

Apparent Horizon





Null vectors

$$\begin{cases} k^a = (1/\sqrt{2})(n^a + s^a) \\ \ell^a = (1/\sqrt{2})(n^a - s^a) \end{cases}$$

Projection operator onto S

$$\begin{aligned} P^a{}_b &= \delta^a{}_b + k^a \ell_b + \ell^a k_b \\ &= \delta^a{}_b + n^a n_b - s^a s_b \end{aligned}$$

Extrinsic curvature along k^a

$$\begin{aligned} \kappa_{ab} &= -P^c{}_a P^d{}_b k_{c;d} \\ &= (1/\sqrt{2})(-\nabla_c s_d + K_{cd}) P^c{}_a P^d{}_b \end{aligned}$$

$$\begin{aligned} \textbf{AH} &\iff \textbf{expansion} \quad \theta = 0 \\ &\iff -\sqrt{2}\kappa^a{}_a = k^a{}_{;a} = 0 \\ &\iff \nabla_a s^a - K + K_{ab} s^a s^b = 0 \end{aligned}$$

In spherically sym. spacetime,

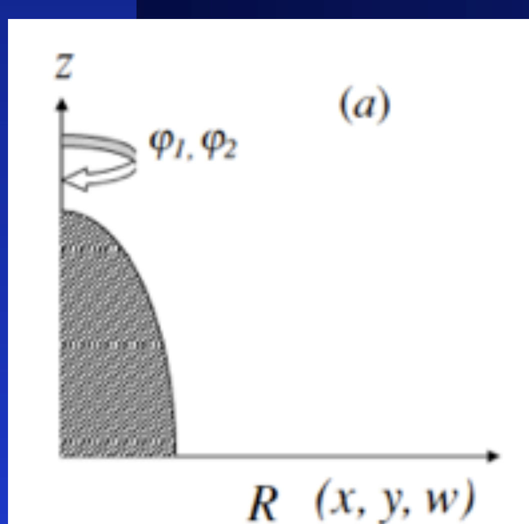
$$dl^2 = \psi^4 a^2 [d\chi^2 + f^2(\chi)\, d\Omega^2]$$

$$\Longrightarrow \quad \theta = \frac{1}{a\psi^6 f^2} \frac{d}{d\chi} [\psi^4 f^2] - \frac{2}{3} K$$

3. Spheroidal matter collapse
A. Initial data construction

- time symmetric, asymptotically flat
- conformal flat
- non-rotating homogeneous dust

- solve the Hamiltonian constraint eq. 512^2 grids
- Apparent Horizon Search
- Define Hoop and check the Hoop Conjecture



$$ds^2 = \psi(R,z)^2 \left[dR^2 + R^2(d\varphi_1^2 + \sin^2\varphi_1 d\varphi_2^2) + dz^2\right]$$

$$R = \sqrt{x^2+y^2+z^2}, \quad \varphi_1 = \tan^{-1}\left(\frac{w}{\sqrt{x^2+y^2}}\right), \quad \varphi_2 = \tan^{-1}\left(\frac{y}{x}\right).$$

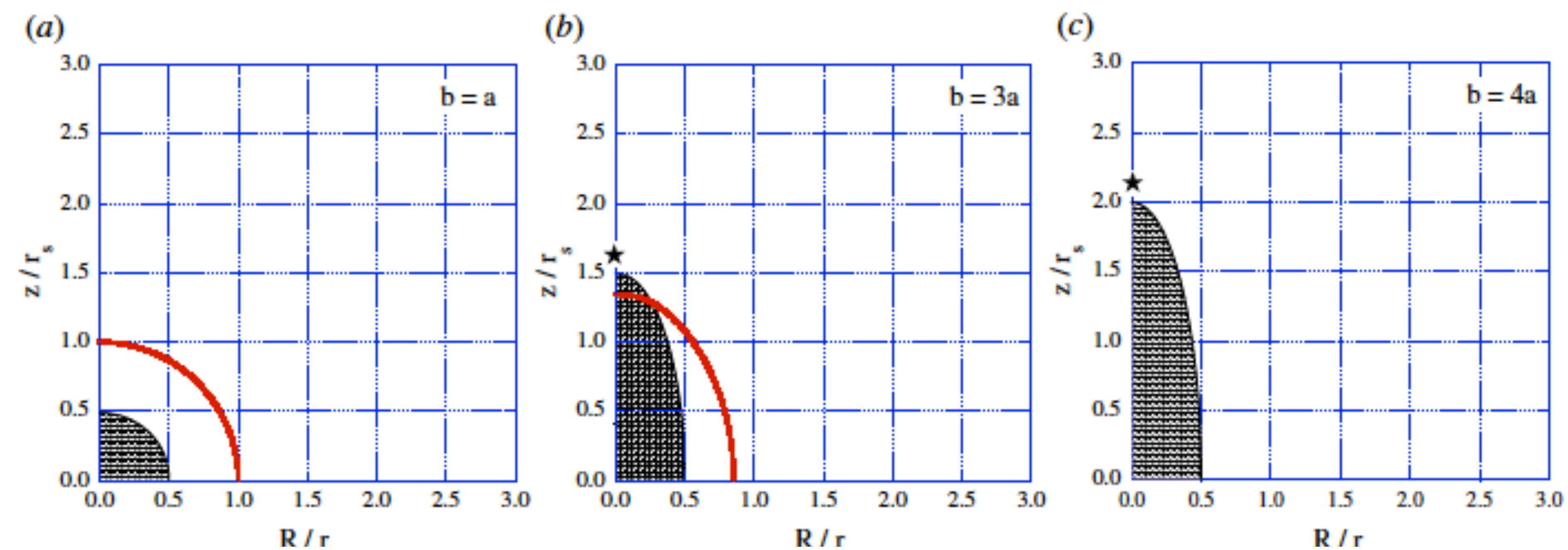
$$\frac{\partial^2\psi}{\partial R^2} + \frac{2}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial z^2} = -4\pi^2 G_5 \rho$$

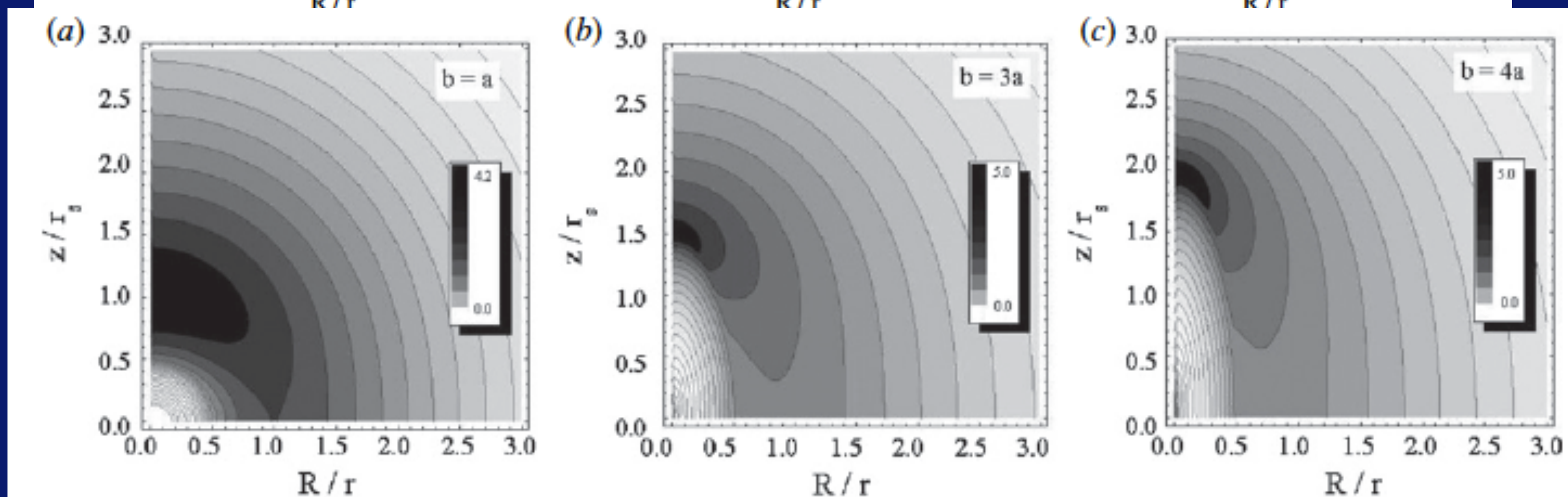
3. Spheroidal matter collapse
B. Initial data sequence

cf. (3-dim.) Nakamura-Shapiro-Teukolsky (1988)

4+1
initial data

Class. Quantum Grav. **27** (2010) 045012 Y Yamada and H Shinkai





Contour Plot of the Kretchmann invariant, $R_{abcd}R^{abcd}$

3. Spheroidal matter collapse
C. Evolution method

- ADM 2+1 Double Axisym Cartoon
- 130^2 x 2^2 grids

- lapse function: Maximal slicing condition
- shift vectors: Minimum distortion condition
- asymptotically flat

- Collisionless Particles (5000)
- the same total mass
- no rotation

- Apparent Horizon Search

ADM evolution eqs. (5D)

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_k\beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$$

$$\begin{aligned} \frac{\partial \gamma_{ij}}{\partial t} &= -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i, \\ \frac{\partial K_{ij}}{\partial t} &= \alpha({}^{(4)}R_{ij} + KK_{ij}) - 2\alpha K_{il}K^{lj} \\ &\quad - 12\pi^2\alpha(S_{ij} + \frac{1}{3}\gamma_{ij}(\rho - S)) \\ &\quad - D_iD_j\alpha + D_i\beta^m K_{mj} + D_j\beta^m K_{mi} + \beta^m D_m K_{ij} \end{aligned}$$

2nd-order differential scheme
Iterative Crank-Nicolson method
Courant factor 0.2

slicing conditions (lapse) α

Maximal slicing condition

$$K=0 \quad \Leftrightarrow \quad \partial_t K = 0 \quad \Leftrightarrow$$

$$\triangle\alpha = \alpha(K_{ij}K^{ij} + \frac{2}{3}\kappa\rho + \frac{1}{3}\kappa S)$$

BC at far region $(\alpha-1)r^2 = \text{const.} \quad \Leftrightarrow \quad \frac{\partial}{\partial x^i}\left[(\alpha-1)r^2\right] = 0.$

" singularity avoidance "

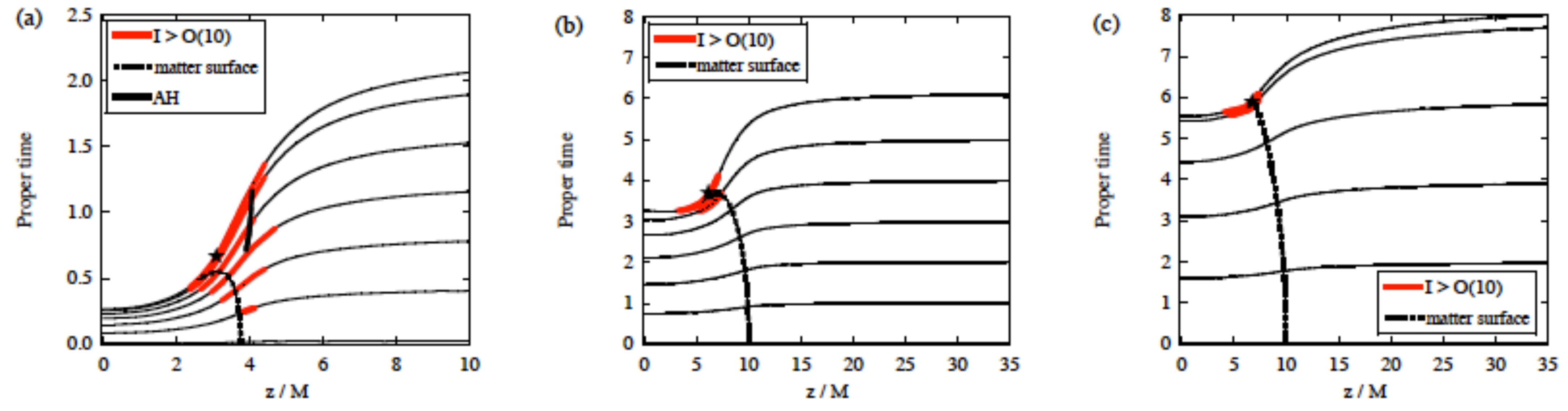


FIG. 4: The snapshots of the hypersurfaces on the z-axis in the proper-time versus coordinate diagram; (a) model 5DSβ, (b) model 5DSδ, and (c) model 4Dδ. The upper most hypersurface is the final data in numerical evolution. We also mark the matter surface and the location of AH if exist. The ranges with $\mathcal{I} \geq 10$ are marked with bold lines and peak value of $\mathcal{I}$ express by asterisks.

slicing conditions (shift) β^i

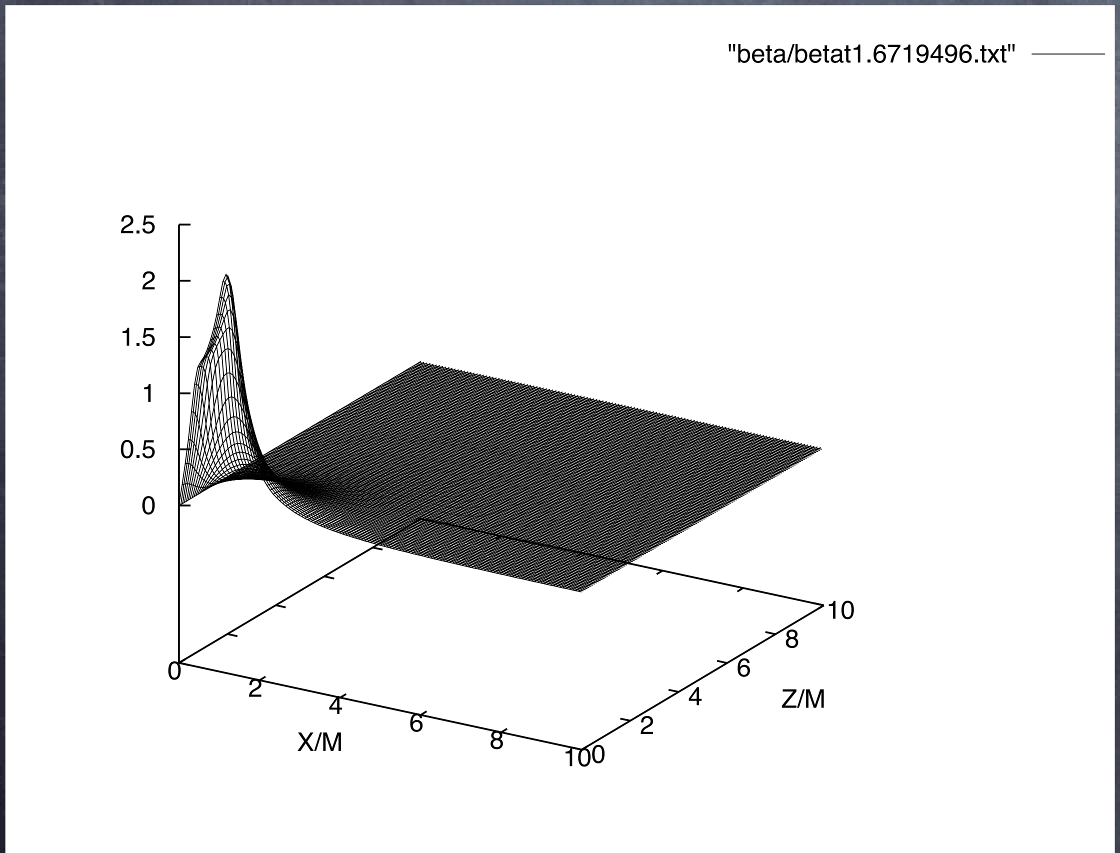
Minimal strain condition

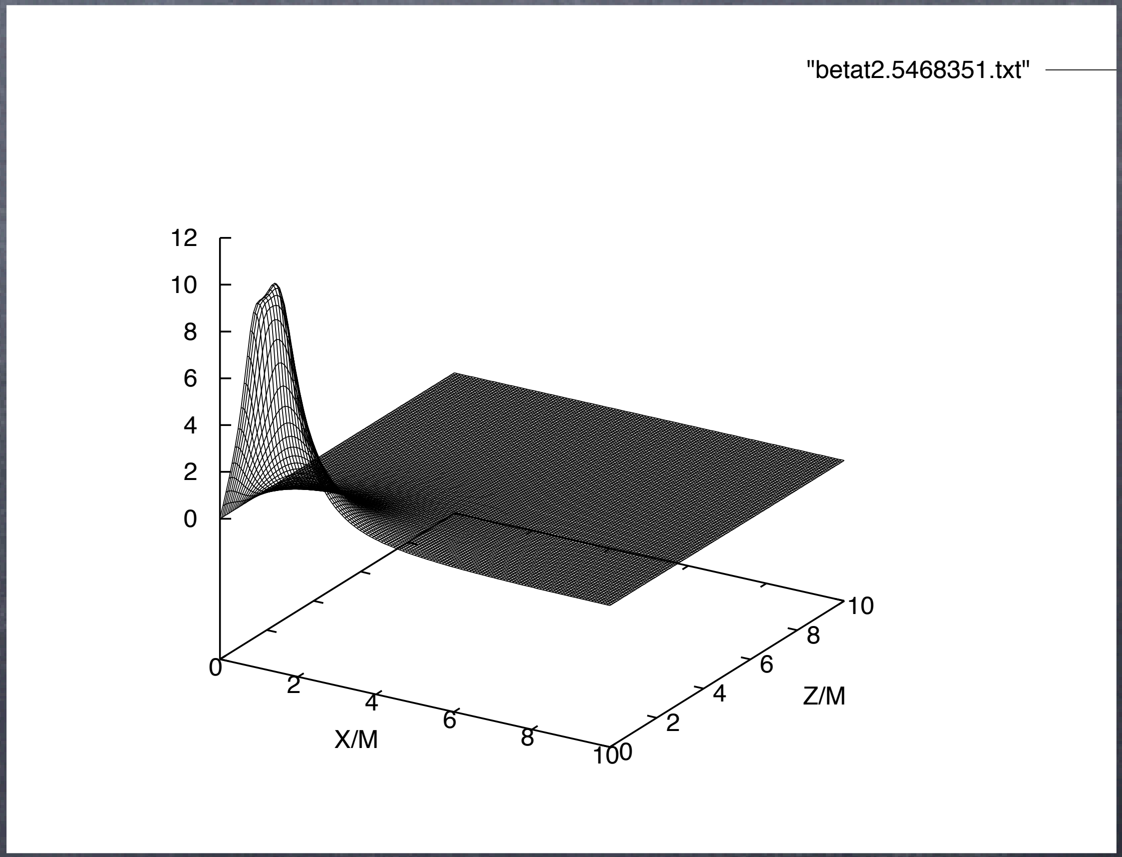
$$\Theta_{\mu\nu} = \perp \nabla_{(\nu} t_{\mu)} = -\alpha K_{\mu\nu} + \frac{1}{2} D_{(\mu}\beta_{\nu)}, \quad \text{where} \quad t^\mu = \alpha n^\mu + \beta^\mu$$

$$D_j \Theta^{ij} = 0 \quad \Leftrightarrow \quad \triangle \beta^i + D^i D_j \beta^j + R_{ij}\beta^j = 2D^j(\alpha K_{ij}).$$

BC at far region $\beta r^2 = \text{const.} \quad \Leftrightarrow \quad \frac{\partial}{\partial x^i}\left[\beta r^2\right] = 0.$

" anti grid-stretching"





matter = particles

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha{}_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

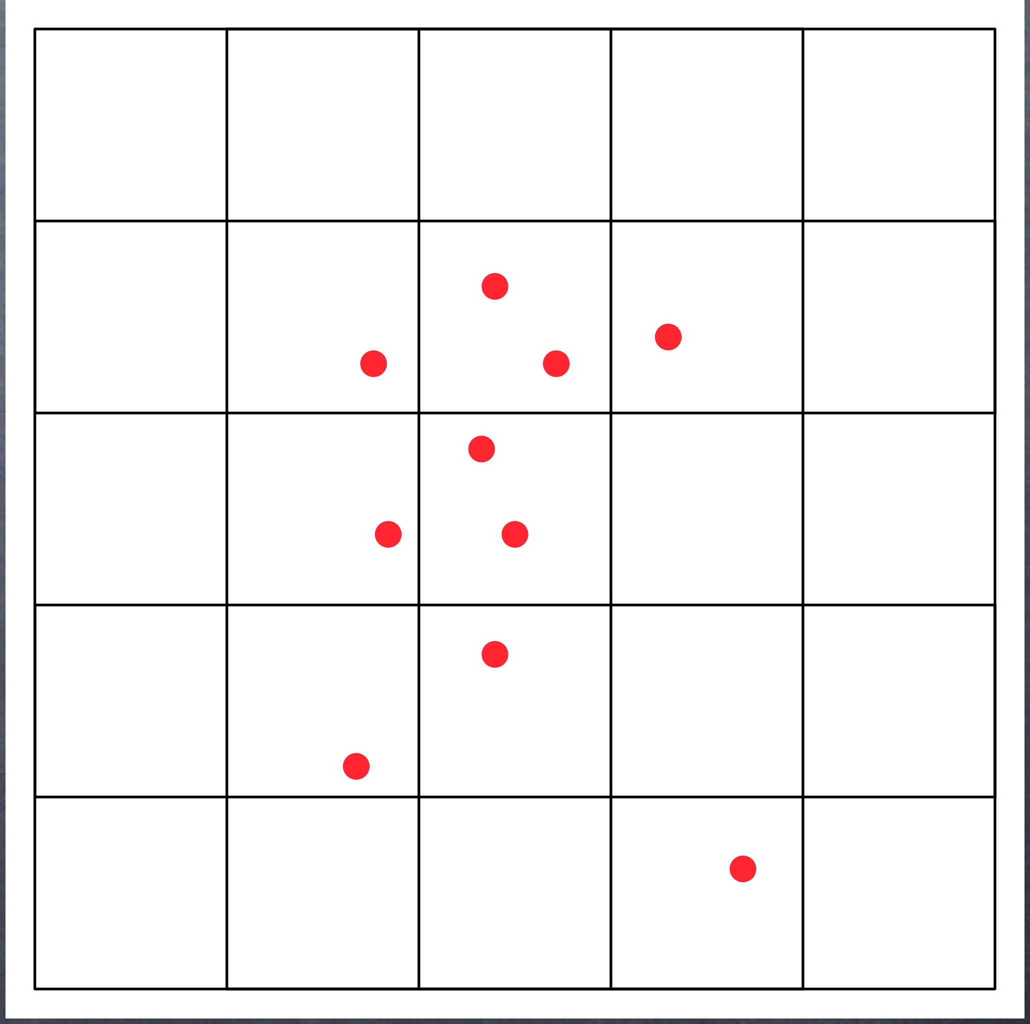
5000 particles

Runge-Kutta method

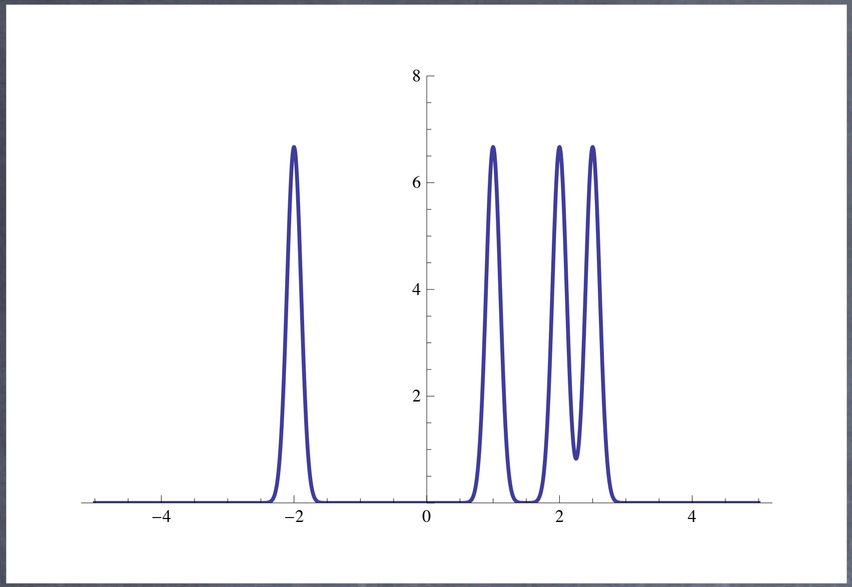
$$T^{ij} = \sum_A m\, n_{(A)}\, u^\mu_{(A)}\, u^\nu_{(A)}$$

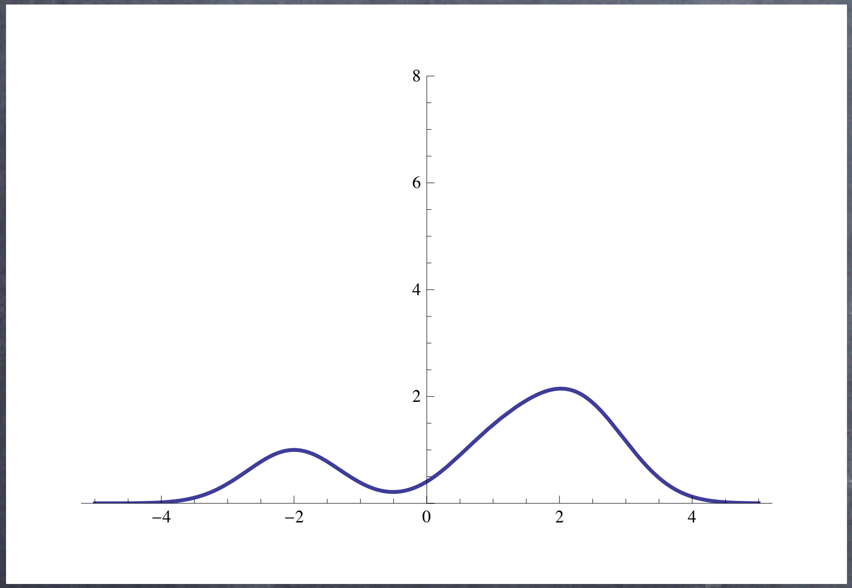
$$\rho = \sum_A m\, n_{(A)} (\alpha u^0_{(A)})^2$$

$$S_{ij} = \sum_A m\, n_{(A)} u_{i(A)} u_{j(A)}$$



matter = particles *cf. SPH*

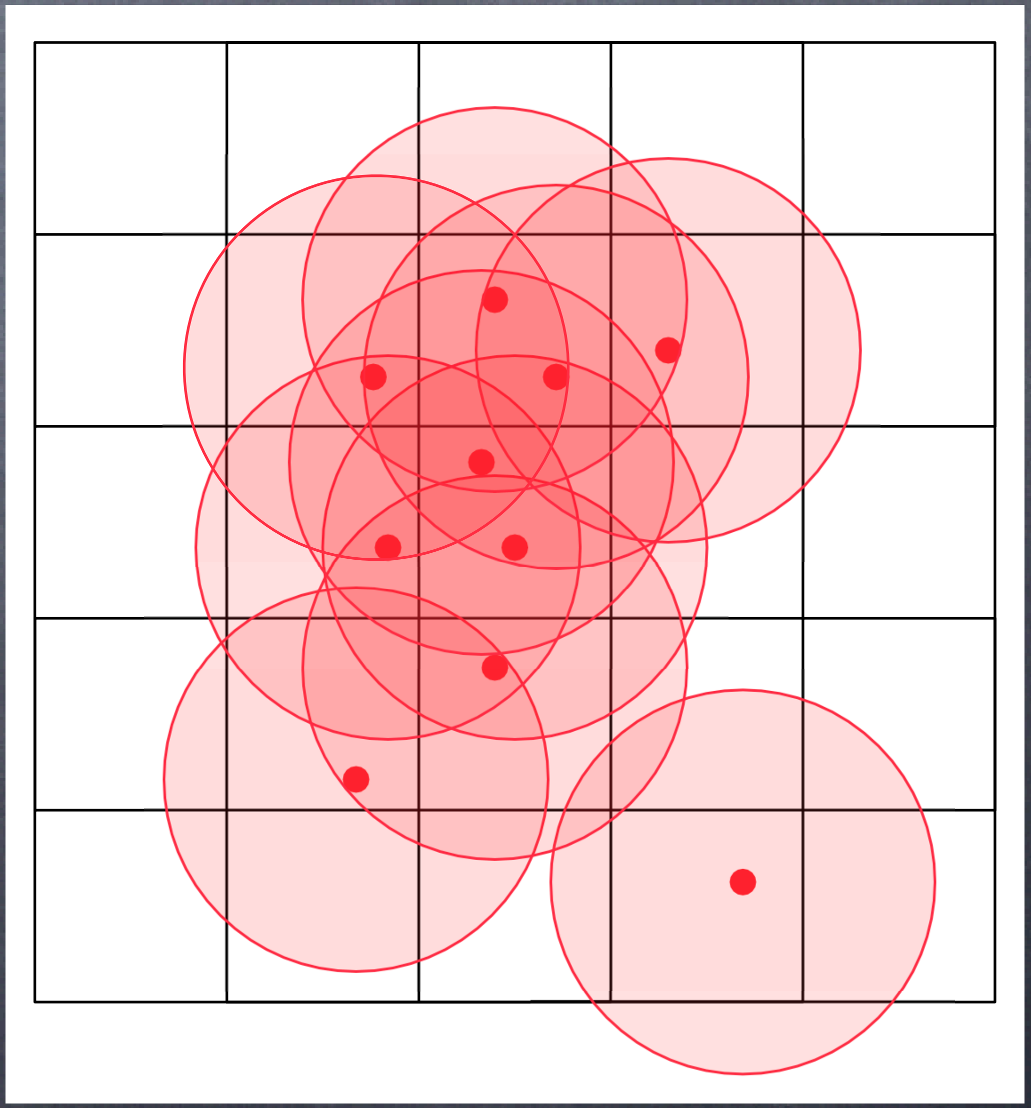




smoothing kernel

$$W(x_i) = \frac{\exp[-(x - x_i)^2/h^2]}{\sqrt{\pi}h}$$

$$W(x_i, y_i) = \frac{\exp[\{-(x - x_i)^2 - (y - y_i)^2\}/h^2]}{\pi h^2}$$

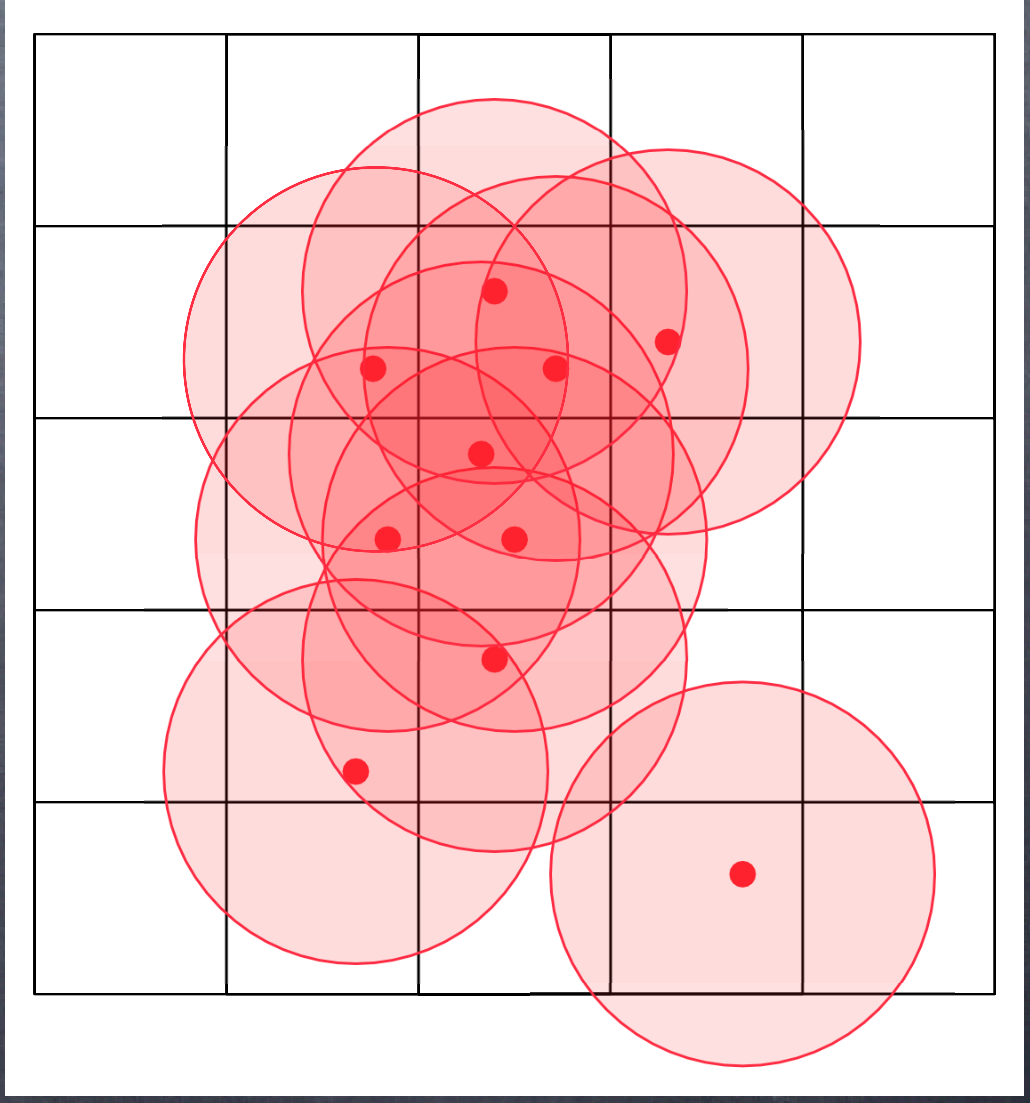


matter = particles

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

$$W(x_i, y_i) = \frac{\exp[\{-(x-x_i)^2 - (y-y_i)^2\}/h^2]}{\pi h^2}$$

$$T^{ij} = \sum_A m\, n_{(A)}\, u^\mu_{(A)}\, u^\nu_{(A)}$$



$$\rho = \sum_A m\, n_{(A)} (\alpha u^0_{(A)})^2\ W$$

$$S_{ij} = \sum_A m\, n_{(A)} u_{i(A)} u_{j(A)}\ W$$

Cartoon method

treating symmetry with Cartesian coord.

Alcubierre, Brandt, Bruegmann, Holz, Seidel, Takahashi, Thornburg, gr-qc/9908012

Axisymmetric system on z

cylindrical coord.

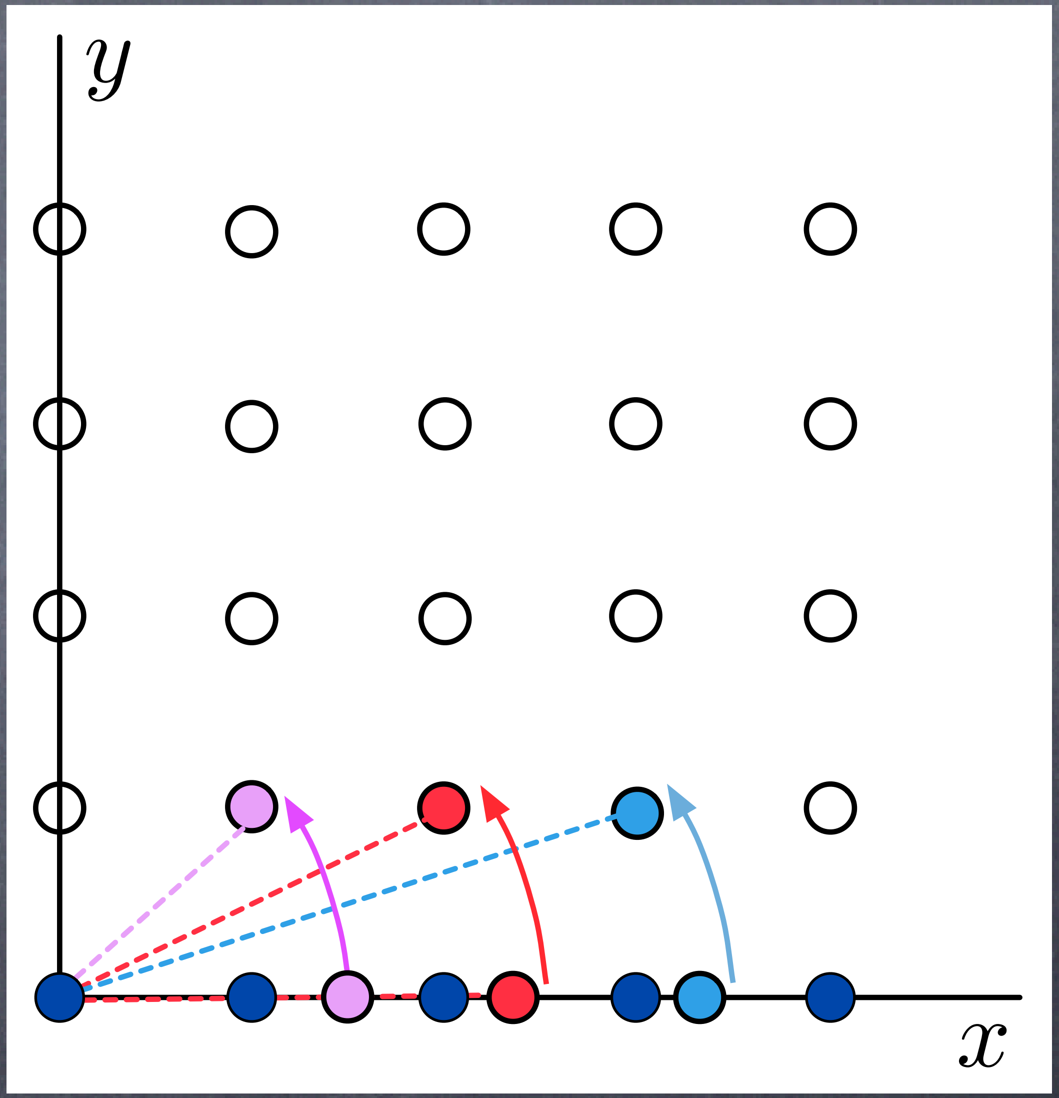
$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$z = z$$

Cartesian coord.

$$(x, y, z)$$

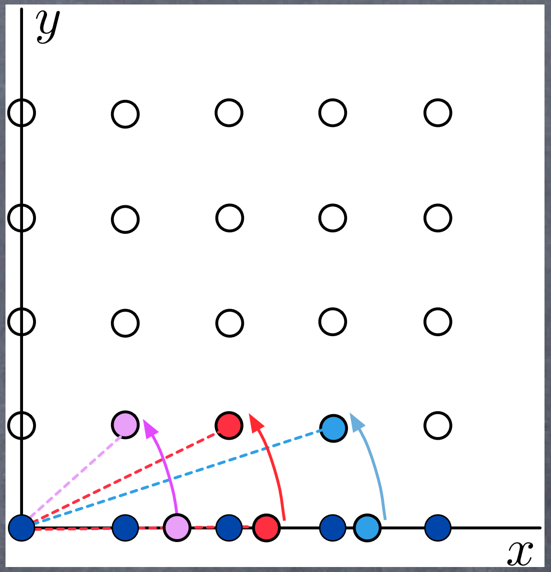


Cartoon method (4-dim.)

treating symmetry with Cartesian coord.

Alcubierre, Brandt, Bruegmann, Holz, Seidel, Takahashi, Thornburg, gr-qc/9908012

Axisymmetric system on z



for scalar

$$\Psi(x,y,z) = \Psi(\rho,0,z)$$

for vector

$$T^z(x,y,z) = T^z(\rho,0,z)$$

$$T^x(x,y,z) = (x/\rho)T^x(\rho,0,z) - (y/\rho)T^y(\rho,0,z)$$
$$T^y(x,y,z) = (y/\rho)T^x(\rho,0,z) + (x/\rho)T^y(\rho,0,z)$$

for 2-rank sym. tensor

$$S^{zz}(x,y,z) = S^{zz}(\rho,0,z)$$

$$S^{zx}(x,y,z) = (x/\rho)S^{zx}(\rho,0,z) - (y/\rho)S^{zy}(\rho,0,z)$$
$$S^{zy}(x,y,z) = (y/\rho)S^{zx}(\rho,0,z) + (x/\rho)S^{zy}(\rho,0,z)$$

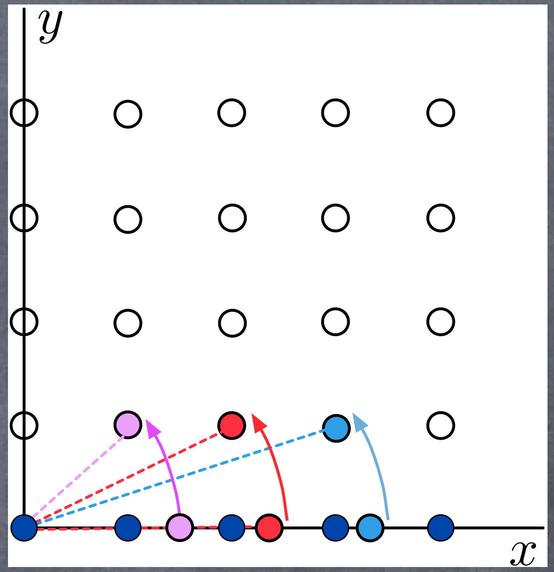
$$S^{xx}(x,y,z) = (x/\rho)^2 S^{xx}(\rho,0,z) + (y/\rho)^2 S^{yy}(\rho,0,z) - (2xy/\rho^2)S^{xy}(\rho,0,z)$$
$$S^{yy}(x,y,z) = (y/\rho)^2 S^{xx}(\rho,0,z) + (x/\rho)^2 S^{yy}(\rho,0,z) + (2xy/\rho^2)S^{xy}(\rho,0,z)$$
$$S^{xy}(x,y,z) = (xy/\rho)[S^{xx}(\rho,0,z) - S^{yy}(\rho,0,z)] + [(x^2-y^2)/\rho^2]S^{xy}(\rho,0,z)$$

Cartoon method (5-dim.)

treating symmetry with Cartesian coord.

Shibata, Yoshino, PRD 80 (2009) 084025

SO(3) sym. system on $(x = y = z, w)$



for scalar

$$\Psi(x,y,z,w) = \Psi(r,0,0,w)$$

for vector

$$T^x(x,y,z,w) = (x/r)T^x(r,0,0,w)$$
$$T^y(x,y,z,w) = (y/r)T^x(r,0,0,w)$$
$$T^z(x,y,z,w) = (z/r)T^x(r,0,0,w)$$
$$T^w(x,y,z,w) = T^w(r,0,0,w)$$

for 2-rank sym. tensor

$$S^{ww}(x,y,z,w) = S^{ww}(r,0,0,w)$$
$$S^{xw}(x,y,z,w) = (x/r)S^{xw}(r,0,0,w)$$
$$S^{yw}(x,y,z,w) = (y/r)S^{xw}(r,0,0,w)$$
$$S^{zw}(x,y,z,w) = (z/r)S^{xw}(r,0,0,w)$$
$$S^{xx}(x,y,z,w) = (x^2/r^2)S^{xx}(r,0,0,w) + \left(1 - x^2/r^2\right) S^{yy}(r,0,0,w)$$
$$S^{yy}(x,y,z,w) = (y^2/r^2)S^{xx}(r,0,0,w) + \left(1 - y^2/r^2\right) S^{yy}(r,0,0,w)$$
$$S^{zz}(x,y,z,w) = (z^2/r^2)S^{xx}(r,0,0,w) + \left(1 - z^2/r^2\right) S^{yy}(r,0,0,w)$$
$$S^{yz}(x,y,z,w) = (yz/r^2)[S^{xx} - S^{yy}](r,0,0,w)$$
$$S^{zx}(x,y,z,w) = (zx/r^2)[S^{xx} - S^{yy}](r,0,0,w)$$
$$S^{xy}(x,y,z,w) = (xy/r^2)[S^{xx} - S^{yy}](r,0,0,w)$$

3. Spheroidal matter collapse
C. Evolution examples (4D, ST1991)

VOLUME 66, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1991

Formation of Naked Singularities: The Violation of Cosmic Censorship

Stuart L. Shapiro and Saul A. Teukolsky

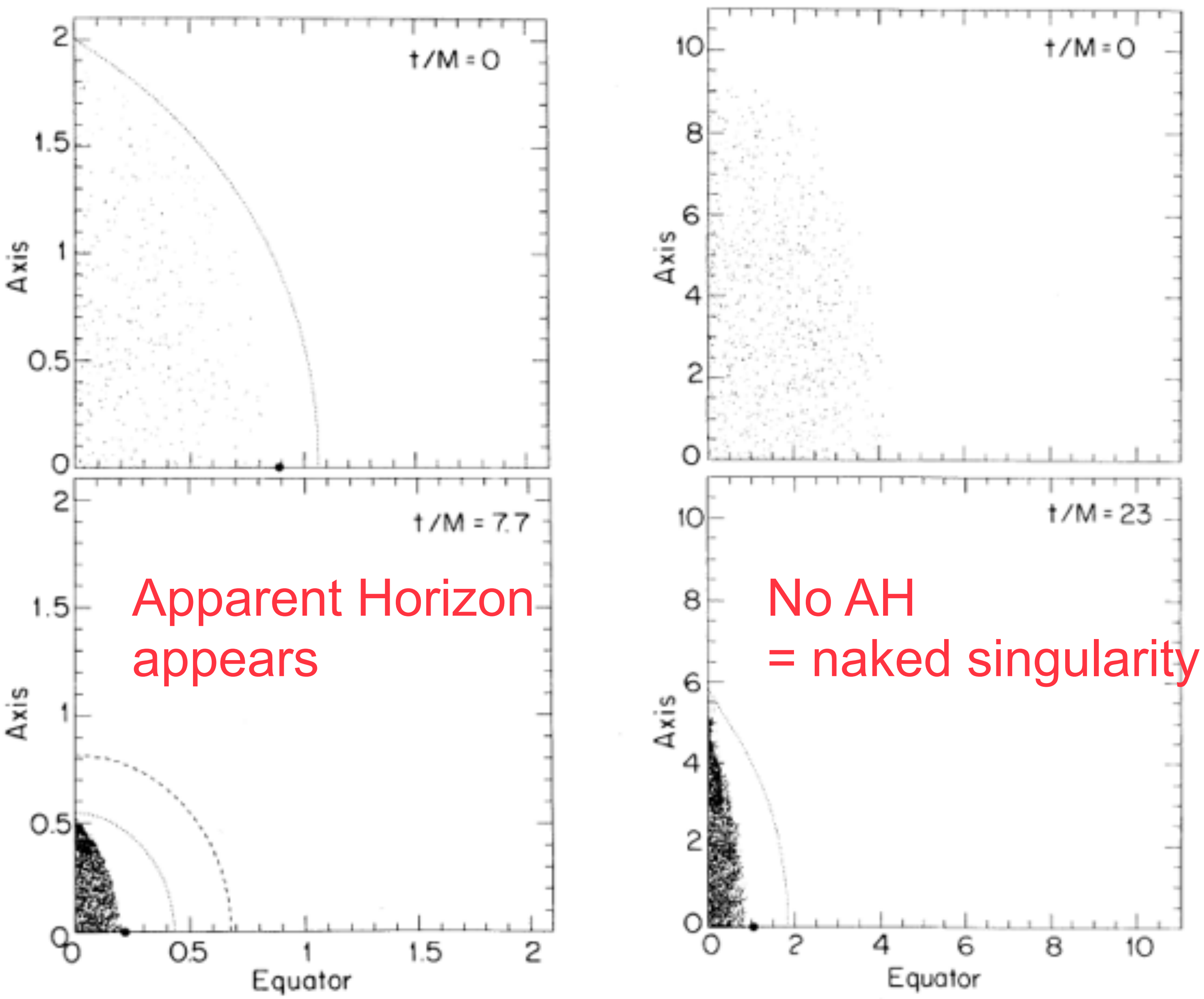


FIG. 1. Snapshots of the particle positions at initial and late times for prolate collapse. The positions (in units of M) are projected onto a meridional plane. Initially the semimajor axis of the spheroid is $2M$ and the eccentricity is 0.9. The collapse proceeds nonhomologously and terminates with the formation of a spindle singularity on the axis. However, an apparent horizon (dashed line) forms to cover the singularity. At $t/M = 7.7$ its area is $\mathcal{A}/16\pi M^2 = 0.98$, close to the asymptotic theoretical limit of 1. Its polar and equatorial circumferences at that time are $\mathcal{C}_{pole}^{AH}/4\pi M = 1.03$ and $\mathcal{C}_{eq}^{AH}/4\pi M = 0.91$. At later times these circumferences become equal and approach the expected theoretical value 1. The minimum exterior polar circumference is shown by a dotted line when it does not coincide with the matter surface. Likewise, the minimum equatorial circumference, which is a circle, is indicated by a solid dot. Here $\mathcal{C}_{eq}^{min}/4\pi M = 0.59$ and $\mathcal{C}_{pole}^{min}/4\pi M = 0.99$. The formation of a black hole is thus consistent with the hoop conjecture.

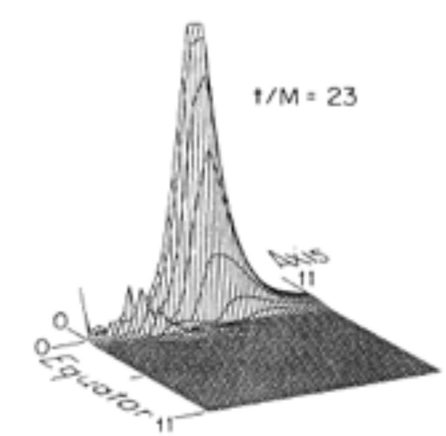
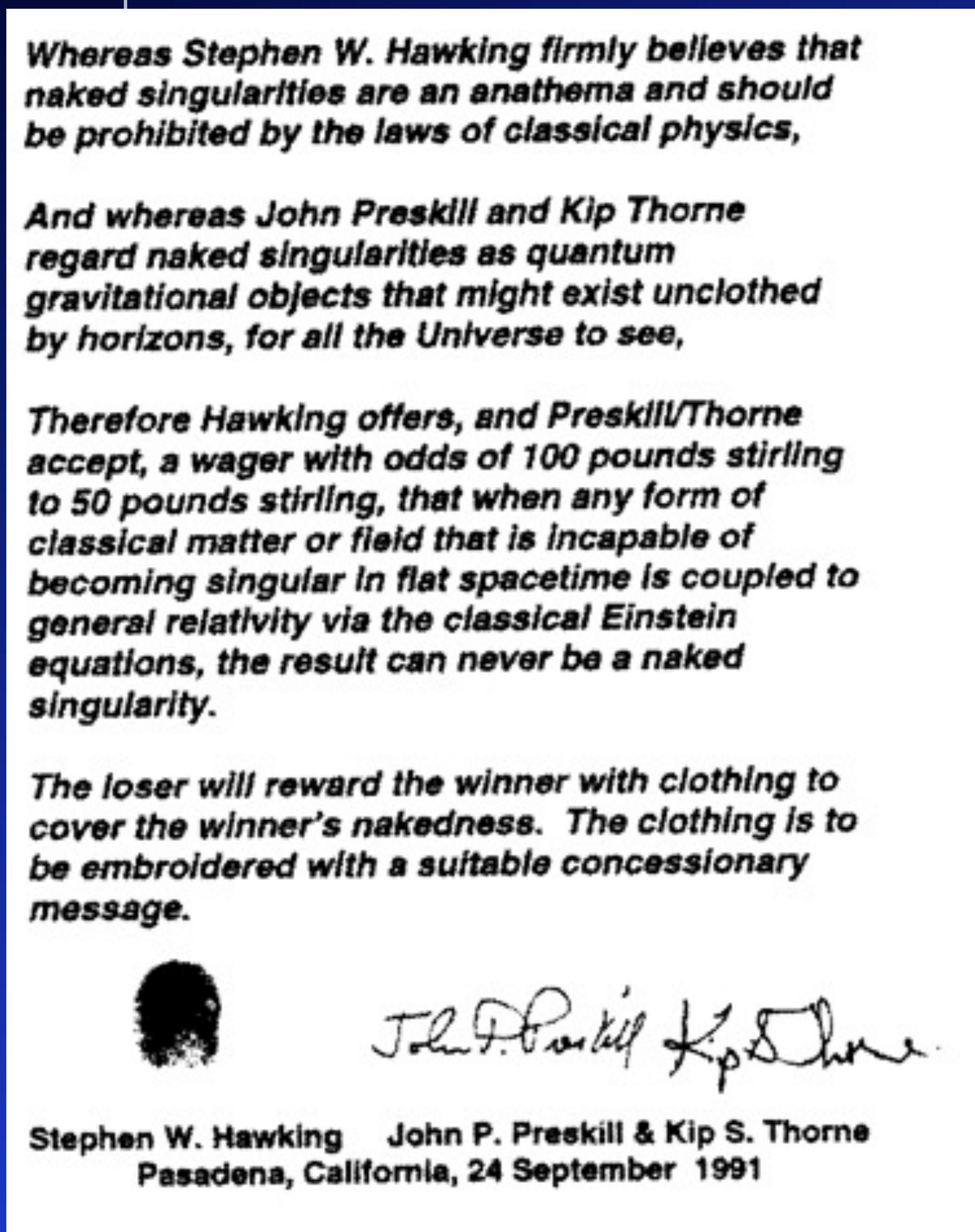


FIG. 4. Profile of I in a meridional plane for the collapse shown in Fig. 2. For the case of 32 angular zones shown here, the peak value of I is $24/M^4$ and occurs on the axis just outside the matter.

ホーキングとソーンの賭け



Whereas Stephen W. Hawking firmly believes that naked singularities are an anathema and should be prohibited by the laws of classical physics,

And whereas John Preskill and Kip Thorne regard naked singularities as quantum gravitational objects that might exist unclothed by horizons, for all the Universe to see,

Therefore Hawking offers, and Preskill/Thorne accept, a wager with odds of 100 pounds stirling to 50 pounds stirling, that when any form of classical matter or field that is incapable of becoming singular in flat spacetime is coupled to general relativity via the classical Einstein equations, the result can never be a naked singularity.

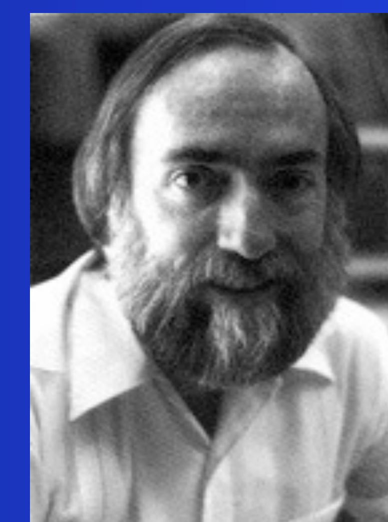
The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable concessionary message.

Stephen W. Hawking   John P. Preskill & Kip S. Thorne

Pasadena, California, 24 September 1991

ホーキング

「裸の特異点は物理法則によって禁止されている」



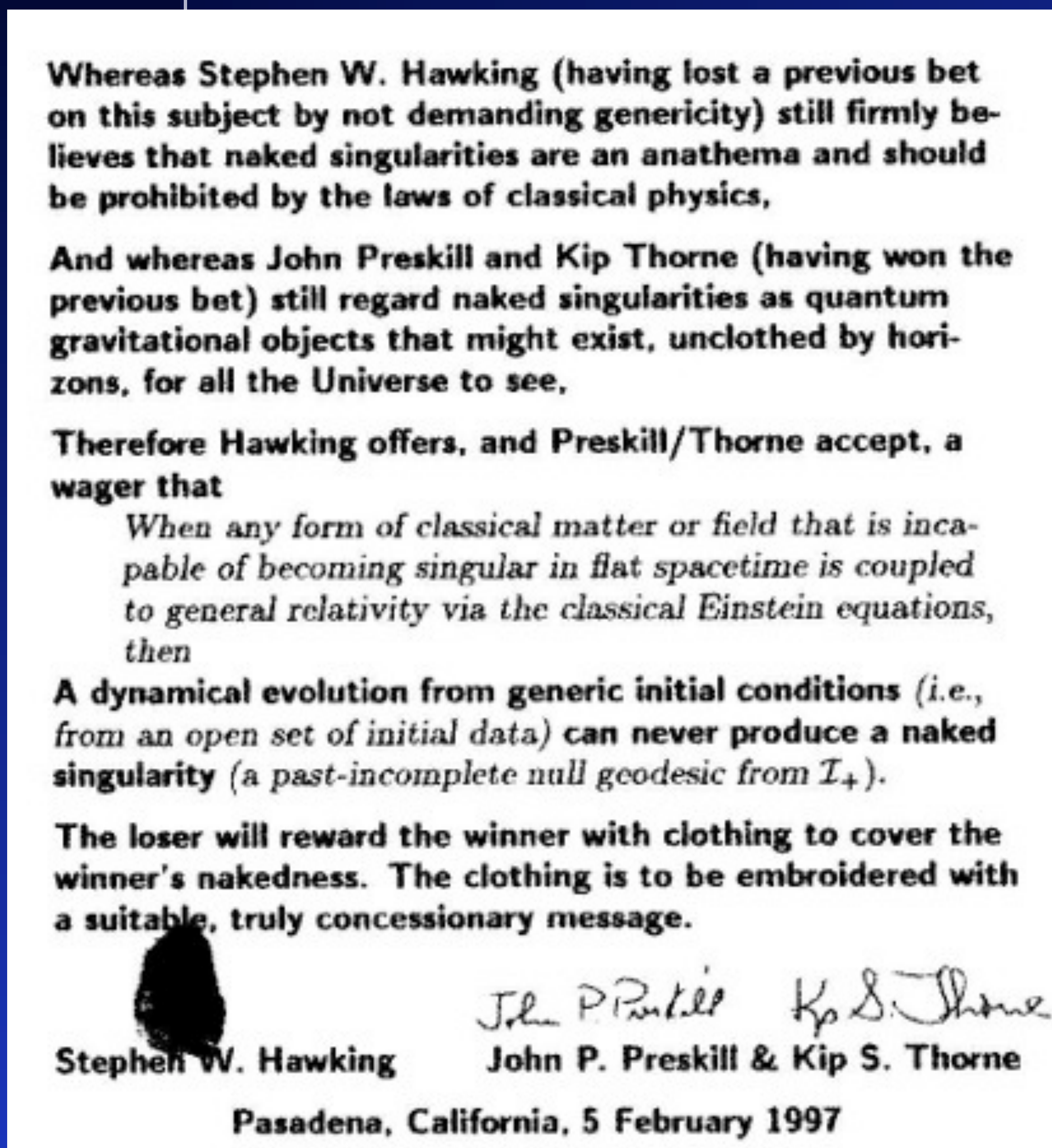
ソーン，プレスキル

「あり得る」

敗者は裸体を覆う着物を勝者に与えること

1991年９月24日

ホーキングとソーンの賭け２



Whereas Stephen W. Hawking (having lost a previous bet on this subject by not demanding genericity) still firmly believes that naked singularities are an anathema and should be prohibited by the laws of classical physics,

And whereas John Preskill and Kip Thorne (having won the previous bet) still regard naked singularities as quantum gravitational objects that might exist, unclothed by horizons, for all the Universe to see,

Therefore Hawking offers, and Preskill/Thorne accept, a wager that

When any form of classical matter or field that is incapable of becoming singular in flat spacetime is coupled to general relativity via the classical Einstein equations, then

A dynamical evolution from generic initial conditions *(i.e., from an open set of initial data)* can never produce a naked singularity *(a past-incomplete null geodesic from $\mathcal{I}_+$)*.

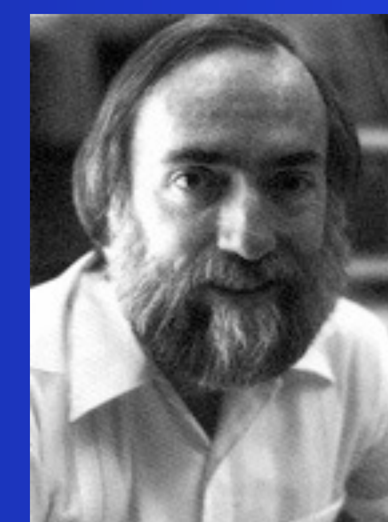
The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable, truly concessionary message.

Stephen W. Hawking　　John P. Preskill & Kip S. Thorne

Pasadena, California, 5 February 1997

ホーキング

「一般的な初期条件では，裸の特異点は発生しない」



ソーン，プレスキル

「あり得る」

敗者は裸体を覆う着物を勝者に与え，その着物には敗北を認める文章を入れること．

1997年2月5日

3. Spheroidal matter collapse
C. Evolution examples (5D, ours)

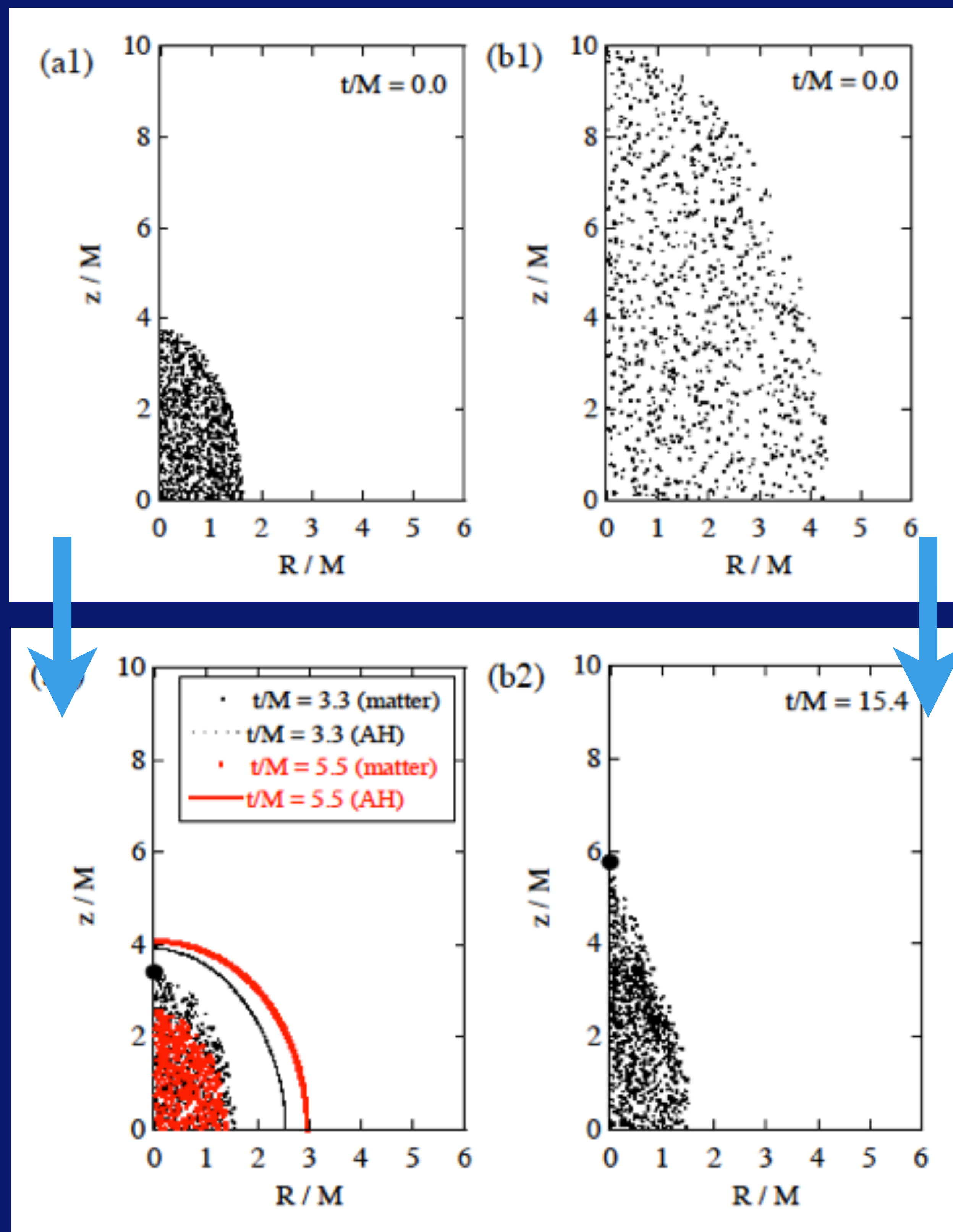


FIG. 2: Snapshots of 5D axisymmetric evolution with the initial matter distribution of $b/M = 4$ [Fig.(a1) and (a2); model 5DSβ in Table I] and 10 [Fig.(b1) and (b2); model 5DSδ]. We see the apparent horizon (AH) is formed at the coordinate time $t/M = 3.3$ for the former model and the area of AH increases, while AH is not observed for the latter model up to the time $t/M = 15.4$ when our code stops due to the large curvature. The big circle indicates the location of the maximum Kretschmann invariant $\mathcal{I}_{\max}$ at the final time at each evolution. Number of particles are reduced to 1/10 for figures.

$$R_{ijkl}R^{ijkl}$$

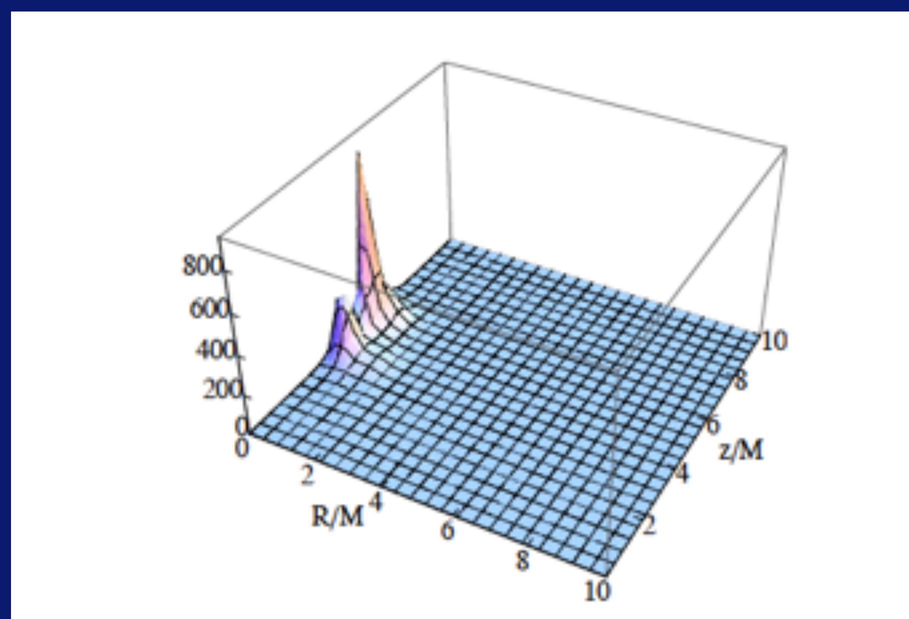
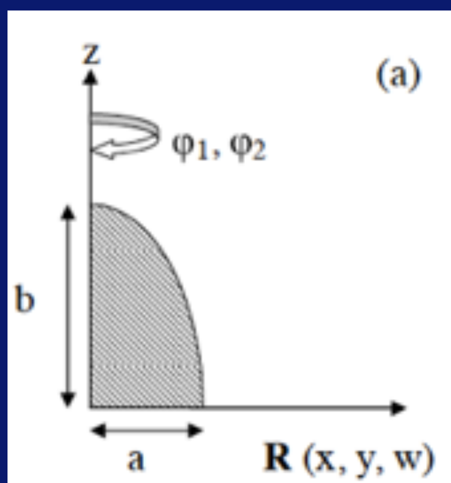
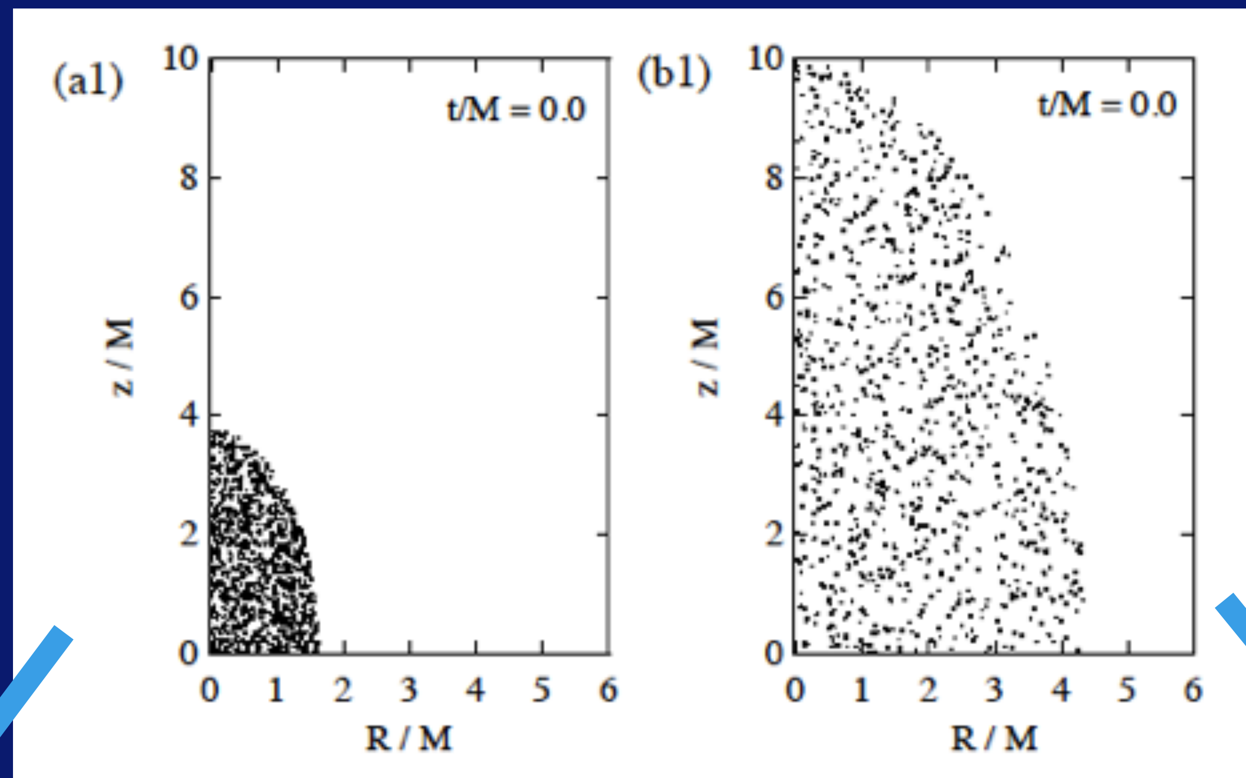


FIG. 3: Kretschmann invariant $\mathcal{I}$ for model 5DSδ at $t/M = 15.4$. The maximum is $O(1000)$, and its location is on z-axis, just outside of the matter.

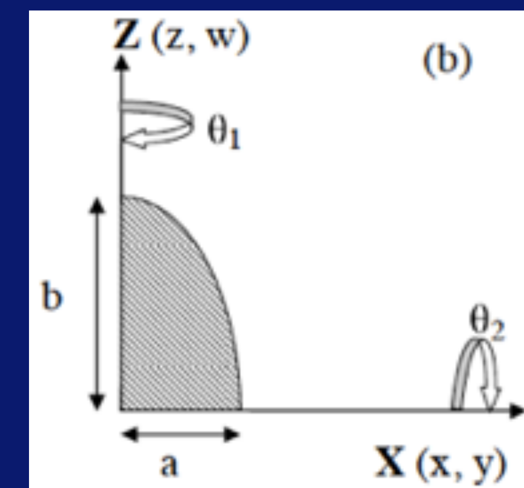
3. Spheroidal matter collapse
C. Evolution examples (5D, ours)

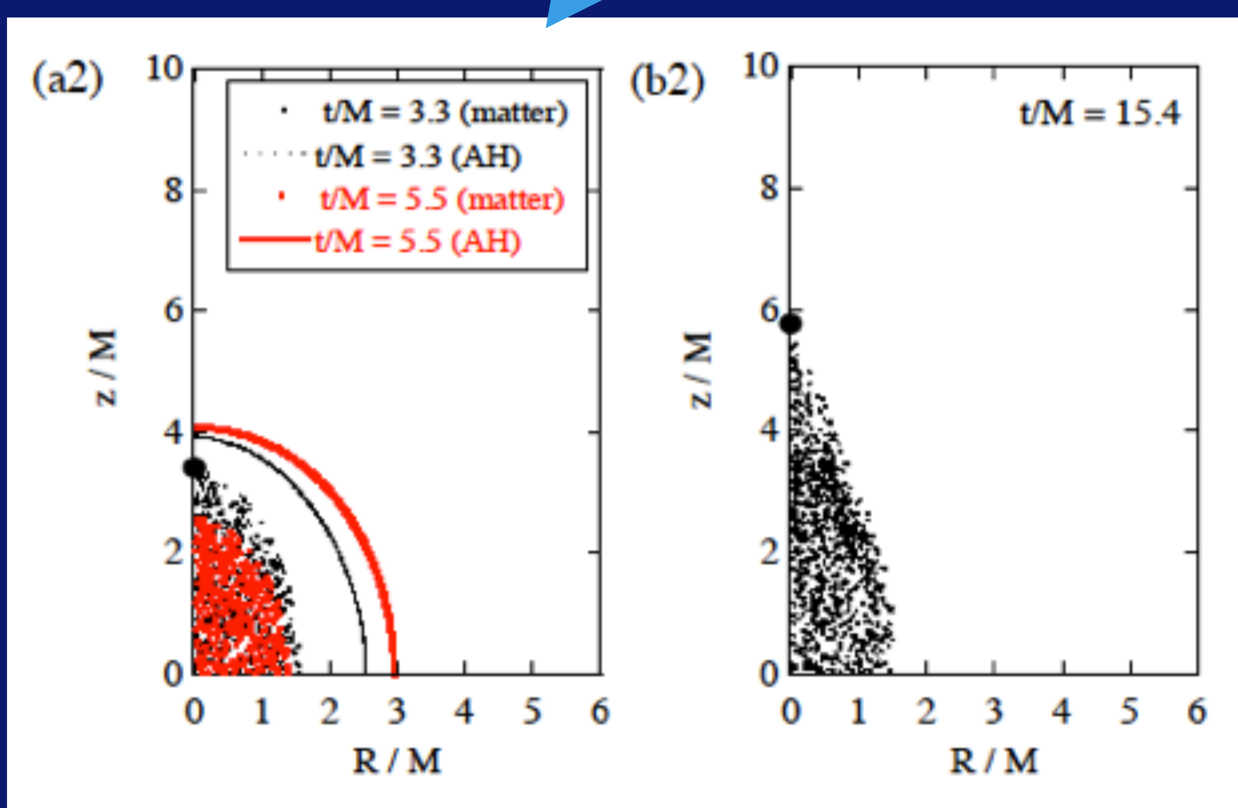
$SO(3)$

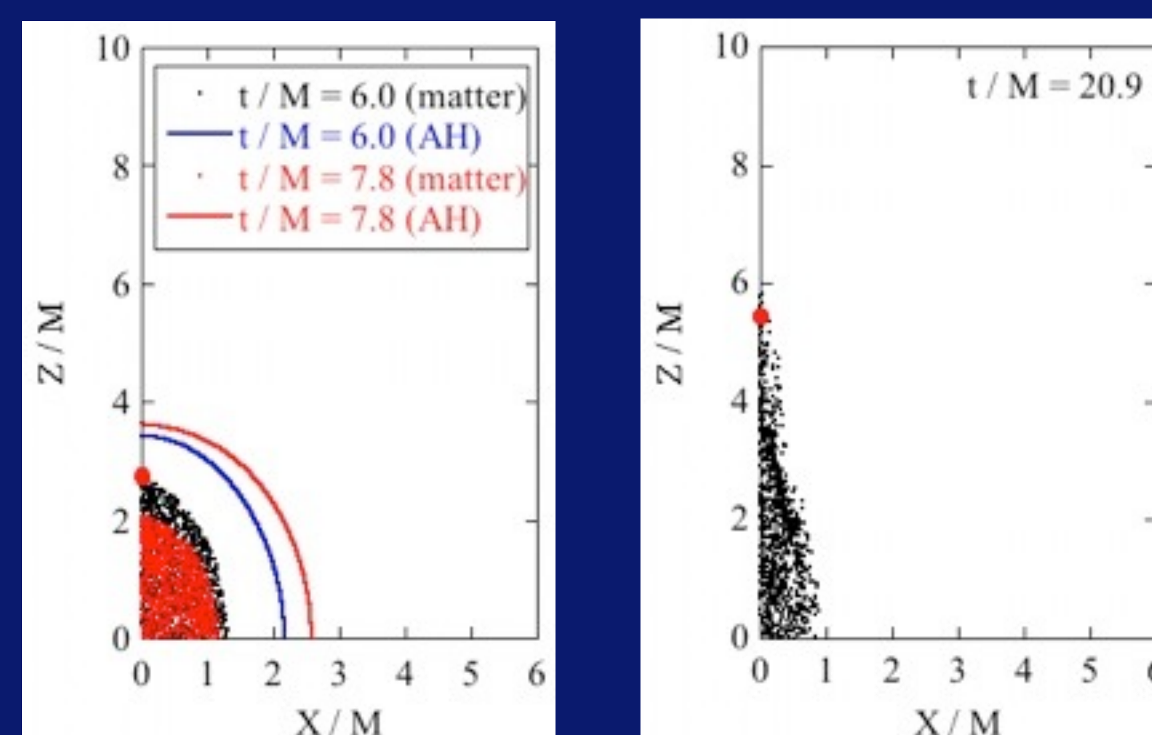




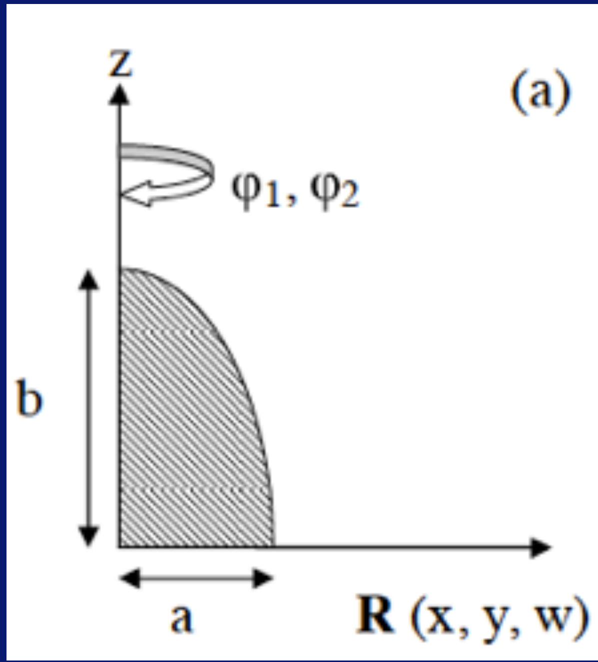
$U(1) \times U(1)$







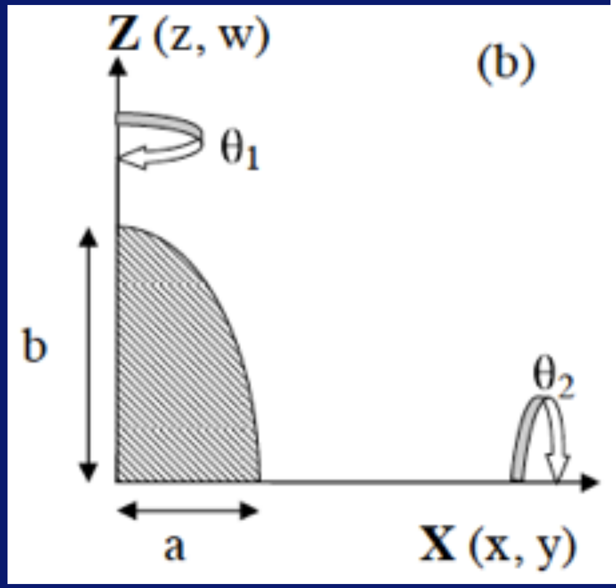
3. Spheroidal matter collapse
D. Comparisons 4D vs. 5D



b/M $(t=0)$	2.50	4.00	6.25	10.00
4D axisym.	4Dα AH-formed $e_{\rm AH} = 0.90$ $e_{\rm f} = 0.92$	4Dβ no $e_{\rm f} = 0.89$	4Dγ no $e_{\rm f} = 0.92$	4Dδ no $e_{\rm f} = 0.96$
5D axisym. SO(3)	5DSα AH-formed $e_{\rm AH} = 0.88$ $e_{\rm f} = 0.82$	5DSβ AH-formed $e_{\rm AH} = 0.88$ $e_{\rm f} = 0.84$	5DSγ no $e_{\rm f} = 0.88$	5DSδ no $e_{\rm f} = 0.96$

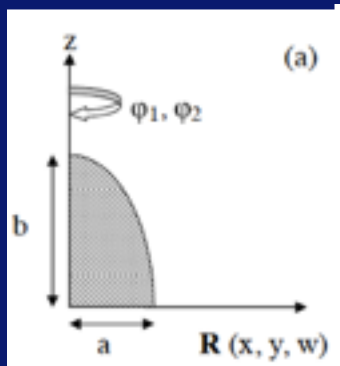
towards spindle

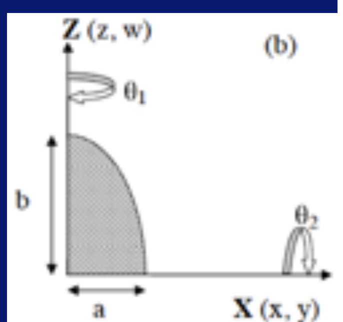
towards spherical



towards spherical towards spindle

3. Spheroidal matter collapse
D. Comparisons 4D vs. 5D



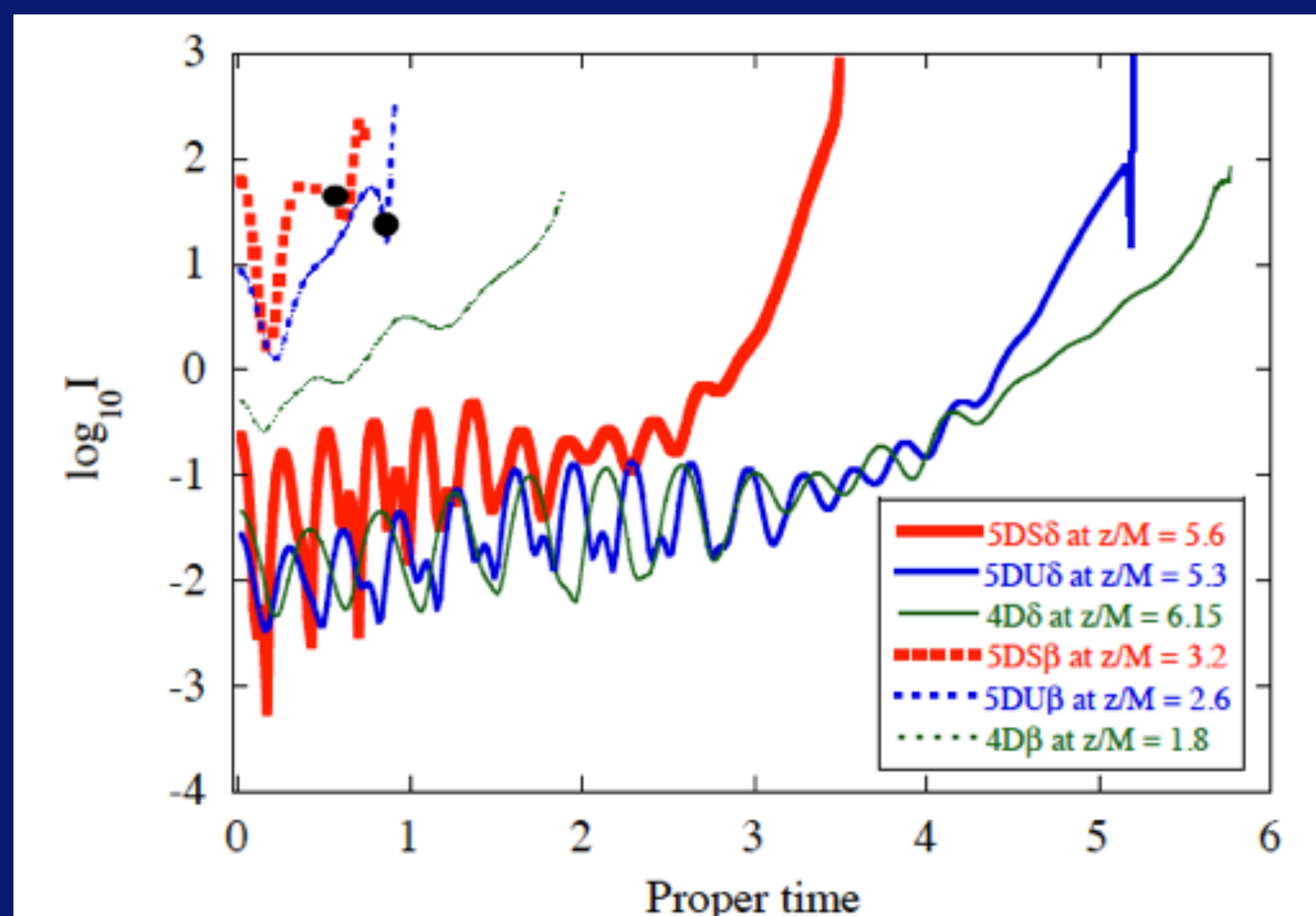


b/M $(t=0)$	2.50	4.00	6.25	10.00
4D axisym.	4Dα AH-formed $e_{\rm AH}=0.90$ $e_{\rm f}=0.92$	4Dβ no $e_{\rm f}=0.89$	4Dγ no $e_{\rm f}=0.92$	4Dδ no $e_{\rm f}=0.96$
5D axisym. SO(3)	5DSα AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.82$	5DSβ AH-formed $e_{\rm AH}=0.88$ $e_{\rm f}=0.84$	5DSγ no $e_{\rm f}=0.88$	5DSδ no $e_{\rm f}=0.96$
5D double axisym. U(1)×U(1)	5DUα AH-formed $e_{\rm AH}=0.86$ $e_{\rm f}=0.79$	5DUβ AH-formed $e_{\rm AH}=0.87$ $e_{\rm f}=0.81$	5DUγ AH-formed $e_{\rm AH}=0.92$ $e_{\rm f}=0.90$	5DUδ no $e_{\rm f}=0.98$

5D collapses
- proceed rapidly.
- towards spherical.
- AH forms in wider ranges.

$$I = R_{abcd}R^{abcd}$$

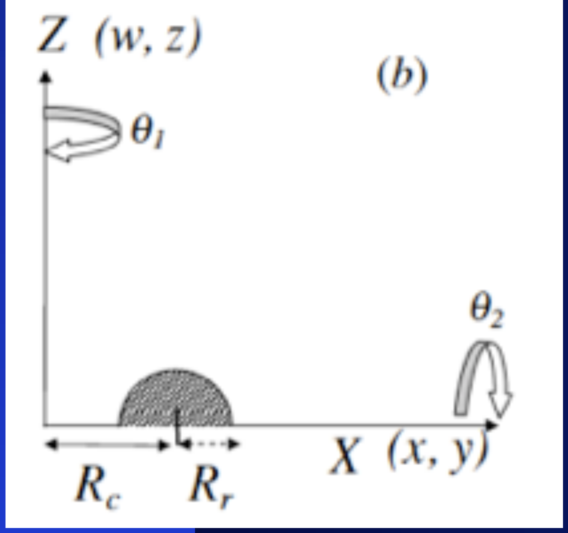
at $I(t_{\rm end})$



4. Ring matter collapse
A. Initial data construction

- time symmetric, asymptotically flat
- conformal flat
- non-rotating homogeneous dust

- solve the Hamiltonian constraint eq. 512^2 grids
- Apparent Horizon Search
  both for Ring Horizon and Common Horizon
- Define Hoop and check the Hoop Conjecture



$$ds^2 = \psi(X,Z)^2(dX^2 + dZ^2 + X^2 d\vartheta_1 + Z^2 d\vartheta_2)$$

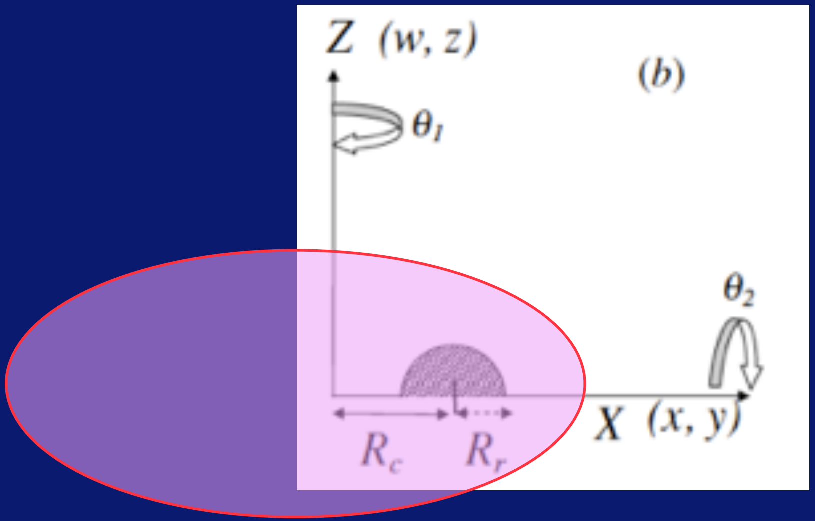
$$X = \sqrt{x^2+y^2},\ \ Z = \sqrt{z^2+w^2},\ \ \vartheta_1 = \tan^{-1}\left(\frac{y}{x}\right),\ \vartheta_2 = \tan^{-1}\left(\frac{z}{w}\right)$$

$$\frac{1}{X}\frac{\partial}{\partial X}\left(X\frac{\partial\psi}{\partial X}\right) + \frac{1}{Z}\frac{\partial}{\partial Z}\left(Z\frac{\partial\psi}{\partial Z}\right) = -4\pi^2 G_5 \rho.$$

4. Ring matter collapse

A. *Apparent Horizon Search, and its Area*

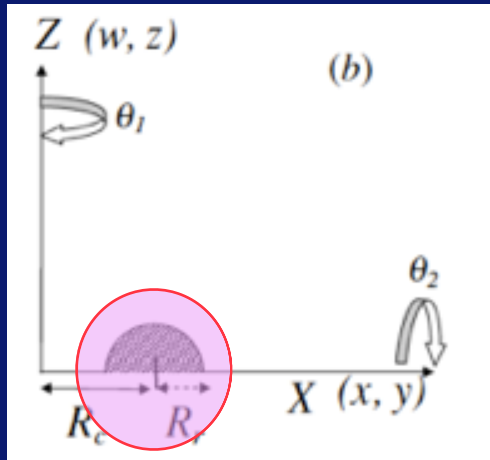
Common Horizon



$$\ddot{r}_m - 4\frac{\dot{r}_m^{\,2}}{r_m} - 3r_m - \frac{r_m^2+\dot{r}_m^{\,2}}{r_m}\left[2\frac{\dot{r}_m}{r_m}\cot(2\phi) - \frac{3}{\psi}(\dot{r}_m\sin\phi + r\cos\phi)\frac{\partial\psi}{\partial X} + \frac{3}{\psi}(\dot{r}_m\cos\phi - r_m\sin\phi)\frac{\partial\psi}{\partial Z}\right] = 0$$

$$A_3^{(T1)} = 4\pi^2\int_0^{\pi/2}\psi^3 r_m^2\cos\phi\sin\phi\sqrt{\dot{r}_m^{\,2}+r_m^2}\,d\phi$$

Ring Horizon

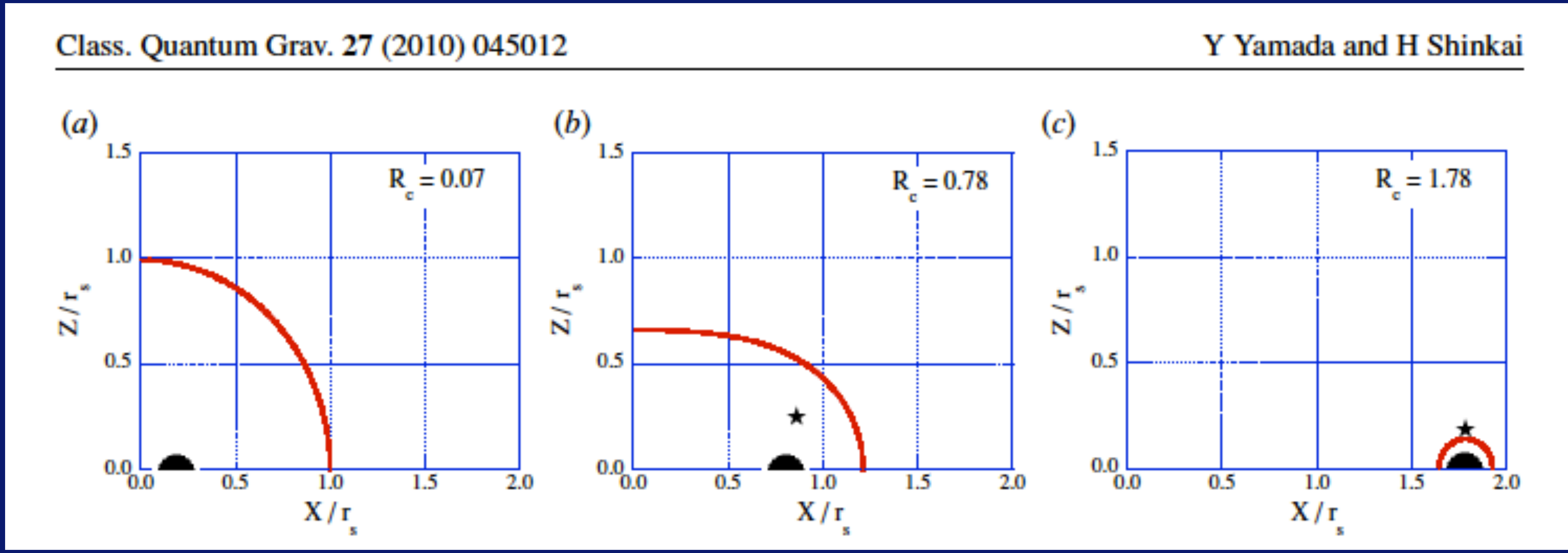


$$\ddot{r}_m - \frac{3\dot{r}_m^{\,2}}{r_m} - 2r_m - \frac{r_m^2+\dot{r}_m^{\,2}}{r_m}\times\left[\frac{\dot{r}_m\sin\xi + r_m\cos\xi}{r_m\cos\xi + R_c} - \frac{\dot{r}_m}{r_m}\cot\xi + \frac{3}{\psi}(\dot{r}_m\sin\xi + r\cos\xi)\frac{\partial\psi}{\partial x} - \frac{3}{\psi}(\dot{r}_m\cos\xi - r\sin\xi)\frac{\partial\psi}{\partial z}\right] = 0$$

$$A_3^{(T2)} = 4\pi^2\int_0^{\pi}\psi^3(R_c + r_m\cos\xi)r_m\sin\xi\sqrt{\dot{r}_m^{\,2}+r_m^2}\,d\xi$$

4. Ring matter collapse
B. Initial data sequence

Apparent Horizons Search



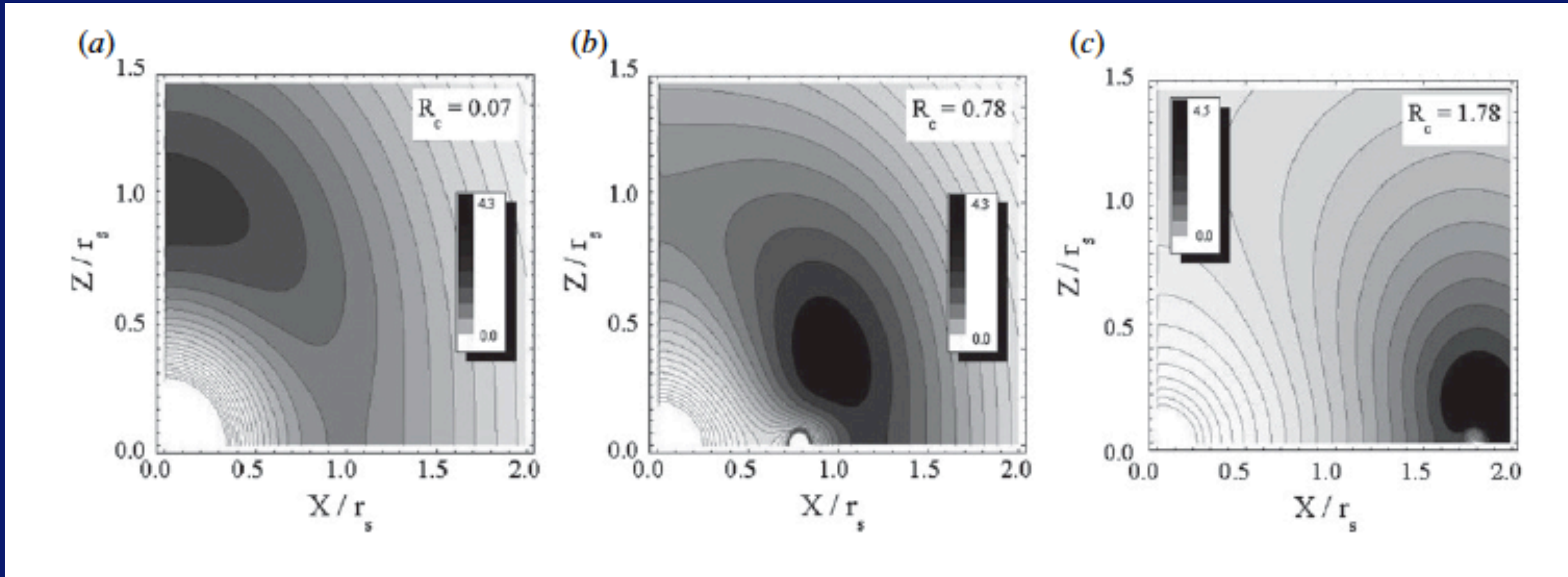
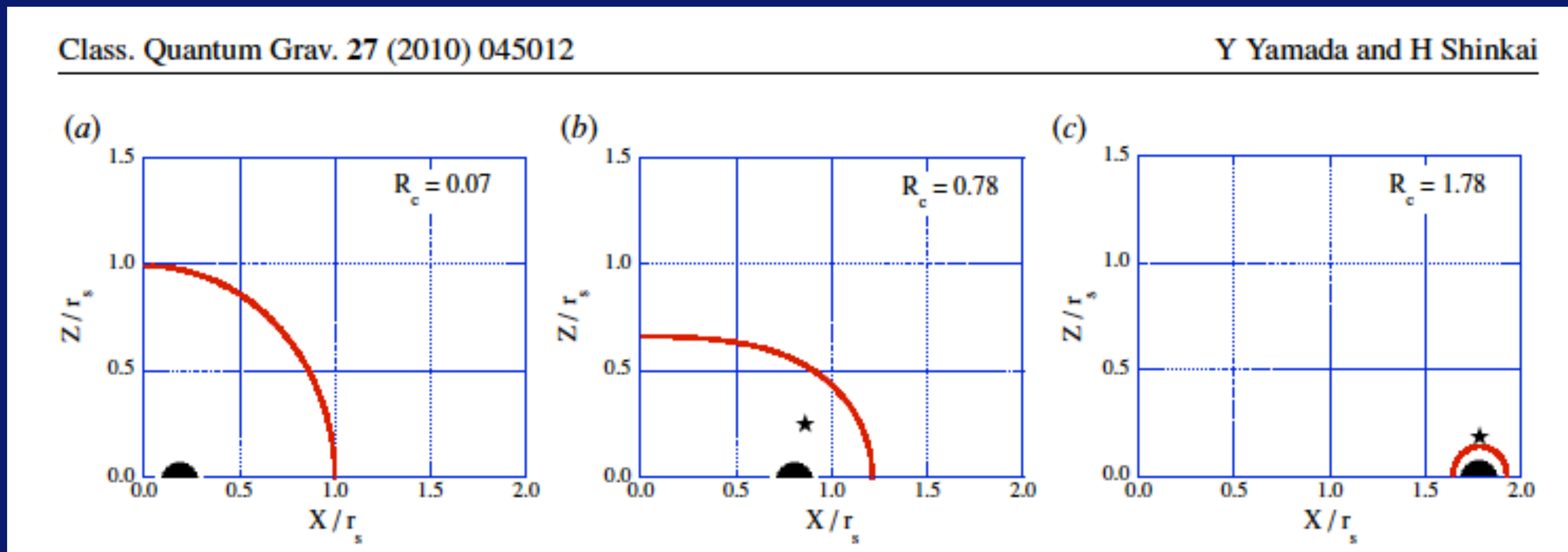
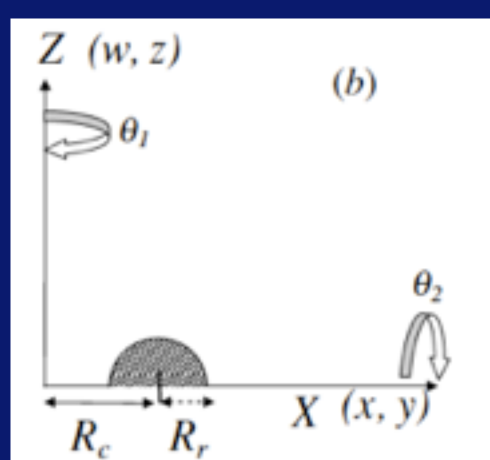


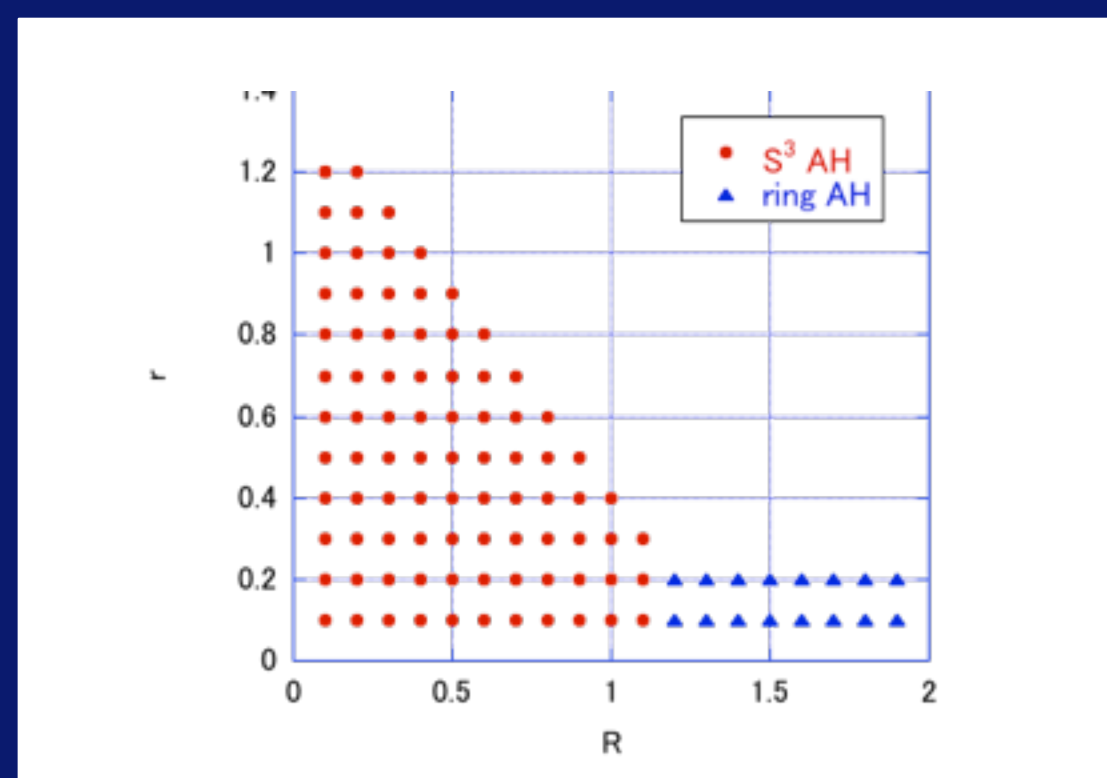
Figure 8. Contours of Kretchmann invariant, $\log_{10} \mathcal{I}^{(4)}$, corresponding to figure 7.

4. Ring matter collapse
B. Initial data sequence

Apparent Horizons Search

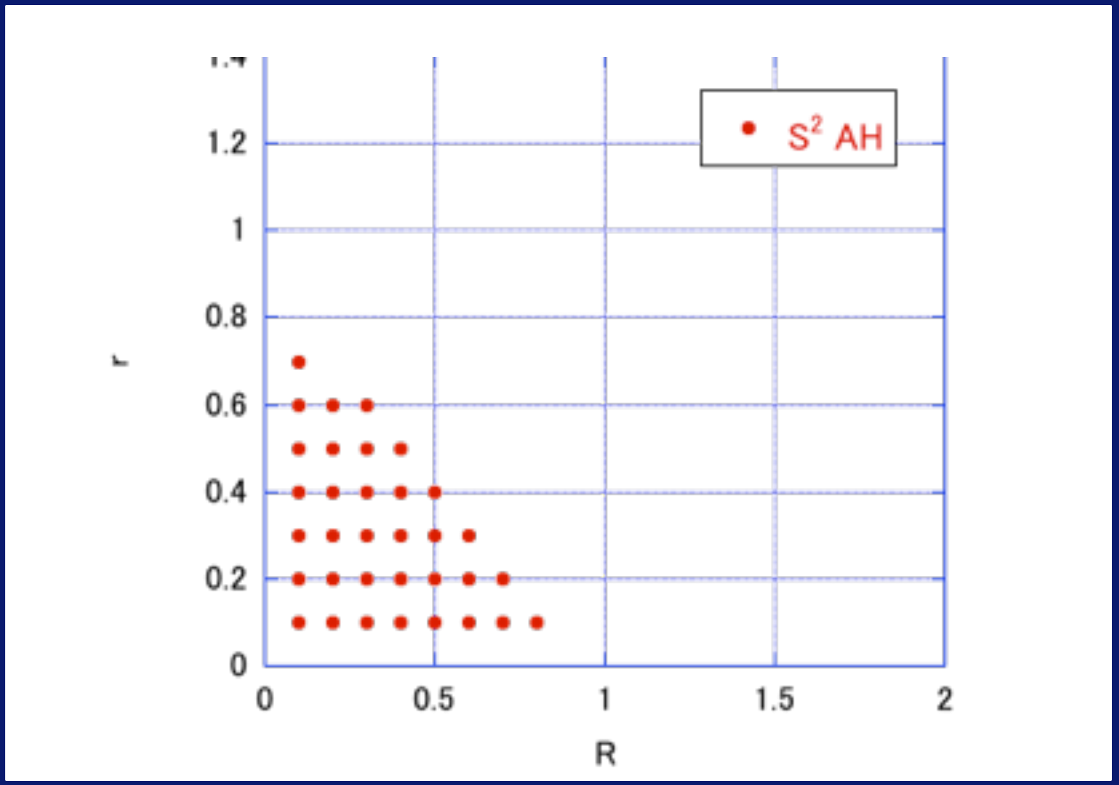




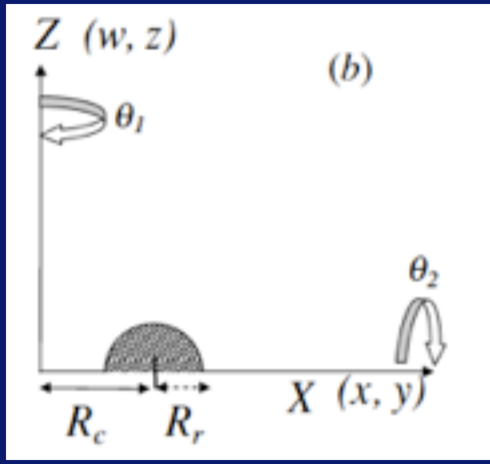


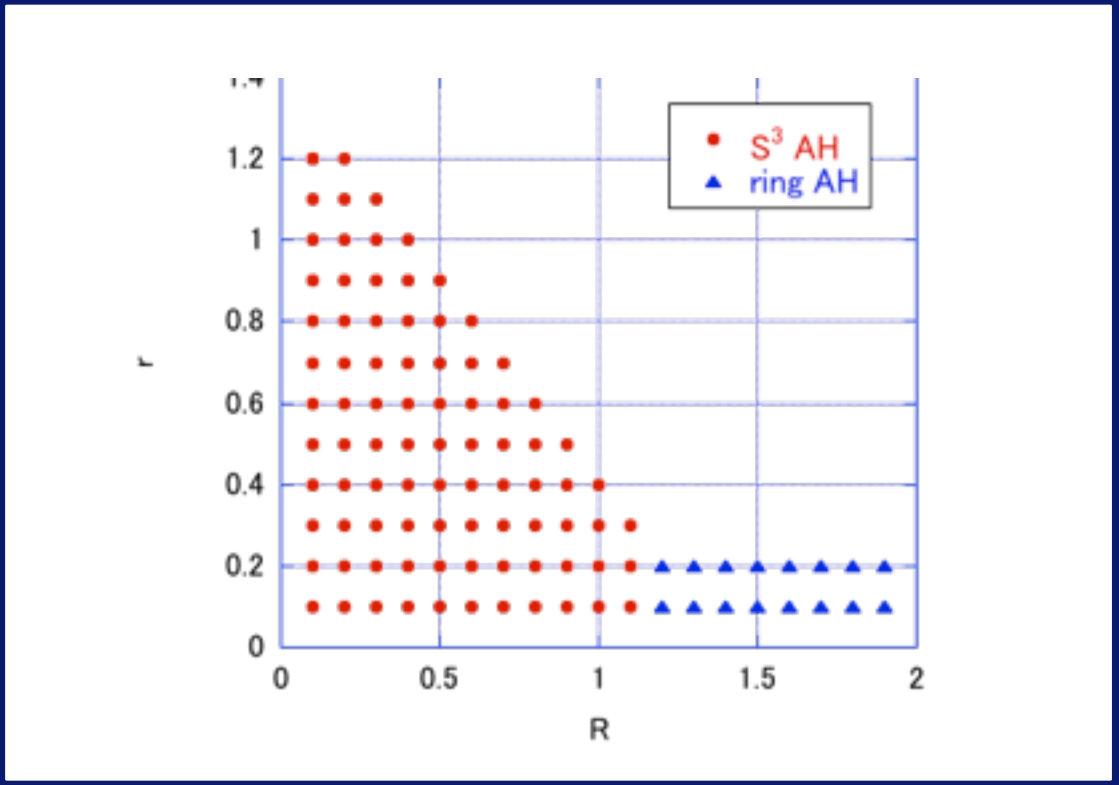
By the way,

there might not be a ring horizon in 4D



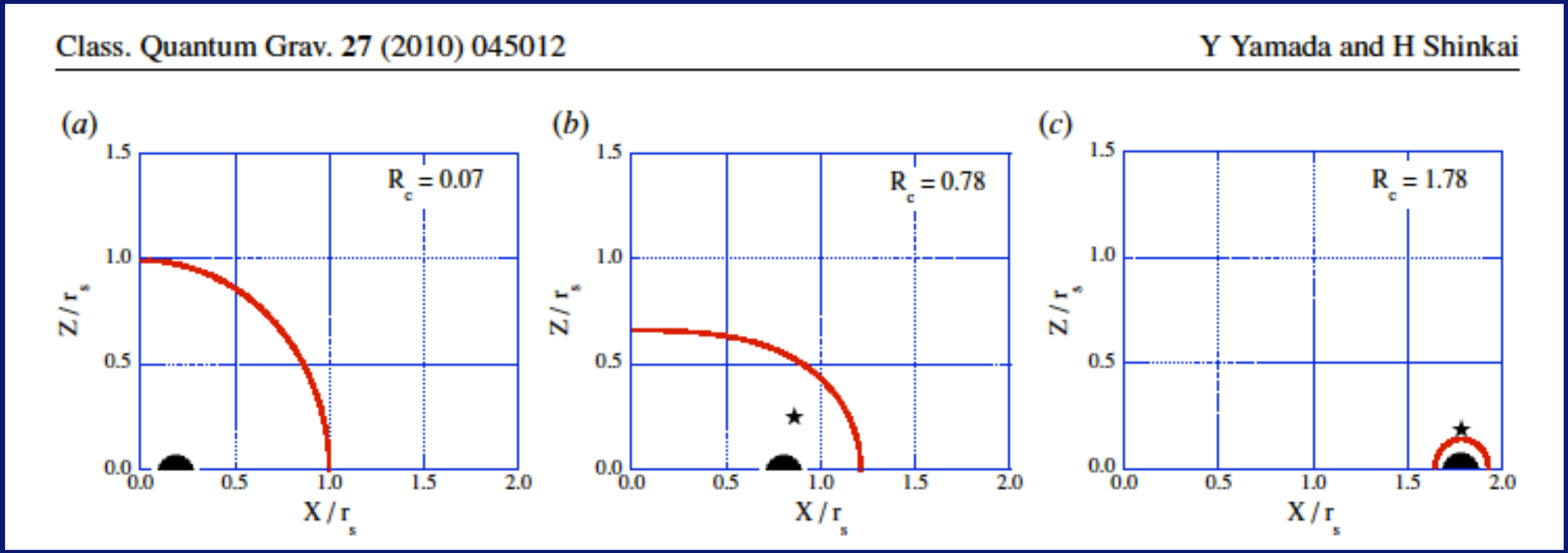
4D



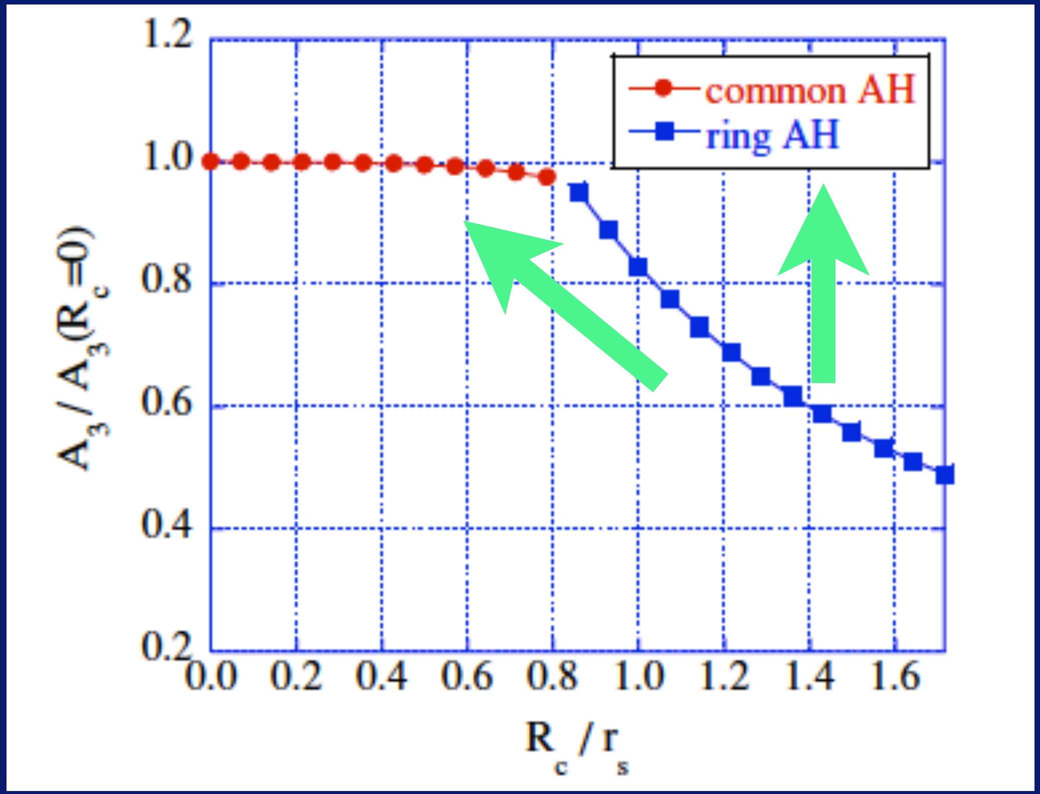


5D

4. Ring matter collapse
B. Initial data sequence



Area of Apparent Horizon



if evolved,

4. Ring matter collapse
C. Evolution method

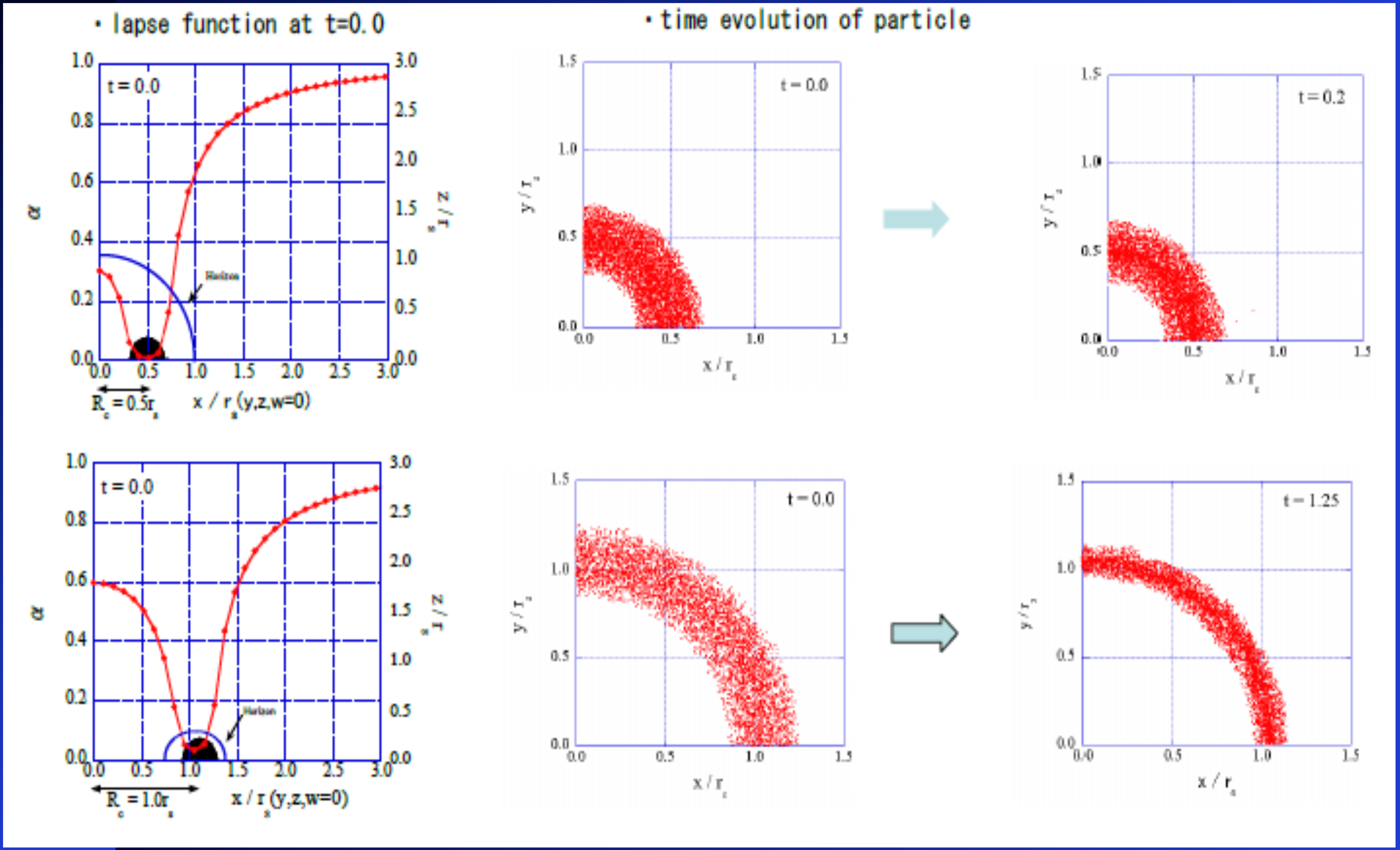
- ADM full 4+1,   ADM 2+1 Double Axisym Cartoon
- 33^4 grids, 130^2 x 2^2 grids

- lapse function: Maximal slicing condition
- shift vectors: zero
- asymptotically flat

- Collisionless Particles (5000)
- the same total mass
- no rotation

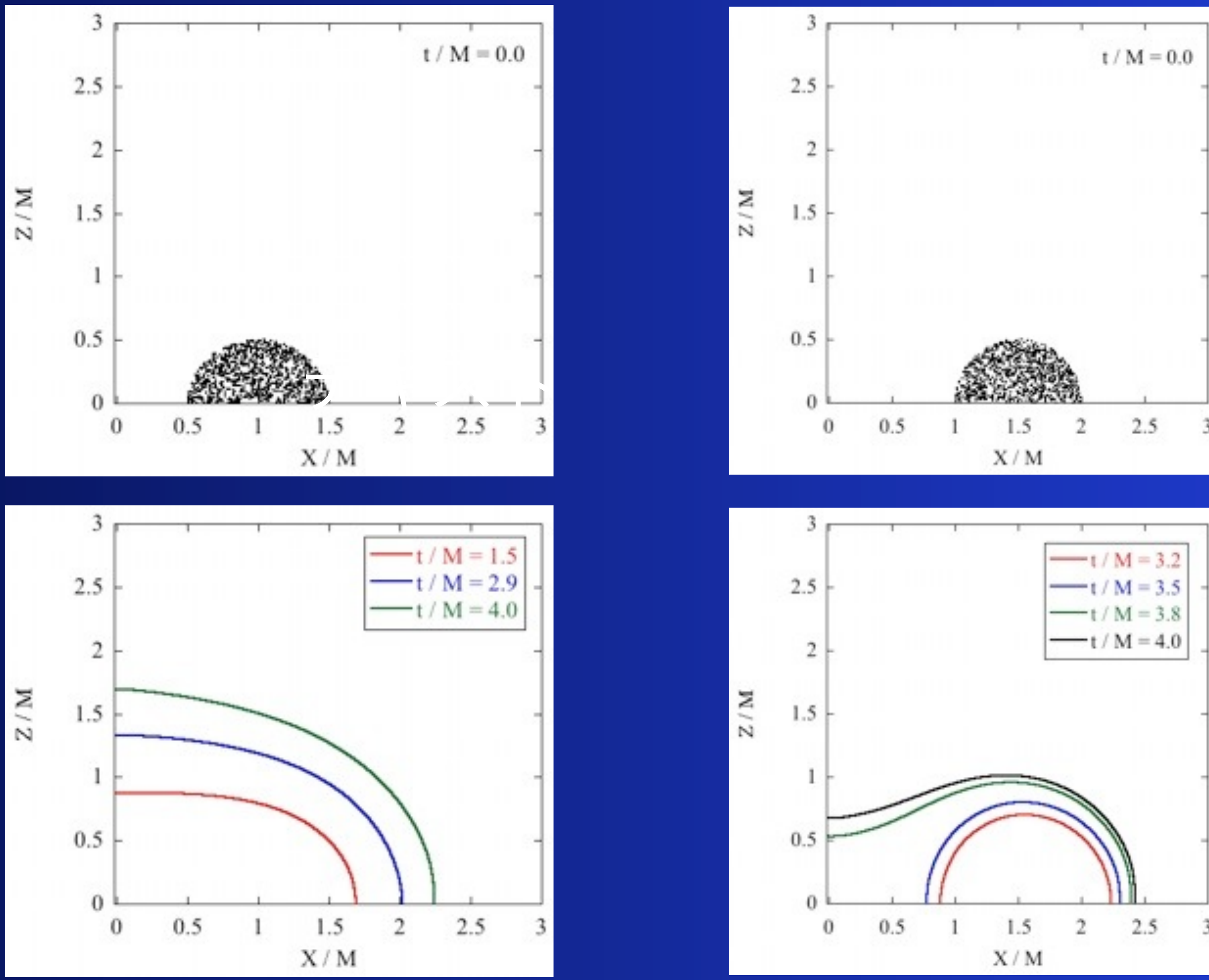
- Apparent Horizon Search
  both for Ring Horizon and Common Horizon

4. Ring matter collapse
D. Evolution examples



4. Ring matter collapse
D. Evolution examples

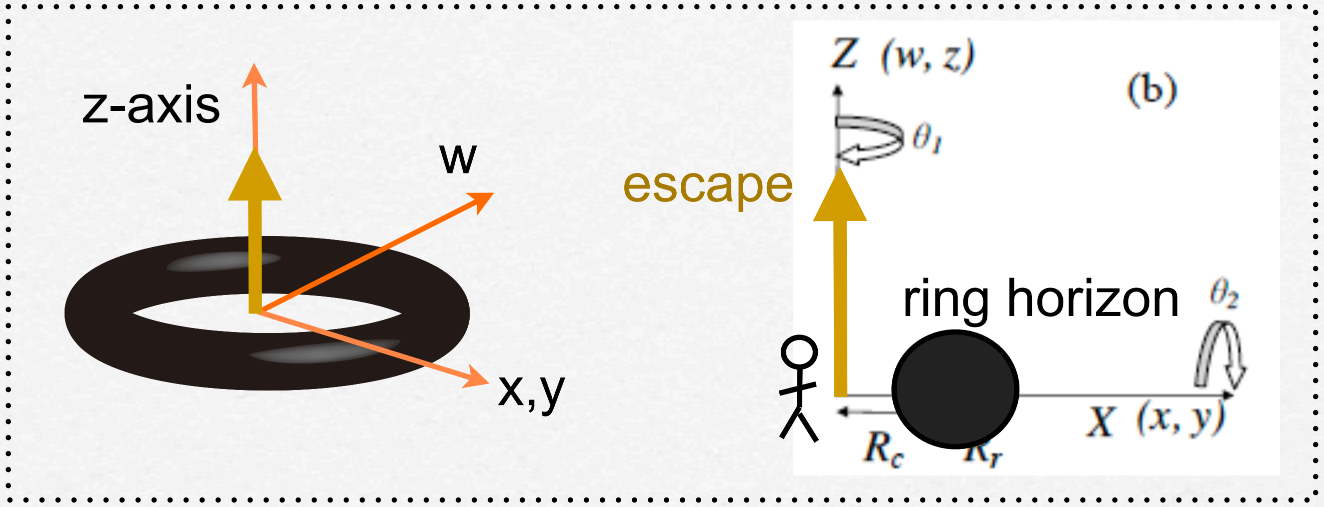
t=0 No Horizon

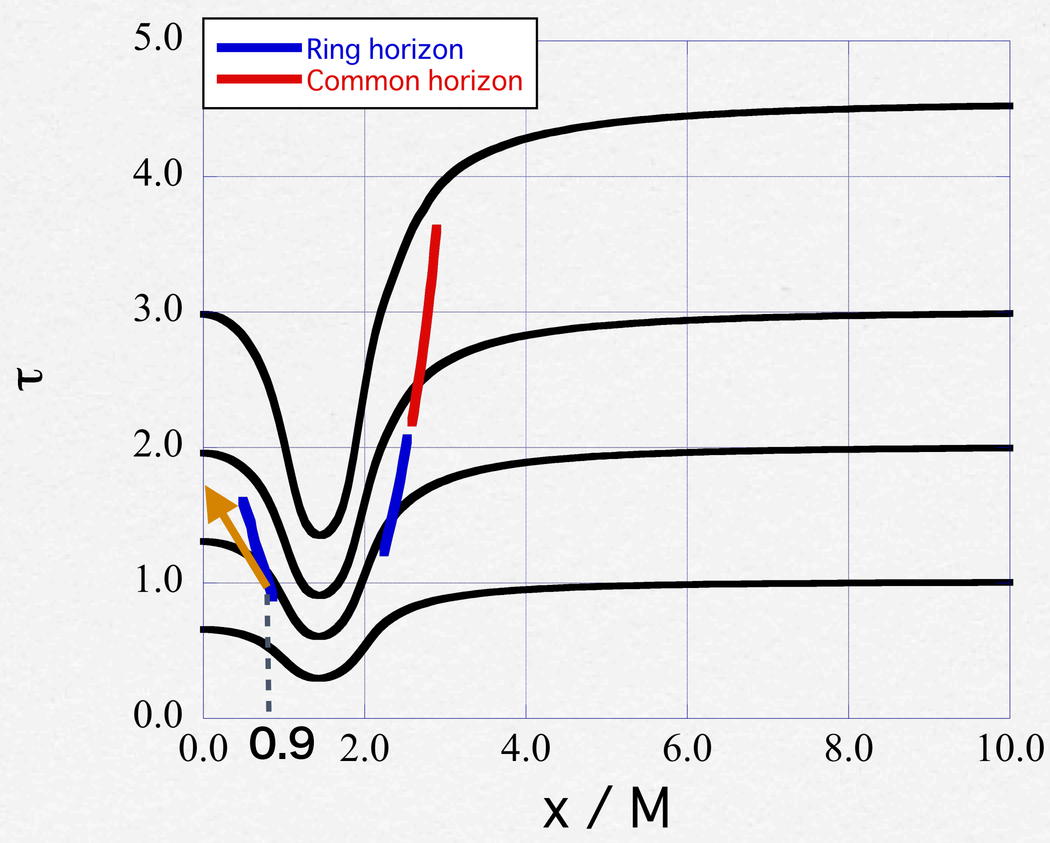


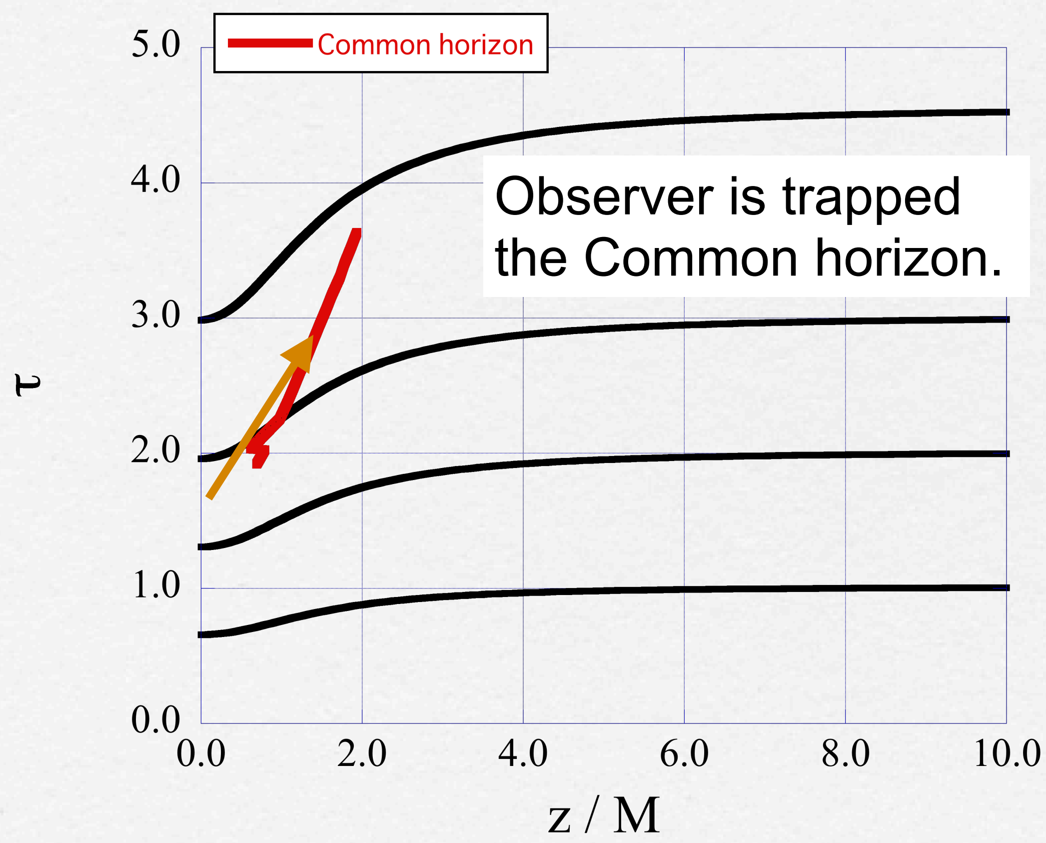
t=1.5 Common Horizon

t=3.2 Ring Horizon
t=3.8 Common Horizon

Is it possible to escape from origin after the observer watched the ring horizon?







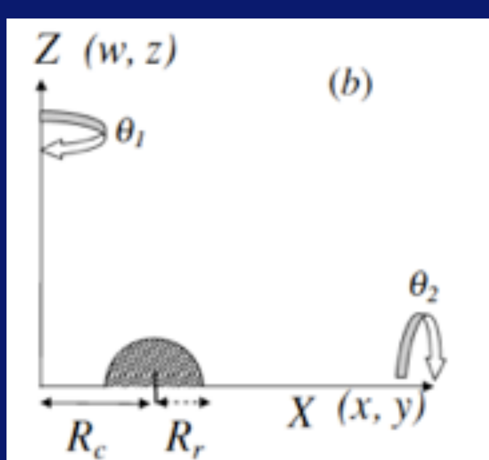
The snapshots of the hypersurfaces on the x and z axis in the propertime versus coordinate diagram.

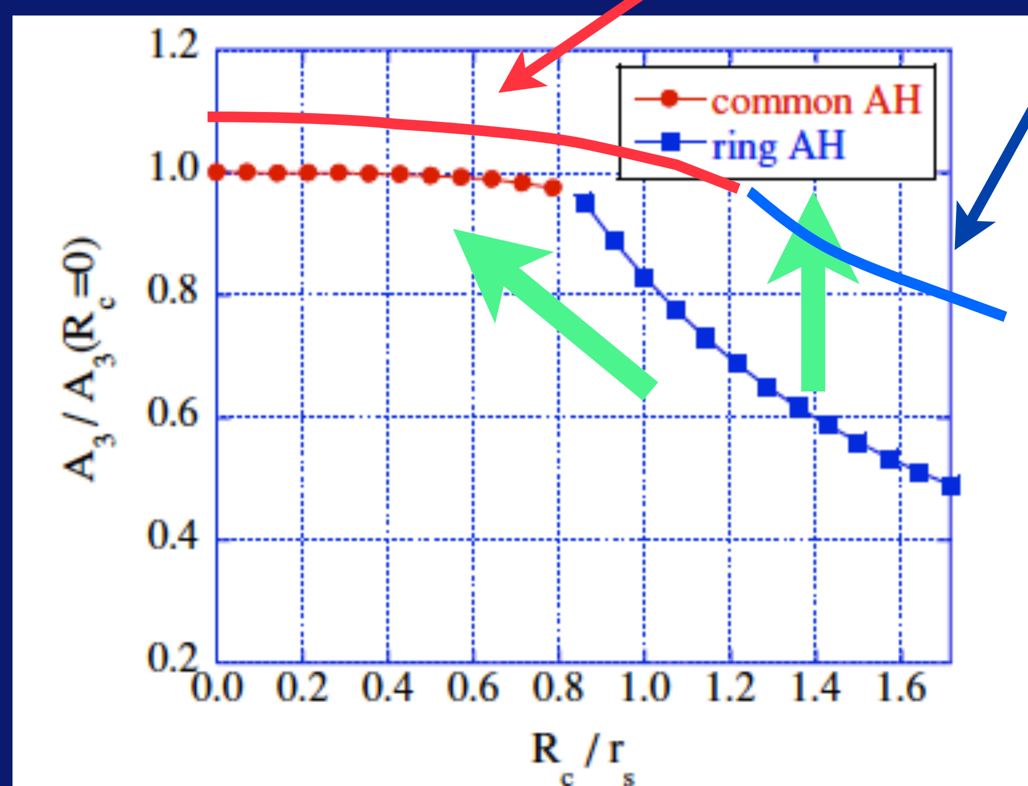
4. Ring matter collapse
E. Interpretation

Area of
Apparent Horizon

expected line
after evolution

if evolved,





with rotation ?

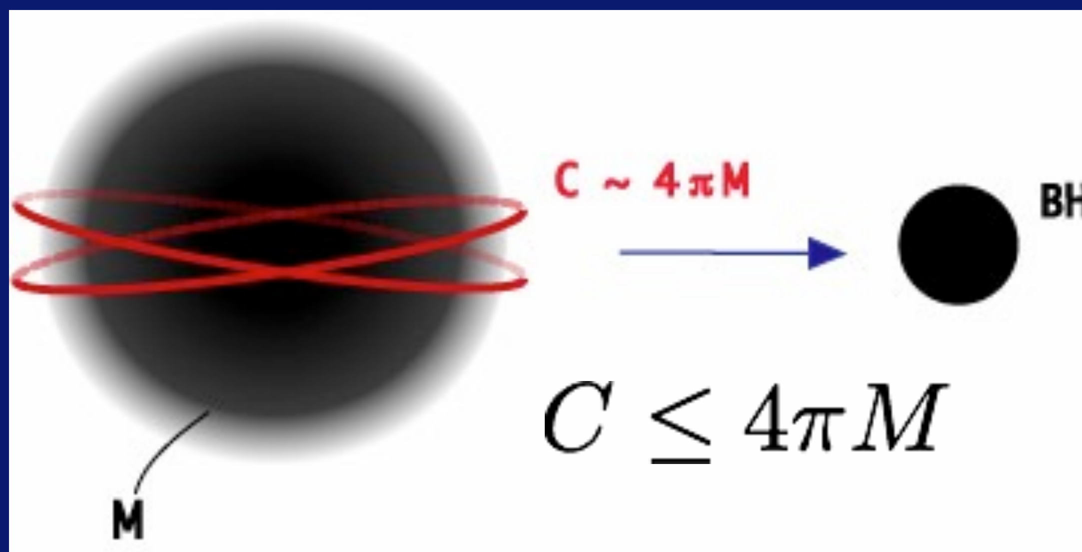
with perturbation?

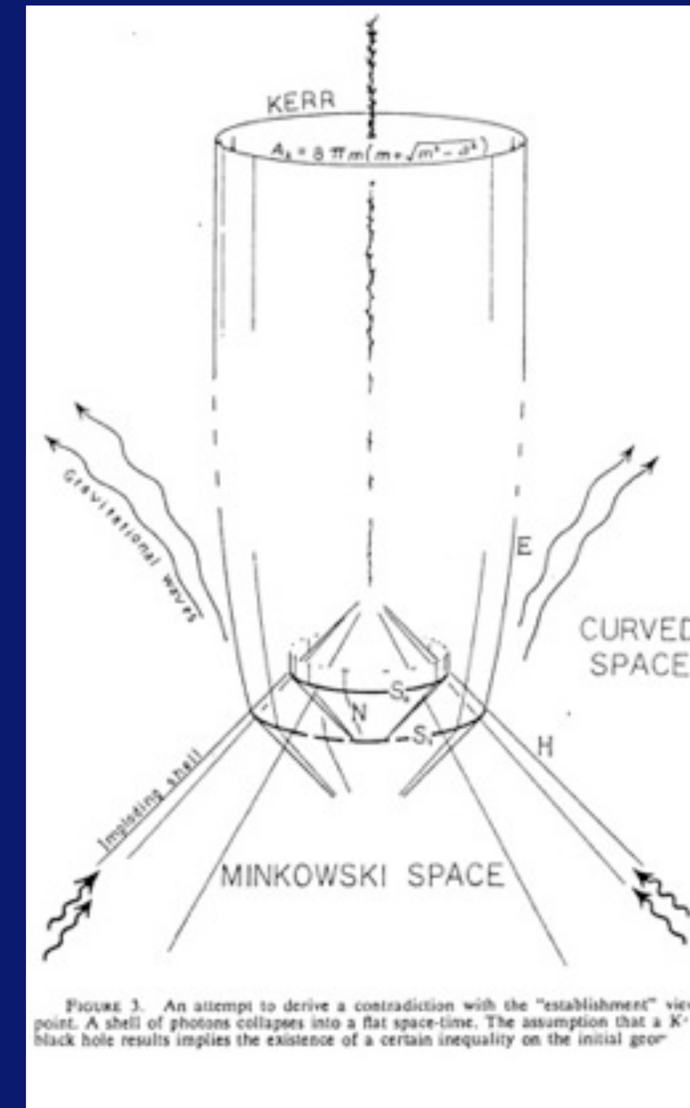
-- under investigation.

5. Hoop Conjecture

A. Hyper-Hoop conjecture ?

Hoop Conjecture Thorne (1972)





Penrose (1969)

$$A \leq 16\pi M^2$$

Hyper-Hoop Conjecture

Ida-Nakao (2002)

$$V_{D-3} \leq G_D M$$

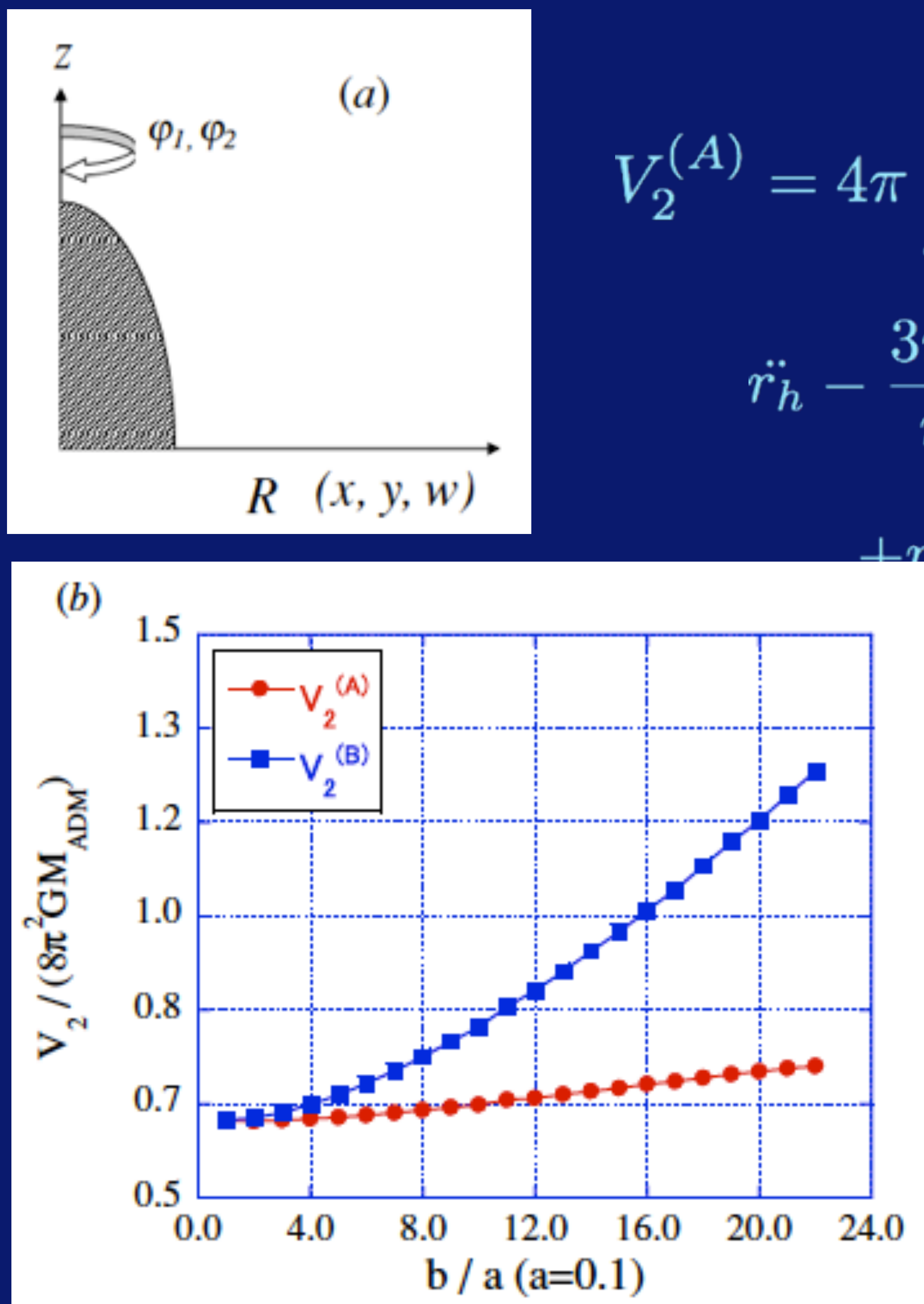
In 5-D, if mass gets compacted in some area, ...

5. Hoop Conjecture

B. Spheroidal Cases

$$V_2 \leq \frac{\pi}{2} 16\pi G_5 M$$

Define Hyper-Hoop as the surface $\delta V_2 = 0$



$$V_2^{(A)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \sin\theta \, d\theta$$

$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h + \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{\dot{r_h}}{r_h}\cot\theta - \frac{2}{\psi}(\dot{r_h}\sin\theta + r_h\cos\theta)\frac{\partial\psi}{\partial z} - \frac{2}{\psi}(r_h\sin\theta - \dot{r_h}\cos\theta)\frac{\partial\psi}{\partial R}\right] = 0$$

$$\int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \cos\theta \, d\theta$$

$$\frac{\dot{r_h}^2}{r_h} - 2r_h \ldots \tan\theta + \frac{2}{\psi}(r_h\sin\theta \ldots \dot{r_h}\cos\theta)\frac{\partial\psi}{\partial R} + \frac{2}{\psi}(r_h\cos\theta + \dot{r_h}\sin\theta)\frac{\partial\psi}{\partial z}\Big] = 0$$

Hyper-Hoop $V_2^{(A)}$ does work for spheroidal horizons.

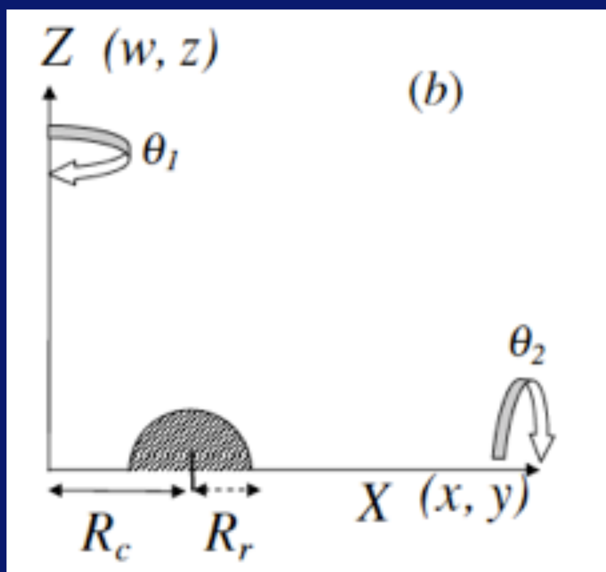
5. Hoop Conjecture

C. Toroidal Cases

$$V_2 \leq \frac{\pi}{2} 16\pi G_5 M$$

$$V_2^{(C)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \cos\phi \, d\phi$$

$$\ddot{r_h} - \frac{3\dot{r_h}^2}{r_h} - 2r_h + \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{\dot{r_h}}{r_h}\cot\phi - \frac{2}{\psi}(\dot{r_h}\sin\phi + r_h\cos\phi)\frac{\partial\psi}{\partial X} - \frac{2}{\psi}(r_h\sin\phi - \dot{r_h}\cos\phi)\frac{\partial\psi}{\partial Z}\right] = 0$$

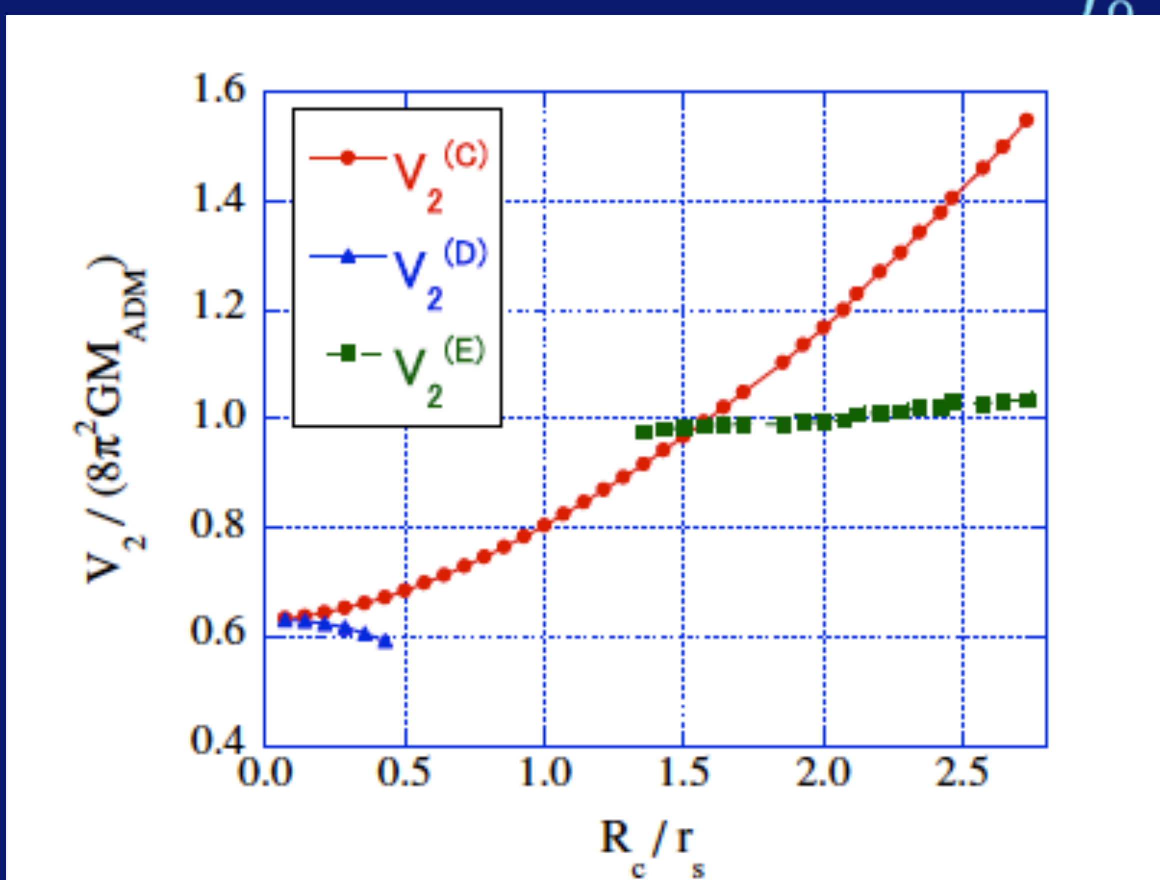


$$V_2^{(D)} = 4\pi \int_0^{\pi/2} \psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\, r_h \sin\phi \, d\phi$$

$$- 2r_h - \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{\dot{r_h}}{r_h}\tan\phi + \frac{2}{\psi}(r_h\sin\phi \ldots s\phi)\frac{\partial\psi}{\partial X} + \frac{2}{\psi}(r_h\cos\phi + \dot{r_h}\sin\phi)\frac{\partial\psi}{\partial Z}\right] = 0$$

$$\psi^2 \sqrt{\dot{r_h}^2 + r_h^2}\,(r_h\cos\xi + R_c)\, d\xi$$

$$- 2r_h - \frac{r_h^2 + \dot{r_h}^2}{r_h}\left[\frac{-R_c + \dot{r_h}\sin\xi}{R_c + r_h\cos\xi} + \frac{2}{\psi}(\dot{r_h}\sin\xi \ldots + r_h\cos\xi)\frac{\partial\psi}{\partial X} + \frac{2}{\psi}(r_h\sin\xi - \dot{r_h}\cos\xi)\frac{\partial\psi}{\partial Z}\right] = 0$$



Hyper-Hoop does not work for ring horizons.

6. Summary and Future Plans

5D vs. 4D Spheroidal collapses (no rotating cases)

Collapse rapidly, towards spherical

Formation of Naked Singularity for highly prolate matter

5D Ring collapses (no rotating cases)

no horizon → common horizon

no horizon ↘ ring horizon ↗ common horizon

Hyper-Hoop prediction for BH formation

works well for formations of spheroidal black holes
but not for rings.

Future Plans:

include rotation, change slicing conditions
search various horizons,
investigate the stability, formation/decay process,....

1. Motivation

- Brane-World models give new viewpoints to gravity and cosmology.

- LHC experiments will (or will not) reveal Higher-Dim BHs in near future.

- Higher-Dim Black Holes (Black Objects) have Rich Structures.

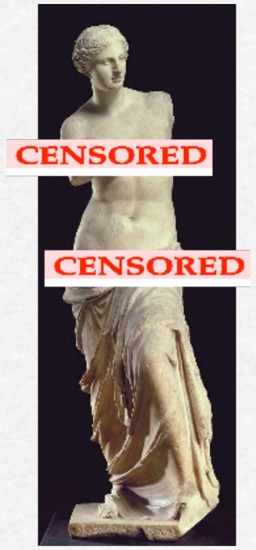
4-dim BHs :		Higher-dim BHs (Black Objects) :
Schwarzschild	→	Schwarzschild-Tangherlini --- unique & stable
Kerr	→	Myers-Perry --- maybe unstable in higher J

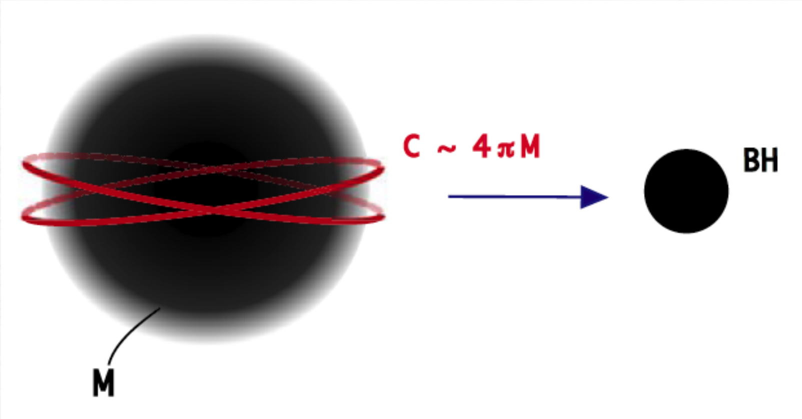

black string
black ring (Emparan-Reall)
black Saturn
di-rings, orthogonal di-rings, ...

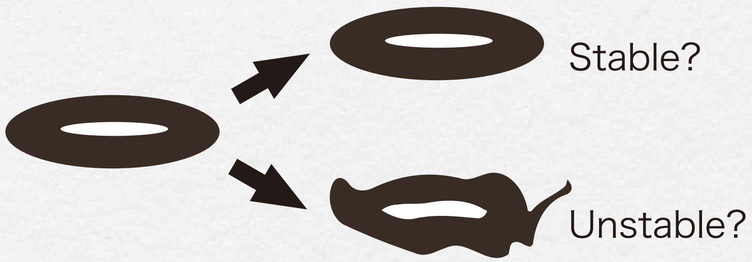
“Black Objects”

black string
black ring
black Saturn
di-rings, orthogonal di-rings ...

Outstanding problems







Cosmic Censorship?

Hoop Conjecture?

Stability?

We plan to investigate numerically as following

- Formation Process.
- Dynamical Features and stability.

2. Our numerical approach

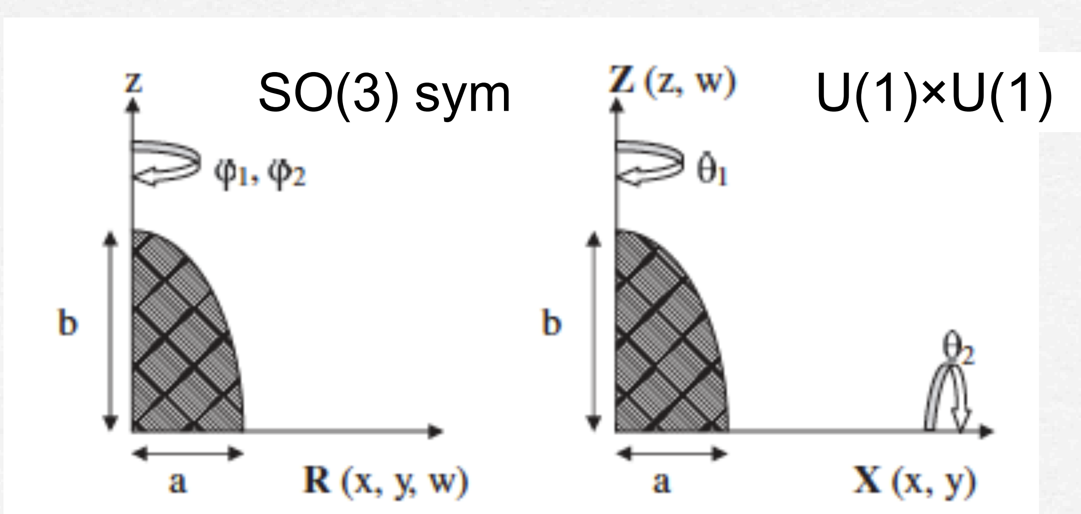
- Evolution of non-rotating matter configurations in 5D.

- Using the (4 + 1) ADM formalism.

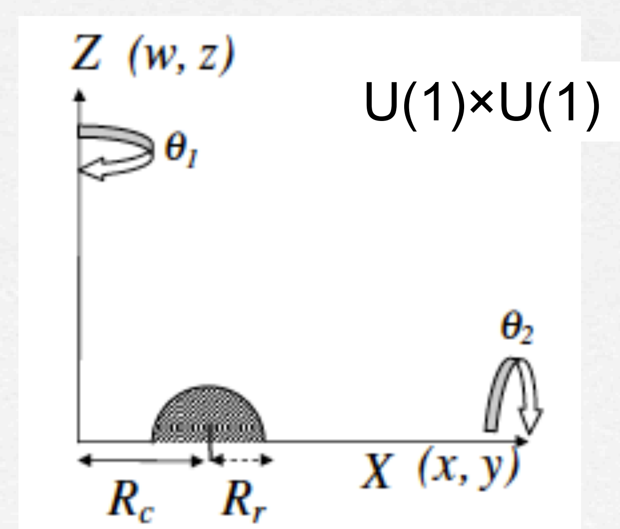
- Express the matter with collisionless particles.

- Search apparent horizons.

- We assume axi-symmetric space-time using the Cartoon method.



Spheroidal matter configuration



Ring matter configuration

The decomposed metric:

$$\begin{aligned} ds^2 &= -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \\ &= (-\alpha^2 + \beta_l \beta^l) dt^2 + 2\beta_i dt dx^i + \gamma_{ij} dx^i dx^j \end{aligned}$$

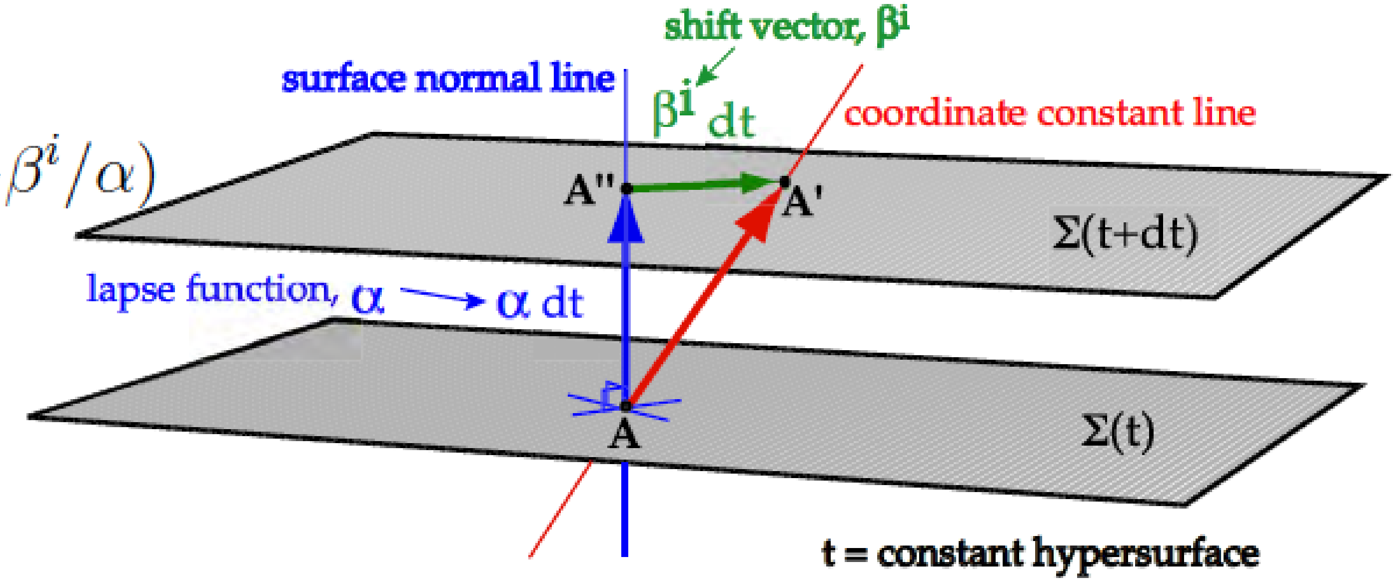
$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_l \beta^l & \beta_j \\ \beta_i & \gamma_{ij} \end{pmatrix}, \qquad g^{\mu\nu} = \begin{pmatrix} -1/\alpha^2 & \beta^j/\alpha^2 \\ \beta^i/\alpha^2 & \gamma^{ij} - \beta^i \beta^j/\alpha^2 \end{pmatrix}$$

where α and β_j are defined as $\alpha \equiv 1/\sqrt{-g^{00}}, \quad \beta_j \equiv g_{0j}$.

- The unit normal vector of the slices, n^μ.

$$\begin{aligned} n_\mu &= (-\alpha, 0, 0, 0) \\ n^\mu &= g^{\mu\nu} n_\nu = (1/\alpha, -\beta^i/\alpha) \end{aligned}$$

- The lapse function, α.
- The shift vector, β^i.



The Standard ADM formulation in $N+1$-dim.

cf. H. Shinkai and G. Yoneda, Gen. Rel. Grav. **36**, 1931 (2004)

The fundamental dynamical variables are (γ_{ij}, K_{ij}), the three-metric and extrinsic curvature. The three-hypersurface Σ is foliated with gauge functions, (α, β^i), the lapse and shift vector.

- The evolution equations:

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_j \beta_i + D_i \beta_j, \tag{1}$$

$$\begin{aligned} \partial_t K_{ij} = {} & \alpha {}^{(N)}R_{ij} + \alpha K K_{ij} - 2\alpha K^{\ell}{}_j K_{i\ell} - D_i D_j \alpha \\ & + \beta^k (D_k K_{ij}) + (D_j \beta^k) K_{ik} + (D_i \beta^k) K_{kj} - \kappa\alpha \left(S_{ij} - \frac{1}{N-1}\gamma_{ij} T \right) - \frac{2\alpha}{N-1}\gamma_{ij}\Lambda, \end{aligned} \tag{2}$$

where $K = K^i{}_i$, and ${}^{(N)}R_{ij}$ and D_i denote N-dimensional Ricci curvature, and a covariant derivative on the three-surface, respectively.

- Constraint equations:

Hamiltonian constr. $\quad \mathcal{H}^{ADM} := {}^{(N)}R + K^2 - K_{ij}K^{ij} - 2\Lambda - 2\kappa\rho \approx 0,$

momentum constr. $\quad \mathcal{M}_i^{ADM} := D_j K^j{}_i - D_i K - \kappa J^i \approx 0,$

where ${}^{(N)}R = {}^{(N)}R^i{}_i$.

Initial data construction

We construct sequences of initial data with

- conformally flat, time symmetric, asymptotically flat
- non-rotating homogeneous dust
- Conformal transformation $\gamma_{ij} = \psi^2 \hat{\gamma_{ij}}$
- The Hamiltonian constraint equation

$$\hat{\triangle}\psi = -4\pi^2 G_5 \rho$$

boundary condition : $\psi = 1 + \frac{M_{ADM}}{r^2}$

Evolution method

- ADM 4+1 Double Axisym Cartoon (150^2 x 2^2 grids)
- asymptotically flat

- Collisionless Particles (5000)
- the same total mass

- Apparent Horizon Search

Spheroidal collapse

- lapse function: Maximal slicing condition
- shift vectors : Minimum strain condition

Ring collapse

- lapse function: K-driver condition
- shift vectors : Zero shift

Evolution equation (ADM 4+1)

metric : $g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_k \beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$ outer boundary $\gamma_{ij} = \delta_{ij} + \frac{\text{const}}{r^2}$

$$\frac{\partial \gamma_{ij}}{\partial t} = -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i$$

$$\frac{\partial K_{ij}}{\partial t} = \alpha({}^{(4)}R_{ij} + K K_{ij}) - 2\alpha K_{il} K^{lj} - \alpha \kappa^2 (S_{ij} + \frac{1}{3}\gamma_{ij}(\rho - S))$$

$$-D_i D_j \alpha + D_i \beta^m K_{mj} + D_j \beta^m K_{mi} + \beta^m D_m K_{ij}$$

2nd-order differential scheme

Iterative Crank-Nicolson method

Courant factor 0.2

Coordinate condition

Spheroidal collapse

- lapse function: Maximal slicing condition

$$K = 0 \Leftrightarrow \partial_t K = 0 \Leftrightarrow \triangle\alpha = \alpha\left(K_{ij}K^{ij} + \frac{2}{3}\kappa\rho + \frac{1}{3}\kappa S\right)$$

boundary condition: $(\alpha - 1)r^2 = \mathrm{const} \Longleftrightarrow \frac{\partial}{\partial x^i}\left[(\alpha-1)r^2\right] = 0$

- shift vectors : Minimum strain condition

$$\Theta_{\mu\nu} = \perp \nabla_{(\nu} t_{\mu)} = -\alpha K_{\mu\nu} + \frac{1}{2} D_{(\mu}\beta_{\nu)}, \quad \text{where} \quad t^\mu = \alpha n^\mu + \beta^\mu$$

$$D_j \Theta^{ij} = 0 \Leftrightarrow \triangle\beta^i + D^i D_j \beta^j + R_{ij}\beta^j = 2D^j(\alpha K_{ij})$$

Ring collapse

- lapse function: K-driver condition

$$\frac{\partial K}{\partial t} = -cK \Leftrightarrow \frac{\partial \alpha}{\partial t} = \epsilon D^2\alpha - \epsilon\alpha\left(K_{ij}K^{ij} + \kappa^2(\rho + S)\right) - \epsilon\beta^i D_i K - \epsilon c K$$

- shift vectors : Zero shift

3. Spheroidal matter collapse (4D)

VOLUME 66, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1991

Formation of Naked Singularities: The Violation of Cosmic Censorship

Stuart L. Shapiro and Saul A. Teukolsky

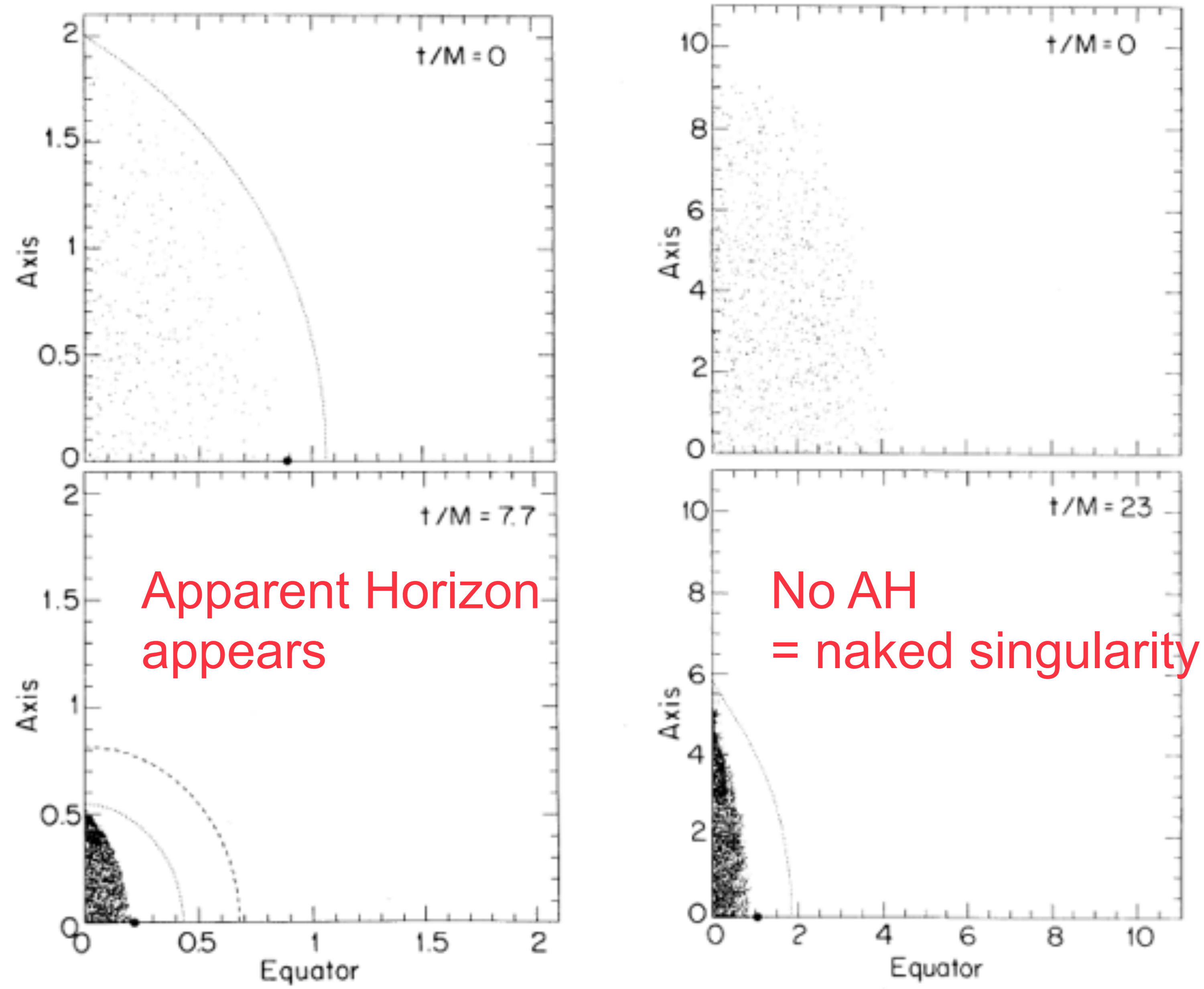


FIG. 1. Snapshots of the particle positions at initial and late times for prolate collapse. The positions (in units of M) are projected onto a meridional plane. Initially the semimajor axis of the spheroid is $2M$ and the eccentricity is 0.9. The collapse proceeds nonhomologously and terminates with the formation of a spindle singularity on the axis. However, an apparent horizon (dashed line) forms to cover the singularity. At $t/M=7.7$ its area is $\mathcal{A}/16\pi M^2=0.98$, close to the asymptotic theoretical limit of 1. Its polar and equatorial circumferences at that time are $\mathcal{C}^{AH}_{pole}/4\pi M=1.03$ and $\mathcal{C}^{AH}_{eq}/4\pi M=0.91$. At later times these circumferences become equal and approach the expected theoretical value 1. The minimum exterior polar circumference is shown by a dotted line when it does not coincide with the matter surface. Likewise, the minimum equatorial circumference, which is a circle, is indicated by a solid dot. Here $\mathcal{C}^{min}_{eq}/4\pi M=0.59$ and $\mathcal{C}^{min}_{pole}/4\pi M=0.99$. The formation of a black hole is thus consistent with the hoop conjecture.

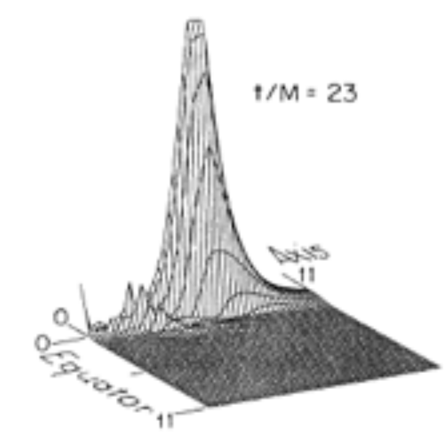


FIG. 4. Profile of I in a meridional plane for the collapse shown in Fig. 2. For the case of 32 angular zones shown here, the peak value of I is $24/M^4$ and occurs on the axis just outside the matter.

3. Spheroidal matter collapse (Yamada and Shinkai, PRD83 (2011) 064006)

SO(3) sym

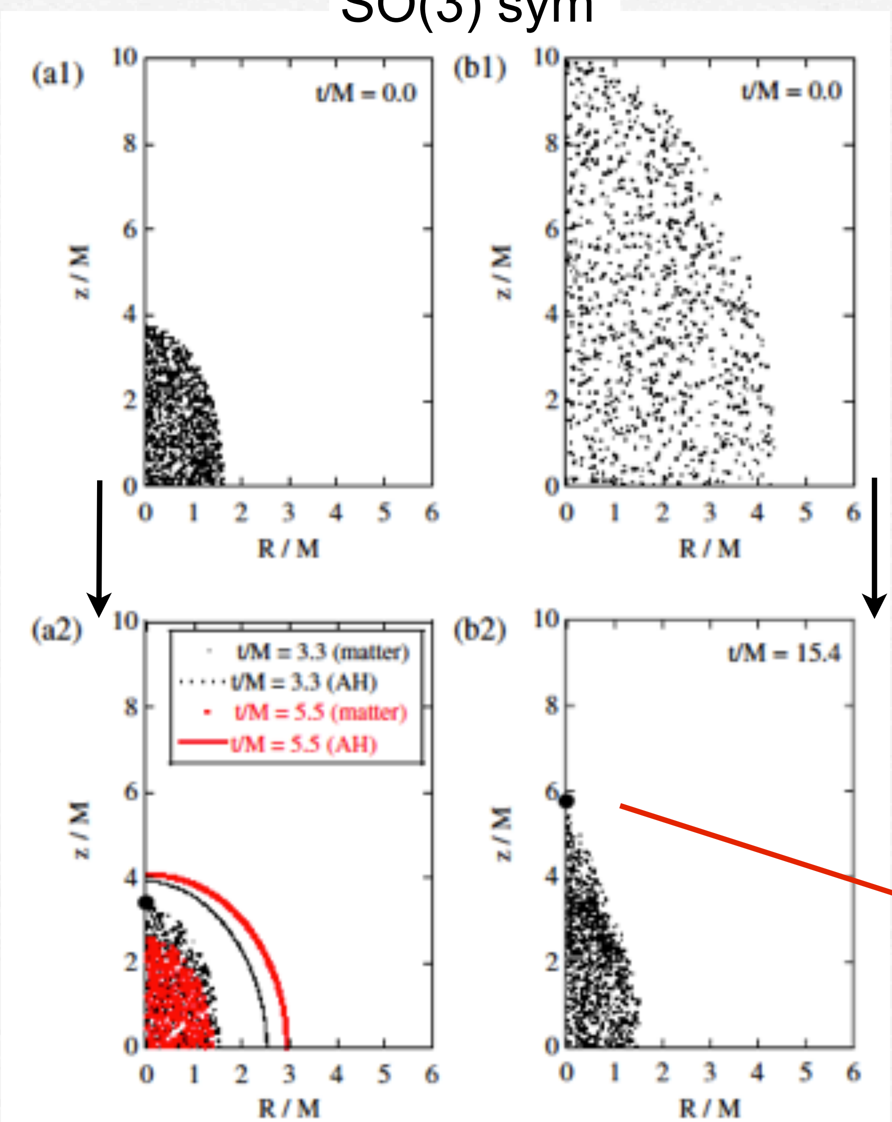


FIG. 2 (color online). Snapshots of 5D axisymmetric evolution with the initial matter distribution of $b/M = 4$ [(a1) and (a2); model 5DSβ in Table I] and 10 [(b1) and (b2); model 5DSδ]. We see the apparent horizon (AH) is formed at the coordinate time $t/M = 3.3$ for the former model and the area of AH increases, while AH is not observed for the latter model up to the time $t/M = 15.4$, when our code stops due to the large curvature. The big circle indicates the location of the maximum Kretschmann invariant $\mathcal{I}_{\max}$ at the final time at each evolution. Number of particles are reduced to 1/10 for figures.

- We prepare several initial data fixing the total ADM mass and the eccentricity of configuration, e = 0.9.

- When the initial matter is highly prolate, AH is not observed.

- The location of $\mathcal{I}_{max}$ is on z-axis, and just outside of the matter (This behavior is similar to the Shapiro-Teukolsky's 4D case).

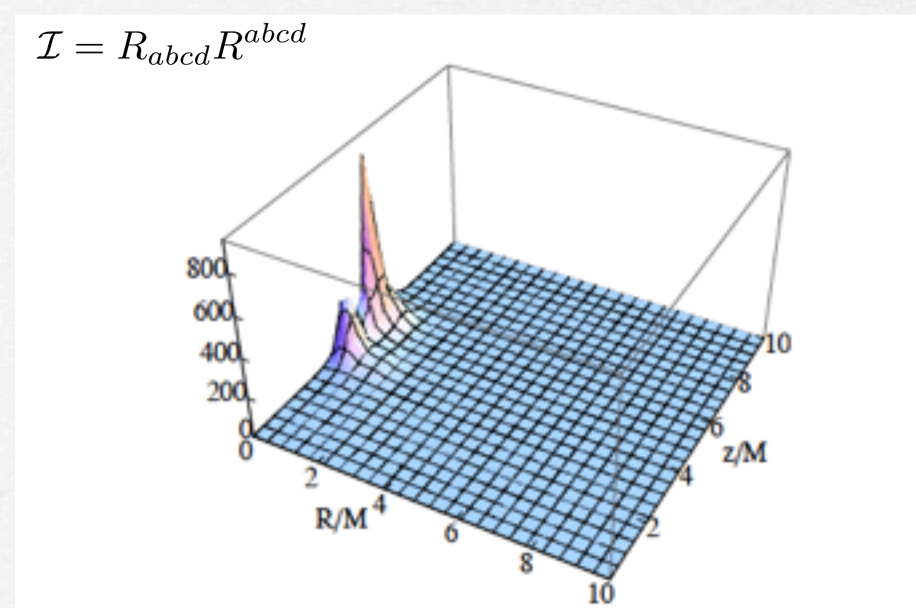
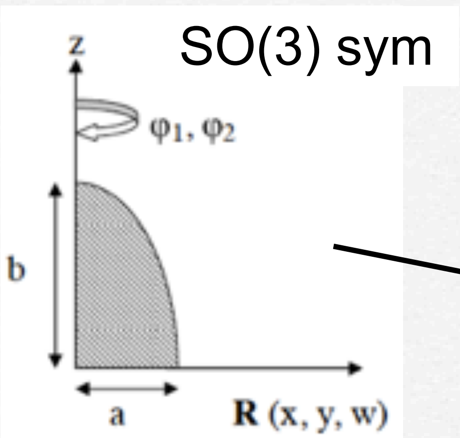
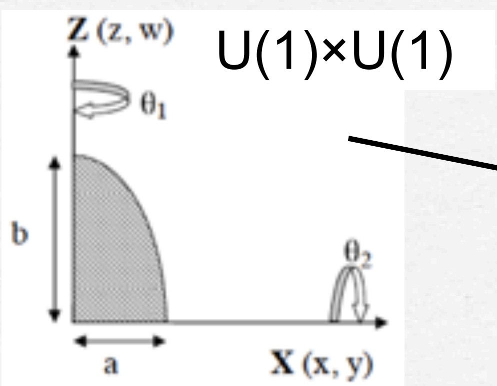


FIG. 3: Kretschmann invariant $\mathcal{I}$ for model 5DSδ at $t/M = 15.4$. The maximum is $O(1000)$, and its location is on z-axis, just outside of the matter.

Eccentricity e = 0.9 (t=0)





$b/M\ (t=0)$	2.50	4.00	6.25	10.00
4D axisym.	4Dα	4Dβ	4Dγ	4Dδ
	AH-formed	no	no	no
	$e_{\rm AH}=0.90$			
	$e_{\rm f}=0.92$	$e_{\rm f}=0.89$	$e_{\rm f}=0.92$	$e_{\rm f}=0.96$
5D axisym. SO(3)	5DSα	5DSβ	5DSγ	5DSδ
	AH-formed	AH-formed	no	no
	$e_{\rm AH}=0.88$	$e_{\rm AH}=0.88$		
	$e_{\rm f}=0.82$	$e_{\rm f}=0.84$	$e_{\rm f}=0.88$	$e_{\rm f}=0.96$
5D double axisym. U(1)×U(1)	5DUα	5DUβ	5DUγ	5DUδ
	AH-formed	AH-formed	AH-formed	no
	$e_{\rm AH}=0.86$	$e_{\rm AH}=0.87$	$e_{\rm AH}=0.92$	
	$e_{\rm f}=0.79$	$e_{\rm f}=0.81$	$e_{\rm f}=0.90$	$e_{\rm f}=0.98$

towards spindle (4D row)

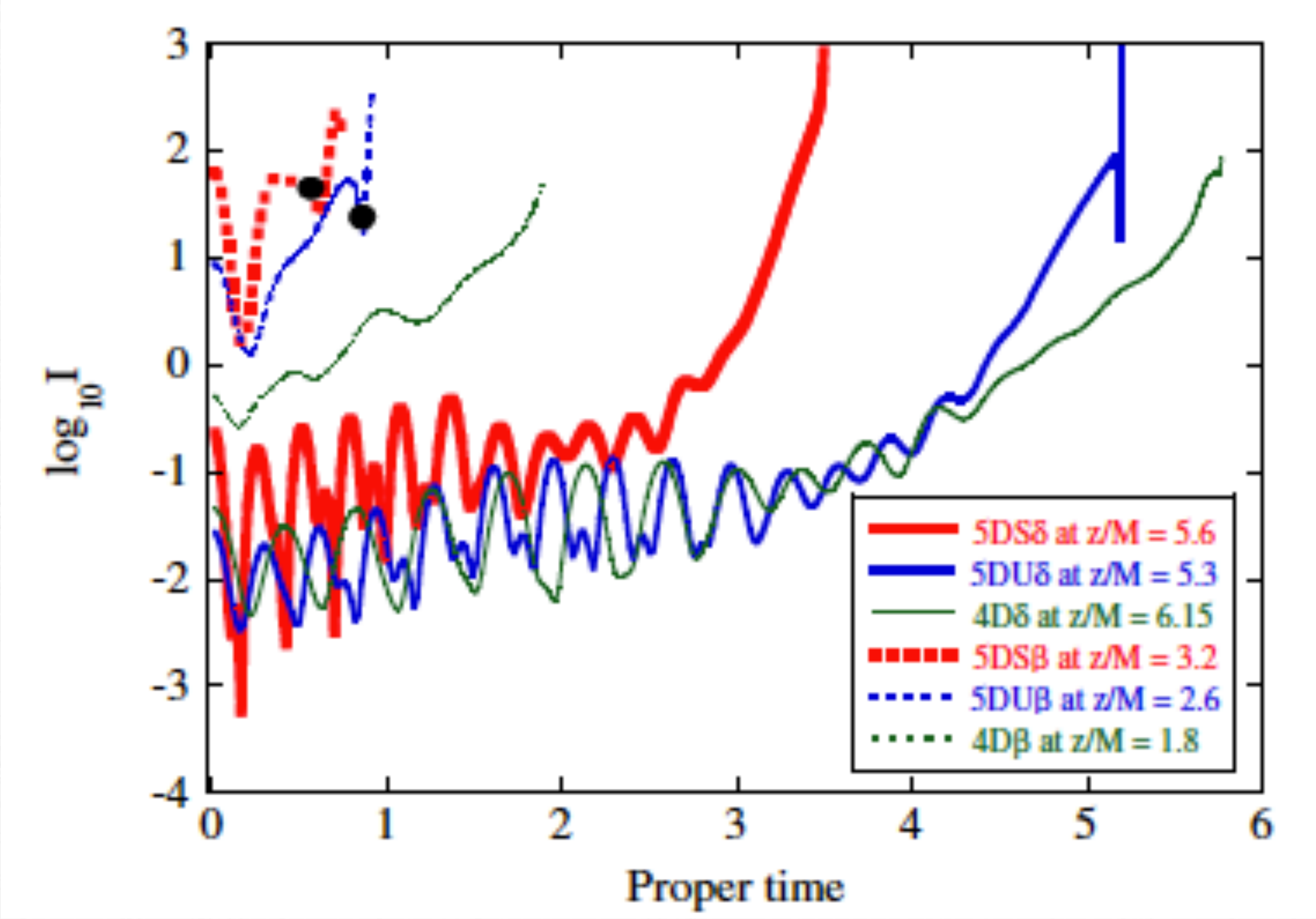
towards spherical (5D SO(3) row)

towards spherical (5D double axisym., 2.50 and 4.00) towards spindle (5D double axisym., 6.25 and 10.00)

- Table shows that the results of their evolutions whether we observed AH or not.

- The eccentricity e of the collapsed matter configurations is also shown; e_{AH} and e_f are at the time of AH formation (if formed), and on the numerically obtained final hypersurface, respectively.

- The 5D collapses proceed towards spherical configurations.



5D collapses
- proceed rapidly.
- towards spherical.
- AH forms in wider ranges.

- In Figure, we plot $\mathcal{I}$ at the point which gives $\mathcal{I}_{max}$ on the final hypersurface as a function of proper time.

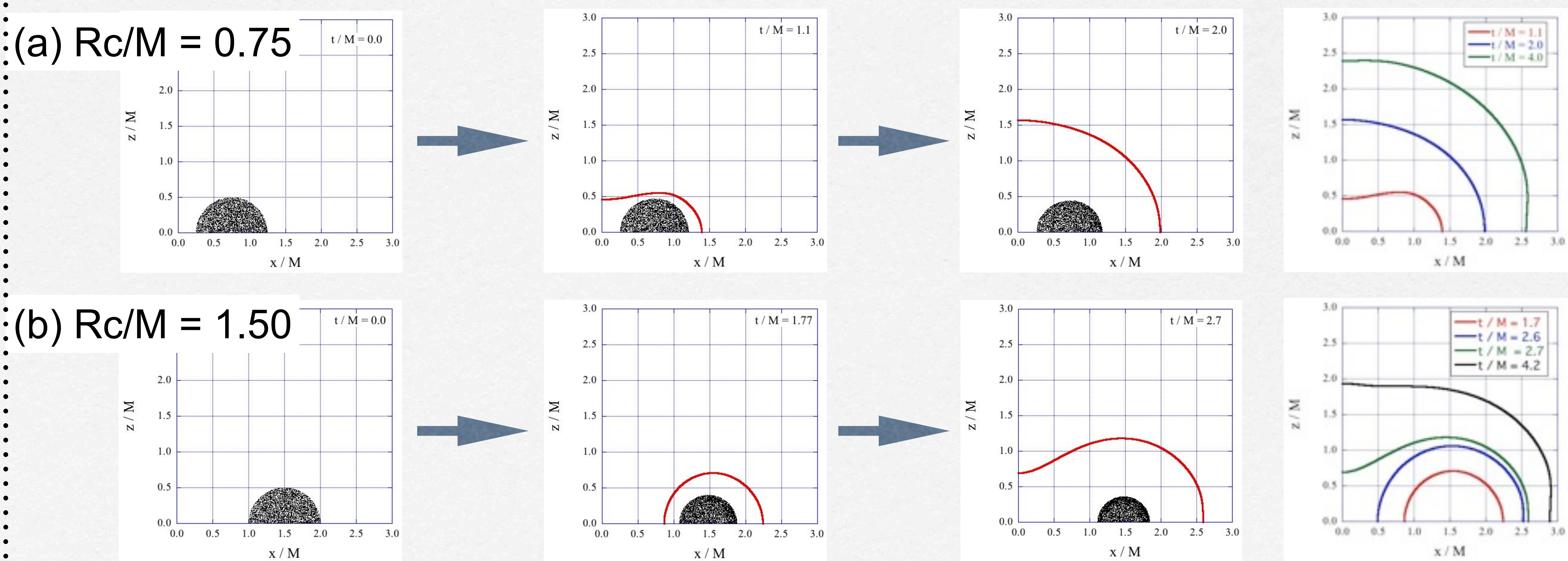
- We see that 5D collapses are generally proceeding more rapidly than 4D collapses.We also see that collapses in 5D doubly-axisymmetric space-time is proceeding more slowly than 5D single-axisymmetric cases.

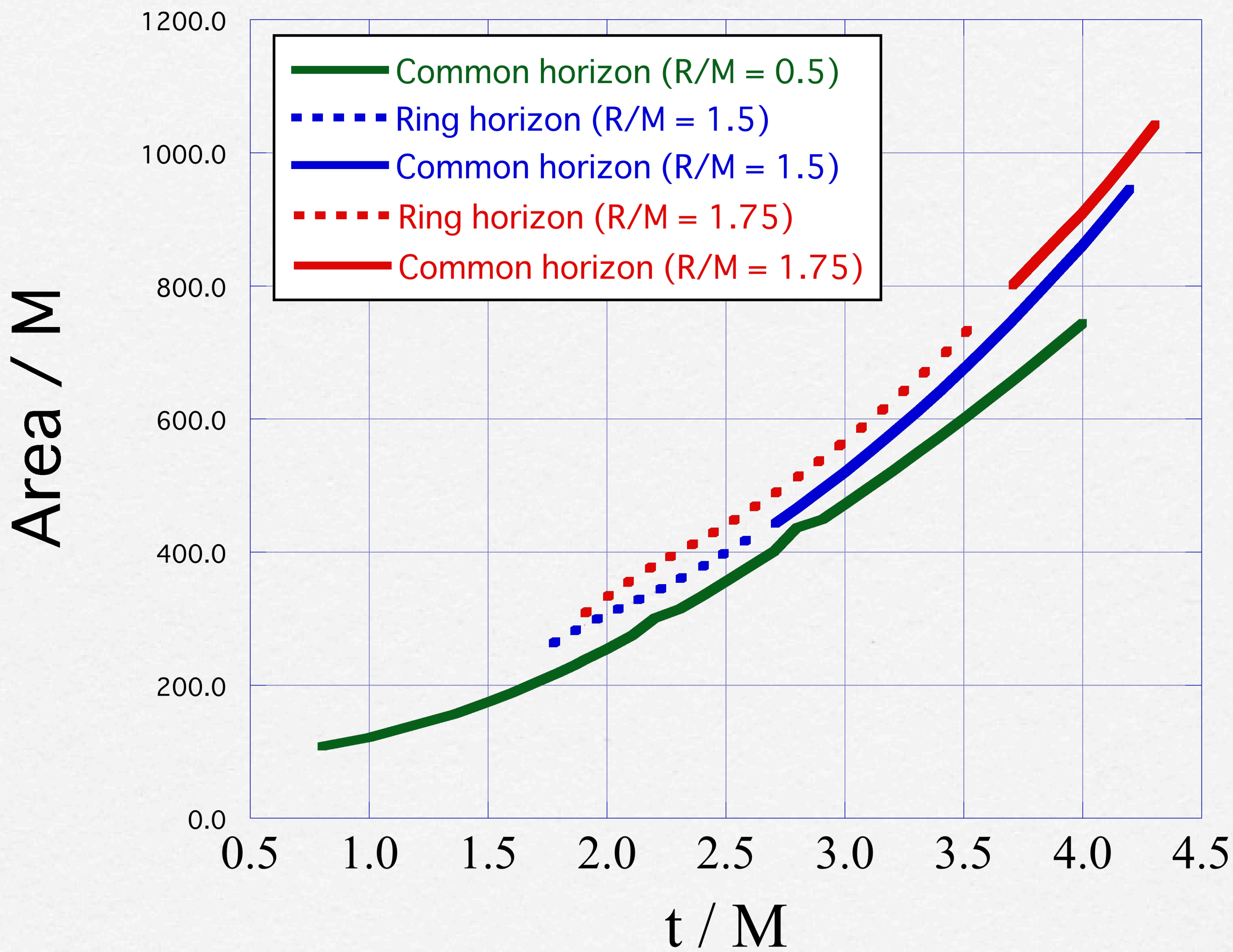
4. Ring Collapse

- Figure is a snapshot of evolutions for the ring matter of which initial radius are (a) Rc/M = 0.75 and (b) 1.50, respectively.

- Both have no AHs on the initial hypersurface, and we searched both spheroidal and toroidal horizons simultaneously at every time steps.

- We observe a formation of spheroidal AH (common horizon) in (a), while we see a formation of toroidal AH (ring horizon) then it switches to common horizon in (b).





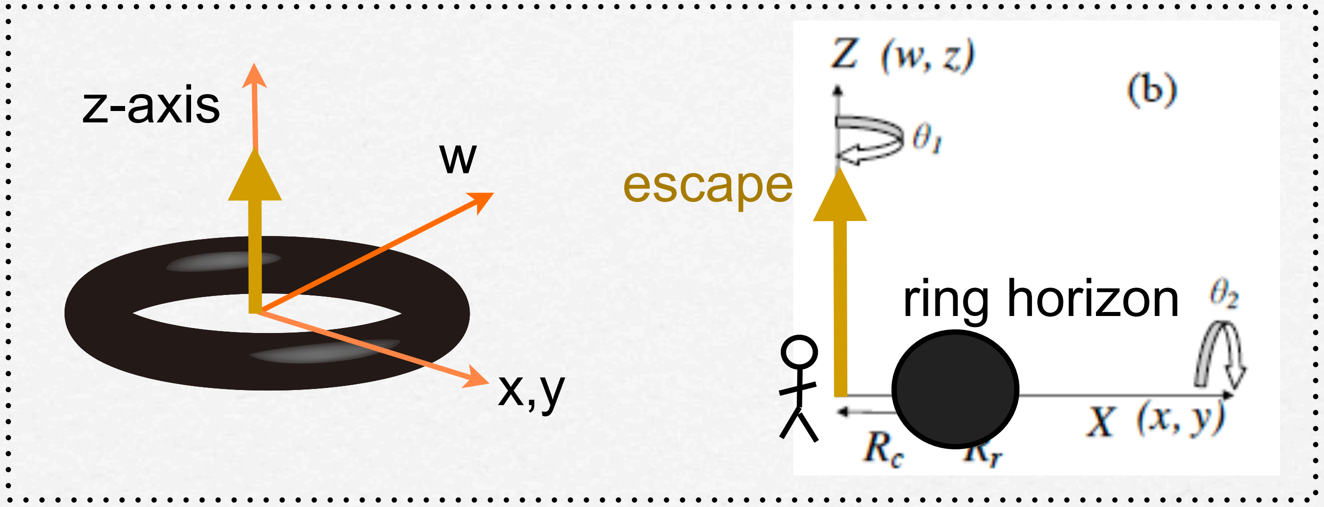
- dashed line (Ring horizon)
- solid line (Common horizon)

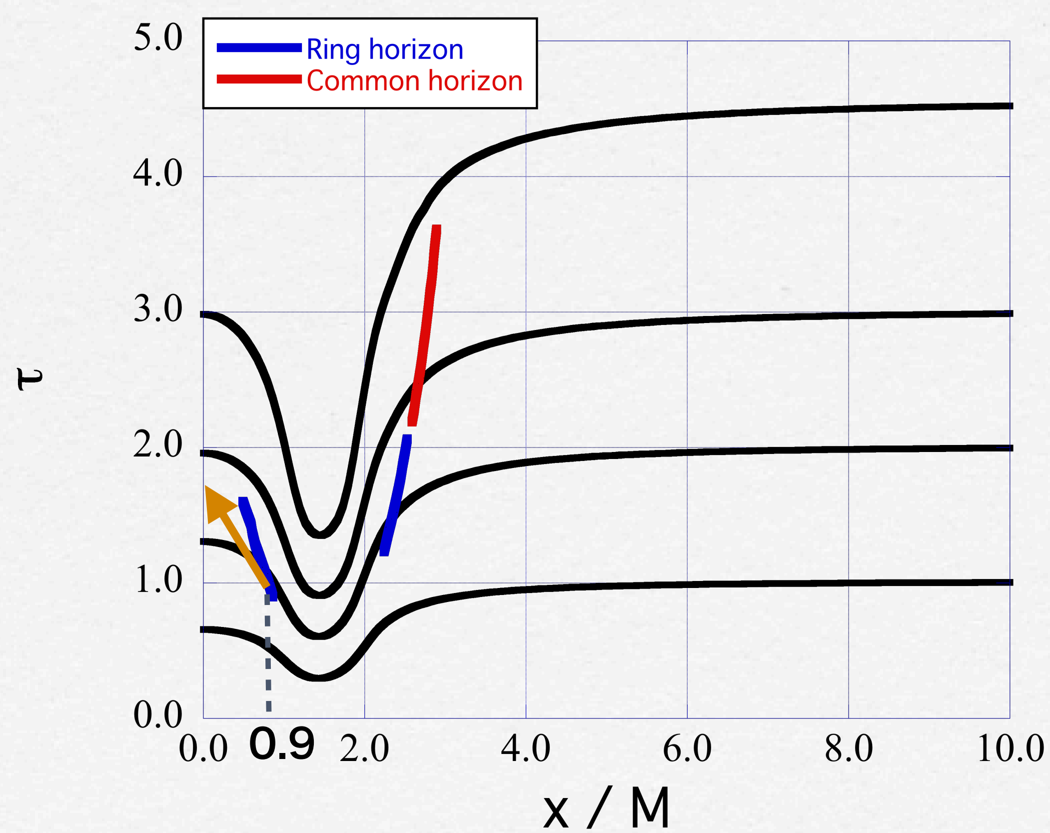
- Figure shows the area of horizon formed during ring collapses of which initial radius are Rc/M = 0.5, 1.5 and 1.75, respectively.

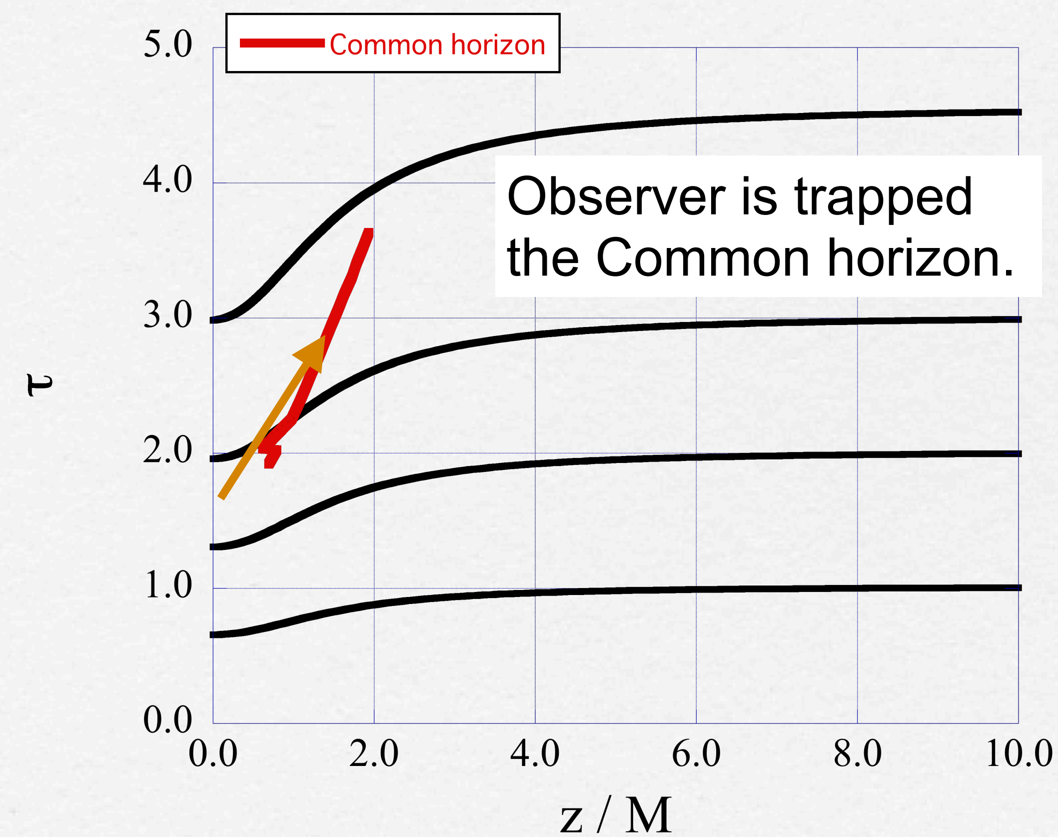
- Area of Ring horizon < Area of Common horizon。

- Both horizon’s area area smoothly connected.

Is it possible to escape from origin after the observer watched the ring horizon?





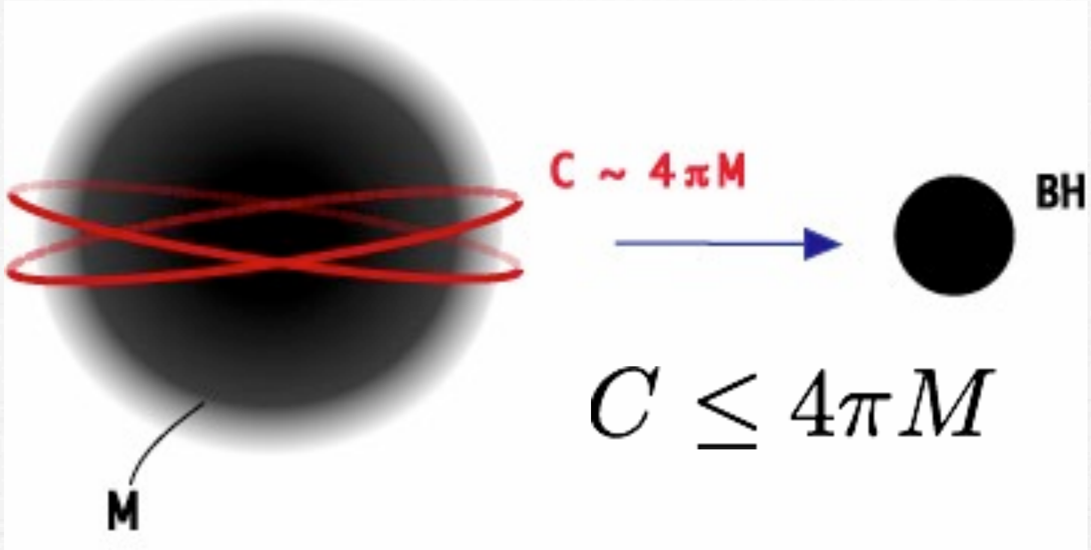


The snapshots of the hypersurfaces on the x and z axis in the propertime versus coordinate diagram.

Validity of Hyper-Hoop Conjecture

Hoop Conjecture Thorne(1972)

Horizons (probably) form when and only when a mass M gets compacted into a region whose circumference in every direction is C ≦ 2π×(2GM/c²) .



Hyper-Hoop Conjecture

$$V_{D-3} \leq G_D M$$

Ida and Nakao, PRD66, 064026 (2002)
Yoo et al, PRD71, 104014 (2005)

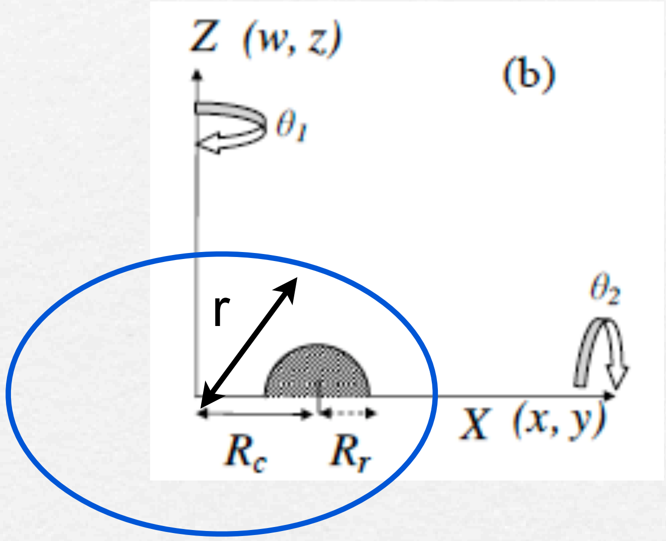
In 5-D, if mass gets compacted in some area, ...

Validity of Hyper-Hoop Conjecture

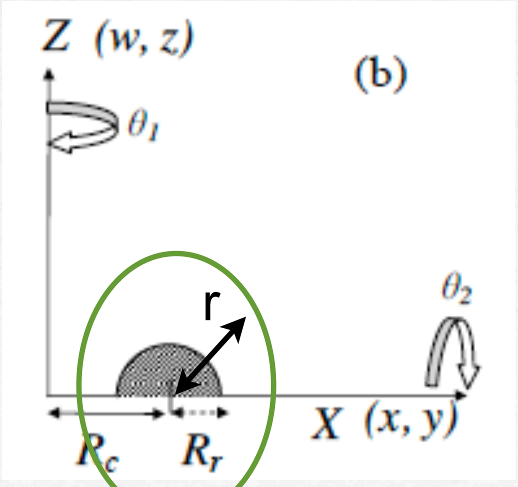
- To verify the Hyper-Hoop conjecture, we calculate the area of characteristic closed two-dimensional submanifold of the horizon.

$$Area(S_1) = 4\pi \int_0^{\pi/2} \sqrt{(r_\xi)^2 + r^2}\, r \sin\xi \sqrt{\gamma}\, d\xi$$

$$Area(S_2) = 4\pi \int_0^{\pi/2} \sqrt{(r_\xi)^2 + r^2}\, r \cos\xi \sqrt{\gamma}\, d\xi$$

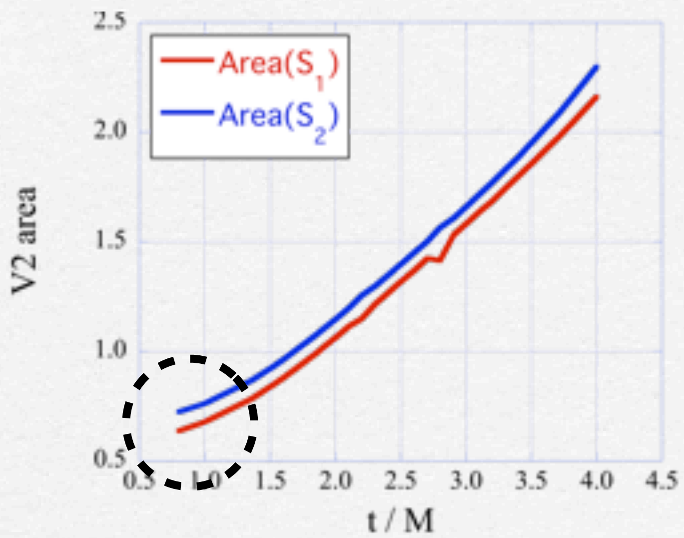


$$Area(S_3) = 2\pi \int_0^{\pi} \sqrt{(r_\xi)^2 + r^2}\, r \sin\xi \sqrt{\gamma}\, d\xi$$



Validity of Hyper-Hoop Conjecture $V_2 \leq 8\pi^2 GM$

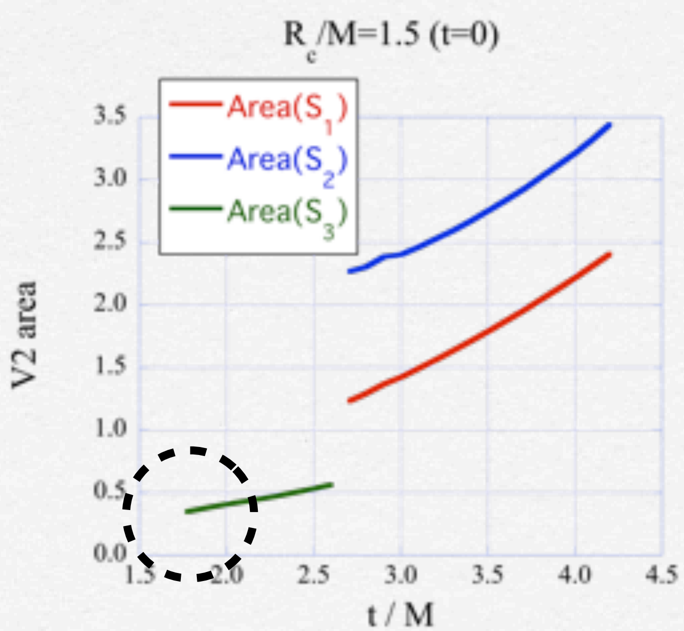
Case 1. no horizon (R_c/M=0.5 at t=0) -> Common horizon

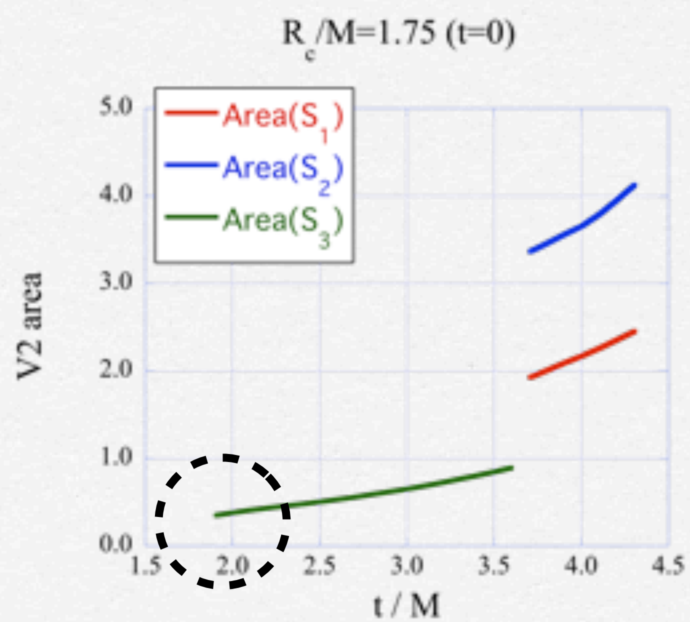


- The ratio less than unity indicates that the validity of the hyper-hoop conjecture.

- The inequality is satisfied when the common horizon is formed.

Case 2. no horizon -> Ring horizon -> Common horizon





- The inequality is satisfied when the ring horizon are formed.

6. Summary and Future works

5D vs. 4D Spheroidal collapses (no rotating cases)

Collapse rapidly, towards spherical
Formation of Naked Singularity for highly prolate matter

5D Ring collapses (no rotating cases)

no horizon → common horizon

↘ ring horizon ↗

Future Plans:

include rotation, change slicing conditions
search various horizons,
investigate the stability, formation/decay process, ...

アインシュタイン方程式の解【シュワルツシルド解】

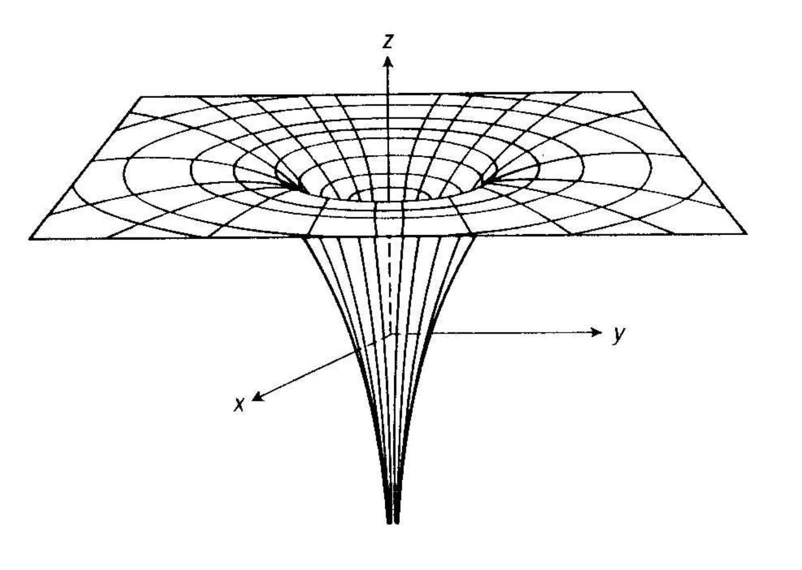
Schwarzschild (1916)
球対称，真空でのEinstein方程式の厳密解

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \frac{dr^2}{1 - \dfrac{2GM}{c^2 r}} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

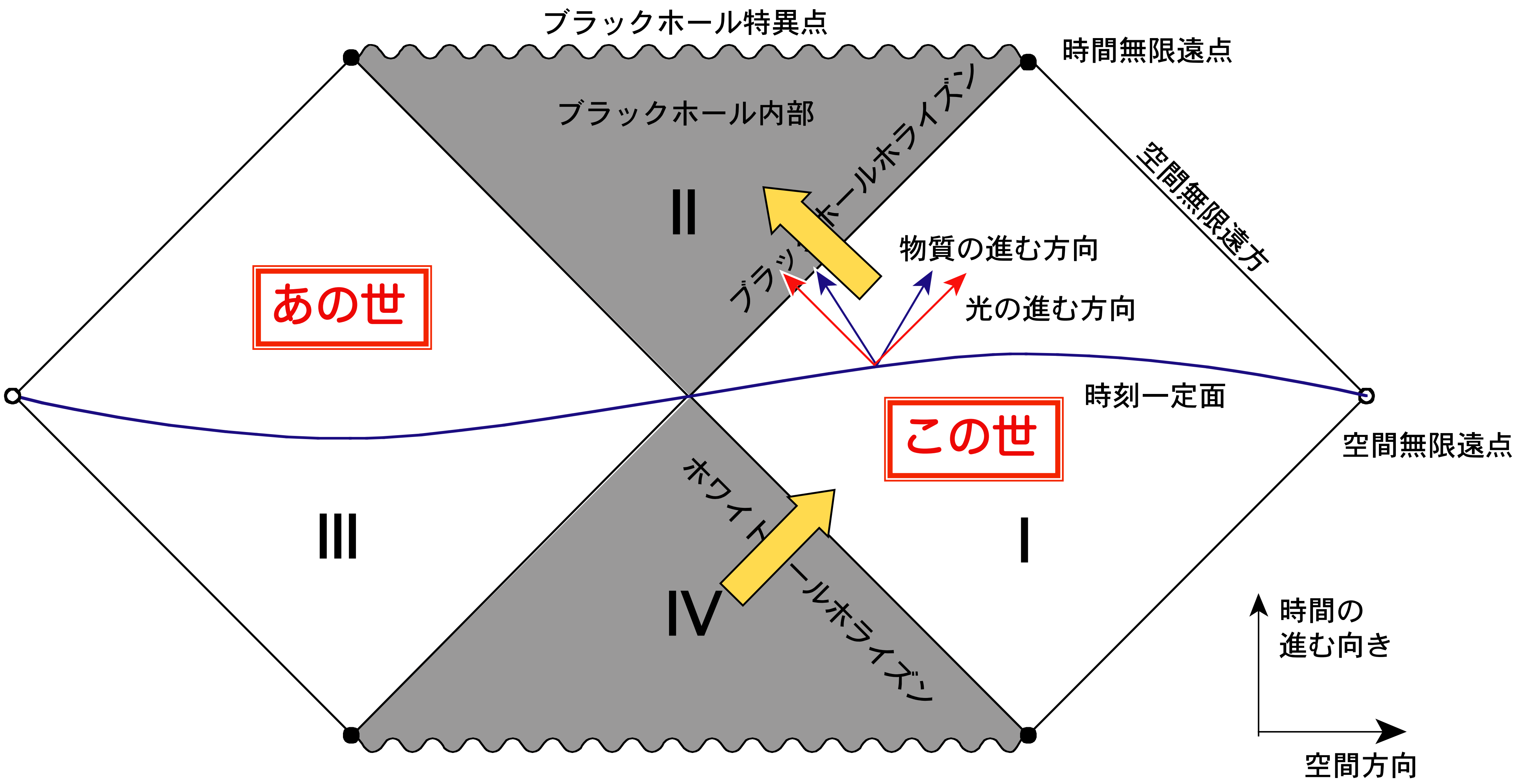
困ったことに，……

r=0 で特異点
⇒ 今でも困ってる

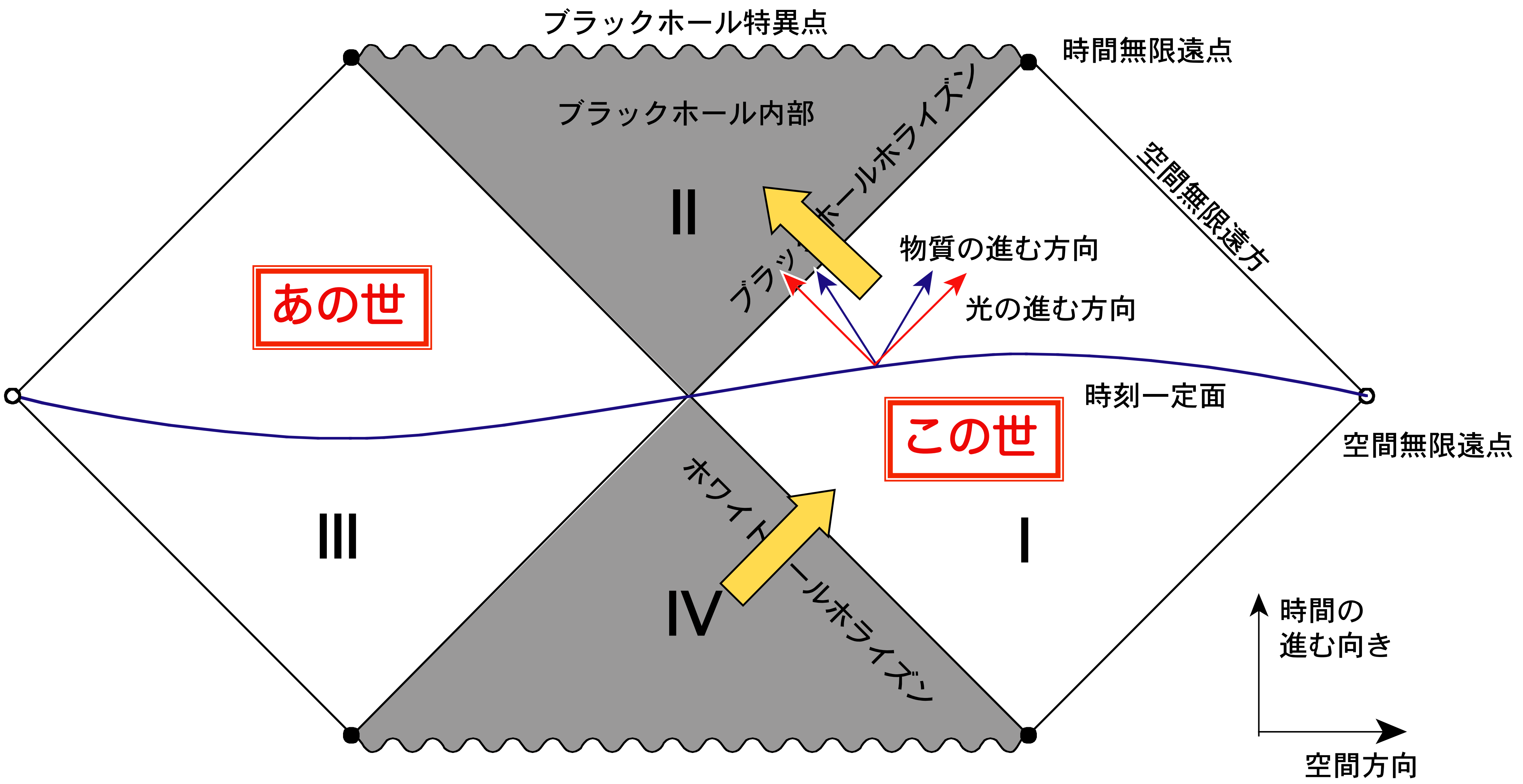
r=2GM/c^2 でも特異点
⇒ ブラックホールの境界
太陽なら2Km，地球なら0.9cm



Schwarzschild Black HoleのPenrose図



Schwarzschild Black HoleのPenrose図



ワームホール

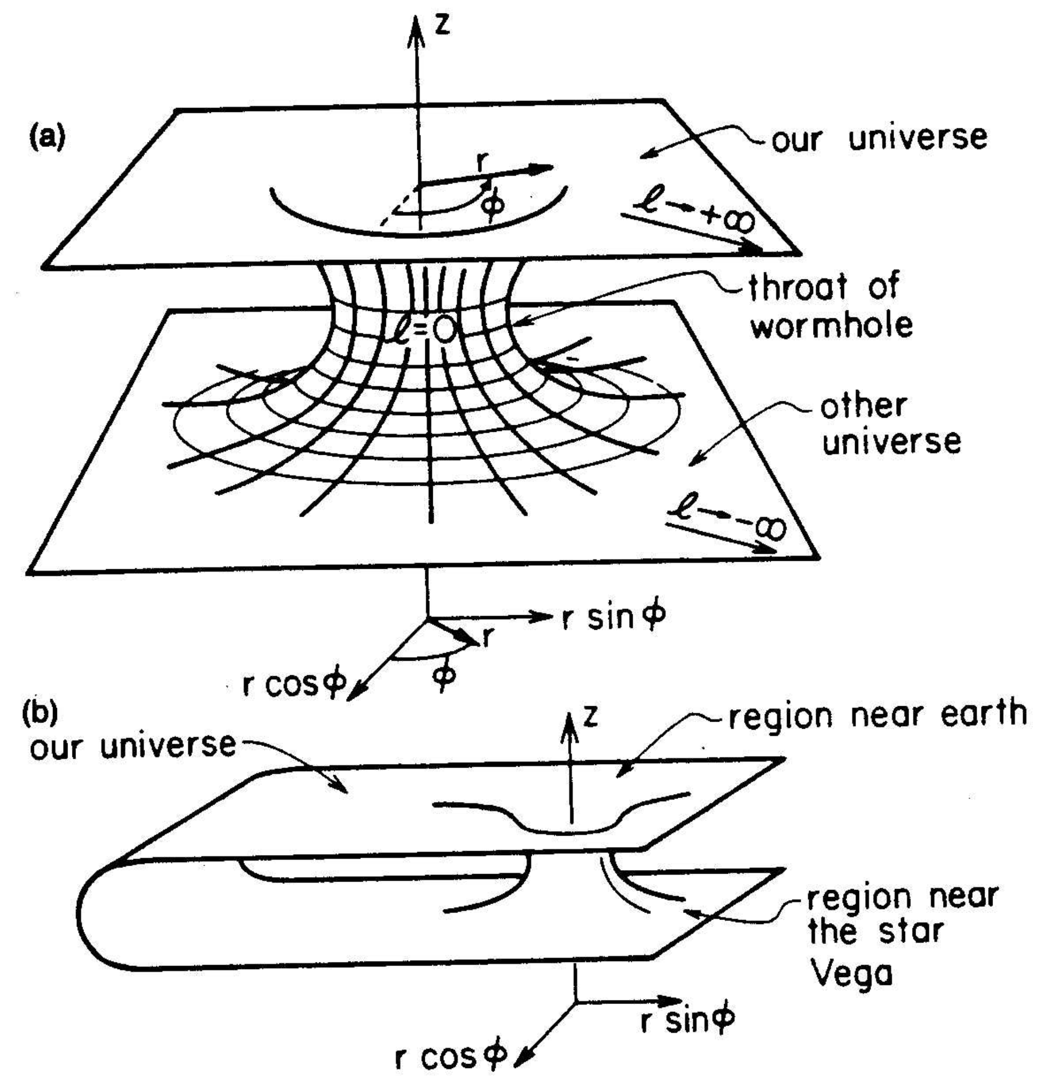
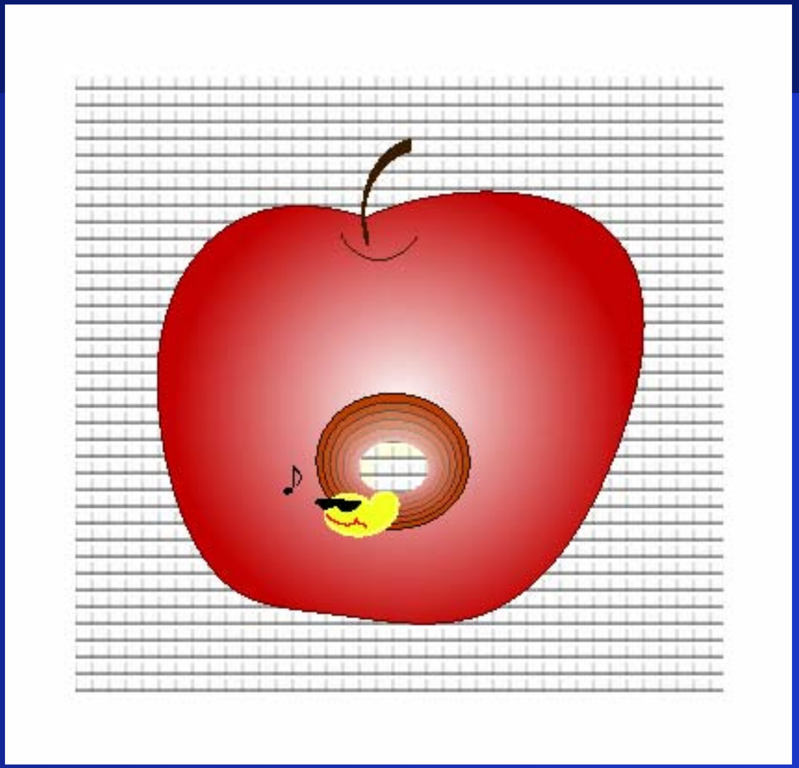


Fig. 1. (a) Embedding diagram for a wormhole that connects two different universes. (b) Embedding diagram for a wormhole that connects two distant regions of our own universe. Each diagram depicts the geometry of an equatorial ($\theta = \pi/2$) slice through space at a specific moment of time ($t =$ const). These embedding diagrams are derived quickly in item (b) of Box 2, and—in a more leisurely fashion—in Sec. III C, where they are also discussed. This figure is adapted from Ref. 1, Fig. 31.5.

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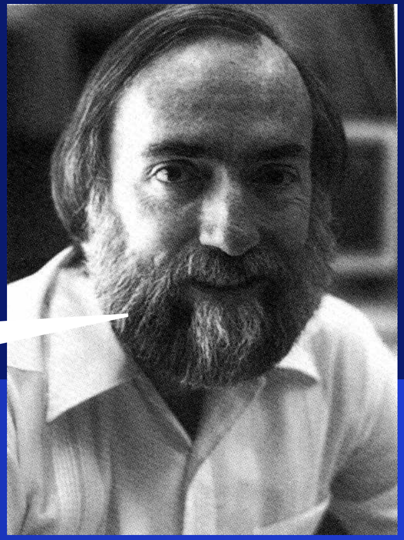
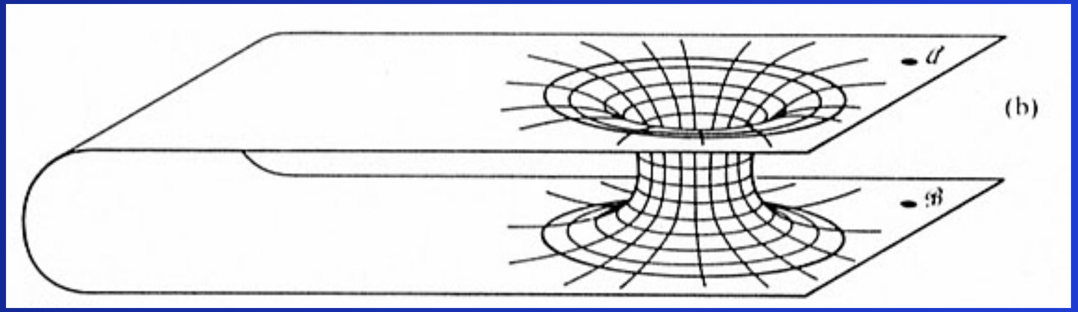
通過可能なワームホール



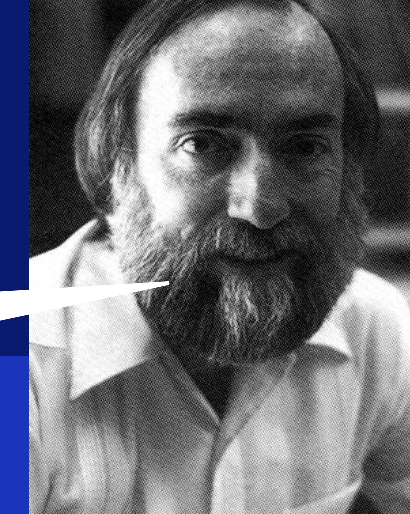
Morris, Thorne, Am. J. Phys 56 (1988) 395
「球対称で静的，一般相対性理論，漸近的平坦，潮汐力が人間に耐えられる大きさ，有限時間に通過可能」なワームホールは，負のエネルギーを考えれば不可能ではない．



Morris, Thorne, Yurtserver, PRL 61 (1988) 3182
片方の出口を光速近くまでに加速することができれば，
旅行者の時間は遅れるので，過去へ旅することができる．

本当に？

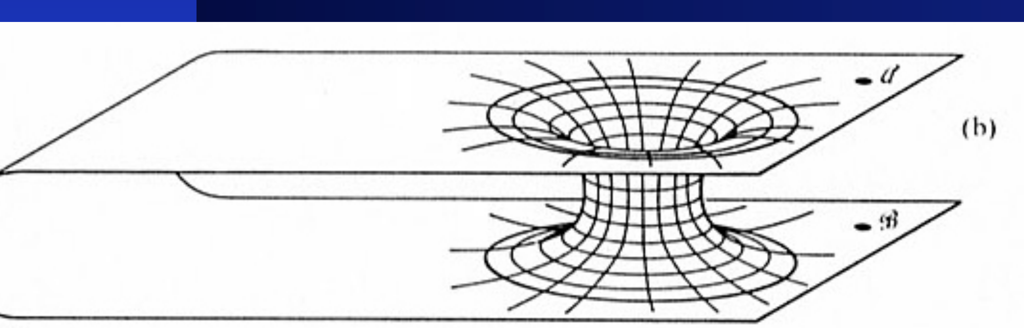
タイムマシンができる！



- もしワームホールが存在して,
- さらにもし負のエネルギーが安定に存在して,
- さらにもしワームホールが通過可能で,
- さらにもし人類が通過可能な技術を持ち,
- さらにもし出口を光速近くで動かすことができるならば,
- さらにもし旅行者が別ルートで同じ場所に戻れれば,

タイムマシンに成り得る

No! 時間順序保護仮説



そもそもワームホールは安定なのか

PHYSICAL REVIEW D **66**, 044005 (2002)

Fate of the first traversible wormhole: Black-hole collapse or inflationary expansion

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(Received 10 May 2002; published 16 August 2002)

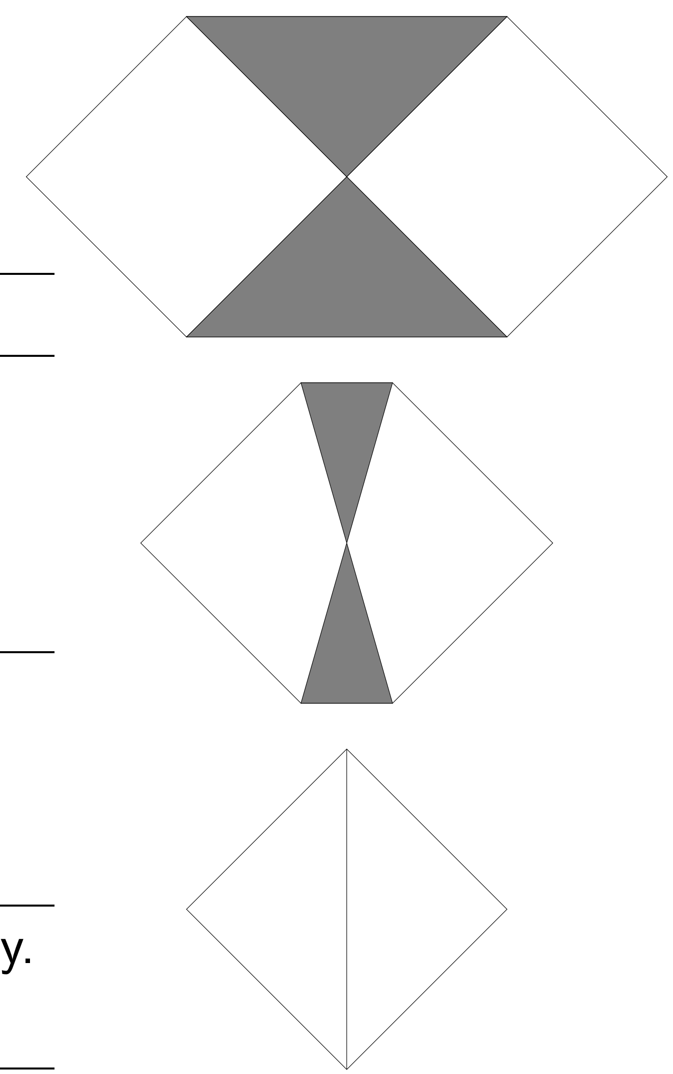
We study numerically the stability of the first Morris-Thorne traversible wormhole, shown previously by Ellis to be a solution for a massless ghost Klein-Gordon field. Our code uses a dual-null formulation for spherically symmetric space-time integration, and the numerical range covers both universes connected by the wormhole. We observe that the wormhole is unstable against Gaussian pulses in either exotic or normal massless Klein-Gordon fields. The wormhole throat suffers a bifurcation of horizons and either explodes to form an inflationary universe or collapses to a black hole if the total input energy, is, respectively, negative or positive. As the perturbations become small in total energy, there is evidence for critical solutions with a certain black-hole mass or Hubble constant. The collapse time is related to the initial energy with an apparently universal critical exponent. For normal matter, such as a traveller traversing the wormhole, collapse to a black hole always results. However, carefully balanced additional ghost radiation can maintain the wormhole for a limited time. The black-hole formation from a traversible wormhole confirms the recently proposed duality between them. The inflationary case provides a mechanism for inflating, to macroscopic size, a Planck-sized wormhole formed in space-time foam.

BH and WH are interconvertible ? (New Duality?)

S.A. Hayward, Int. J. Mod. Phys. D 8 (1999) 373

- They are very similar – both contain (marginally) trapped surfaces and can be defined by trapping horizons (TH)
- Only the causal nature of the THs differs, whether THs evolve in plus / minus density.

	Black Hole	Wormhole
Locally defined by	Achronal(spatial/null) outer TH ⇒ 1-way traversable	Temporal (timelike) outer THs ⇒ 2-way traversable
Einstein eqs.	Positive energy density normal matter (or vacuum)	Negative energy density "exotic" matter
Appearance	occur naturally	Unlikely to occur naturally. but constructible ???



2 Fate of Morris-Thorne (Ellis) wormhole?

- "Dynamical wormhole" defined by local trapping horizon
- spherically symmetric, both normal/ghost KG field
- apply dual-null formulation in order to seek horizons
- Numerical simulation

2.1 ghost/normal Klein-Gordon fields

Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{16\pi} - \frac{1}{4\pi}\underbrace{\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)}_{\text{normal}} + \frac{1}{4\pi}\underbrace{\left(\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)}_{\text{ghost}}\right]$$

The field equations

$$G_{\mu\nu} = 2\left[\psi_{,\mu}\psi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\psi)^2 + V_1(\psi)\right)\right] - 2\left[\phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}(\nabla\phi)^2 + V_2(\phi)\right)\right]$$

$$\Box\psi = \frac{dV_1(\psi)}{d\psi}, \qquad \Box\phi = \frac{dV_2(\phi)}{d\phi}. \quad \text{(Hereafter, we set } V_1(\psi) = 0, V_2(\phi) = 0\text{)}$$

2.2 dual-null formulation, spherically symmetric spacetime

S A Hayward, CQG 10 (1993) 779, PRD 53 (1996) 1938, CQG 15 (1998) 3147

- The spherically symmetric line-element:

$$ds^2 = r^2 dS^2 - 2e^{-f} dx^+ dx^-,$$

where $r = r(x^+, x^-)$, $f = f(x^+, x^-)$, $\cdots$

- The Einstein equations:

$$\begin{aligned}
\partial_\pm \partial_\pm r + (\partial_\pm f)(\partial_\pm r) &= -r(\partial_\pm \psi)^2 + r(\partial_\pm \phi)^2, \\
r\partial_+ \partial_- r + (\partial_+ r)(\partial_- r) + e^{-f}/2 &= 0, \\
r^2 \partial_+ \partial_- f + 2(\partial_+ r)(\partial_- r) + e^{-f} &= +2r^2(\partial_+ \psi)(\partial_- \psi) - 2r^2(\partial_+ \phi)(\partial_- \phi), \\
r\partial_+ \partial_- \phi + (\partial_+ r)(\partial_- \phi) + (\partial_- r)(\partial_+ \phi) &= 0, \\
r\partial_+ \partial_- \psi + (\partial_+ r)(\partial_- \psi) + (\partial_- r)(\partial_+ \psi) &= 0.
\end{aligned}$$

- To obtain a system accurate near $\Im^\pm$, we introduce the conformal factor $\boxed{\Omega = 1/r}$. We also define first-order variables, the conformally rescaled momenta

expansions	$\vartheta_\pm = 2\partial_\pm r = -2\Omega^{-2}\partial_\pm \Omega \qquad (\theta_\pm = 2r^{-1}\partial_\pm r)$	(1)
inaffinities	$\nu_\pm = \partial_\pm f$	(2)
momenta of ϕ	$\wp_\pm = r\partial_\pm \phi = \Omega^{-1}\partial_\pm \phi$	(3)
momenta of ψ	$\pi_\pm = r\partial_\pm \psi = \Omega^{-1}\partial_\pm \psi$	(4)

The set of equations (cont.):

$$\begin{aligned}
\partial_\pm \vartheta_\pm &= -\nu_\pm \vartheta_\pm - 2\Omega \pi_\pm^2 + 2\Omega \wp_\pm^2, & (5)\\
\partial_\pm \vartheta_\mp &= -\Omega(\vartheta_+ \vartheta_-/2 + e^{-f}), & (6)\\
\partial_\pm \nu_\mp &= -\Omega^2(\vartheta_+ \vartheta_-/2 + e^{-f} - 2\pi_+\pi_- + 2\wp_+\wp_-), & (7)\\
\partial_\pm \wp_\mp &= -\Omega \vartheta_\mp \wp_\pm/2, & (8)\\
\partial_\pm \pi_\mp &= -\Omega \vartheta_\mp \pi_\pm/2. & (9)
\end{aligned}$$

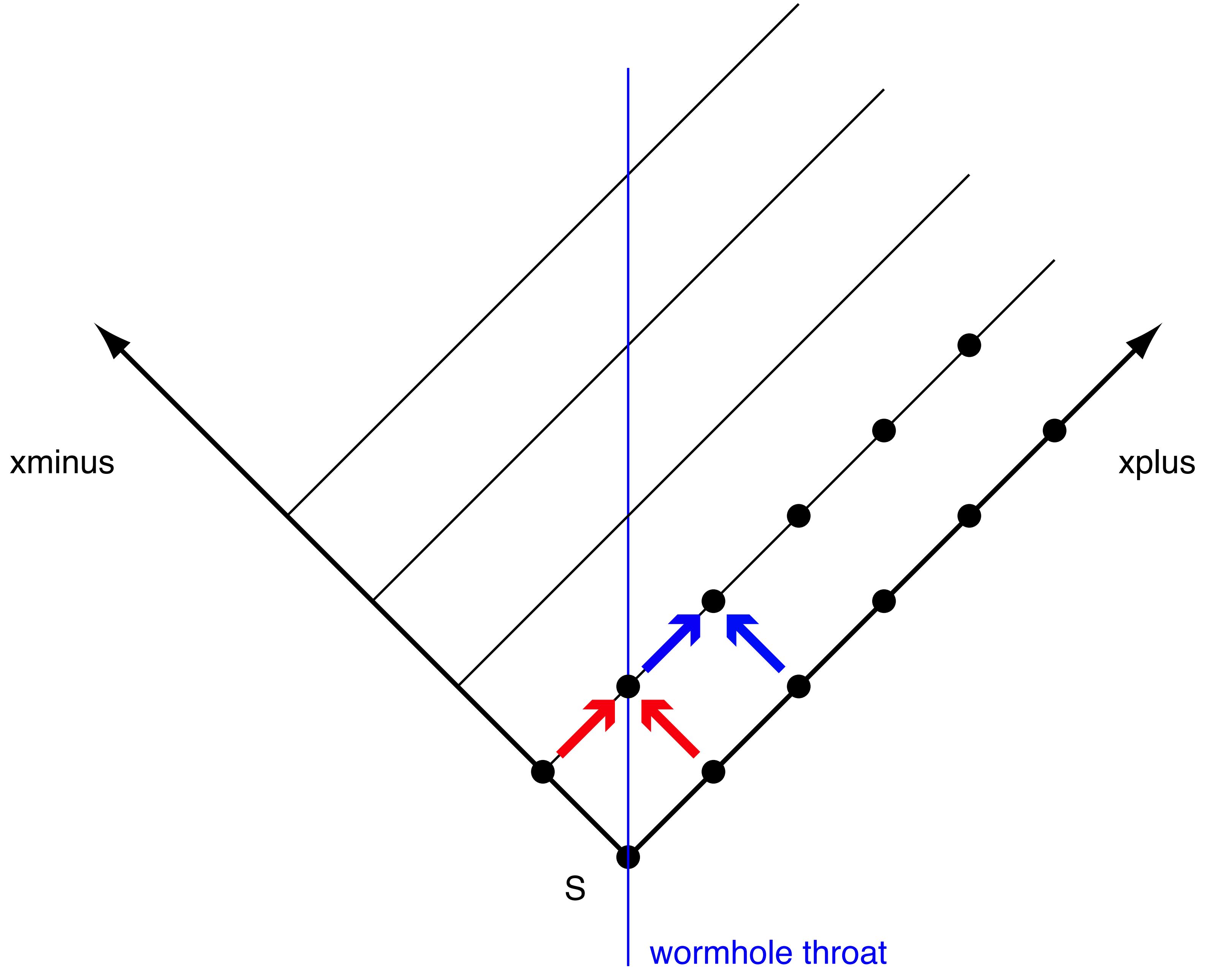
and remember the identity: $\partial_+\partial_- = \partial_-\partial_+$:

2.3 Initial data on $x^+ = 0$, $x^- = 0$ slices and on S

Generally, we have to set :

$$\begin{aligned}
(\Omega, f, \vartheta_\pm, \phi, \psi) &\qquad \text{on } S\text{: } x^+ = x^- = 0\\
(\nu_\pm, \wp_\pm, \pi_\pm) &\qquad \text{on } \Sigma_\pm\text{: } x^\mp = 0,\ x^\pm \geq 0
\end{aligned}$$

Grid Structure for Numerical Evolution



Ghost pulse input – Bifurcation of the horizons

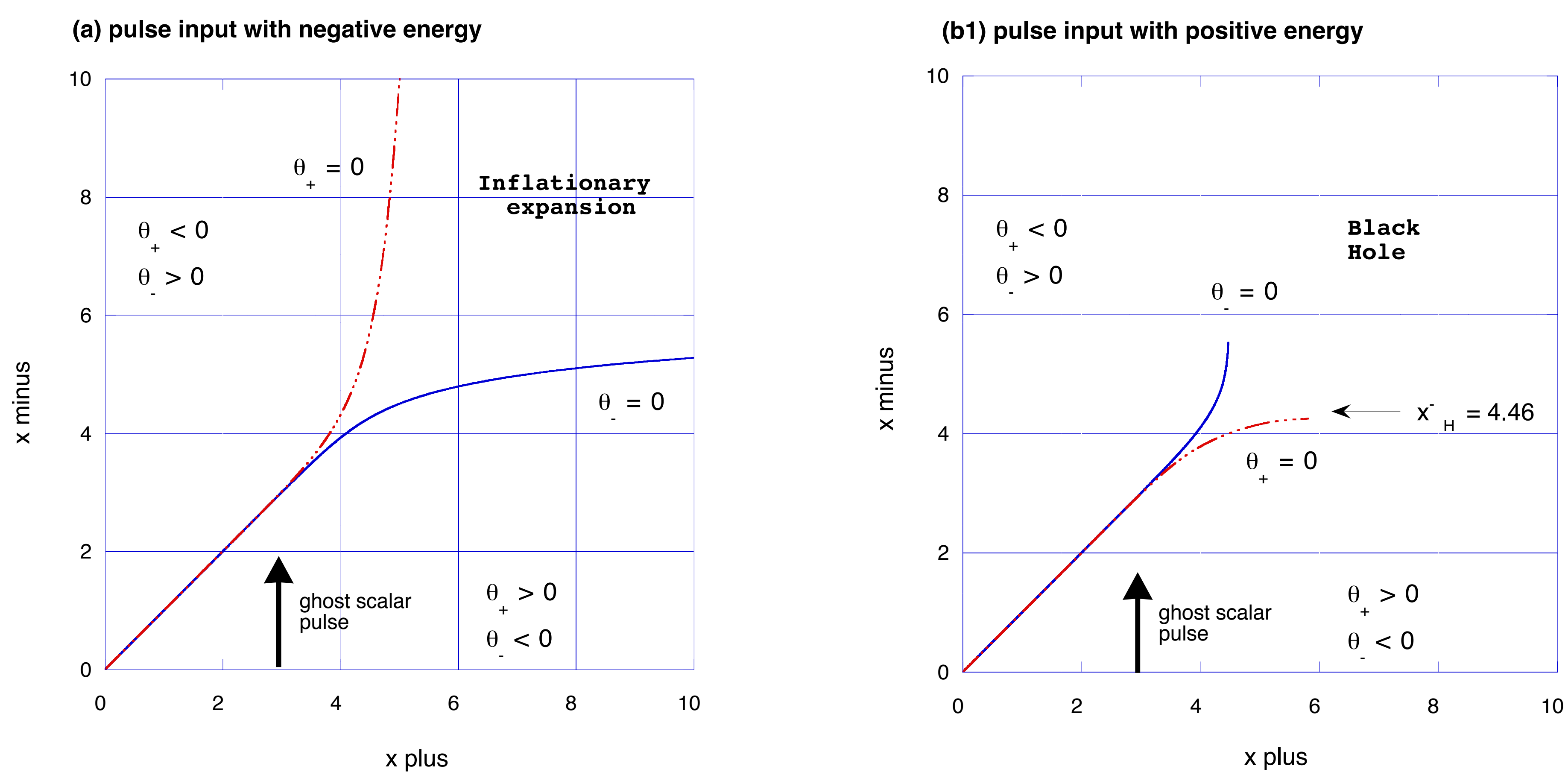
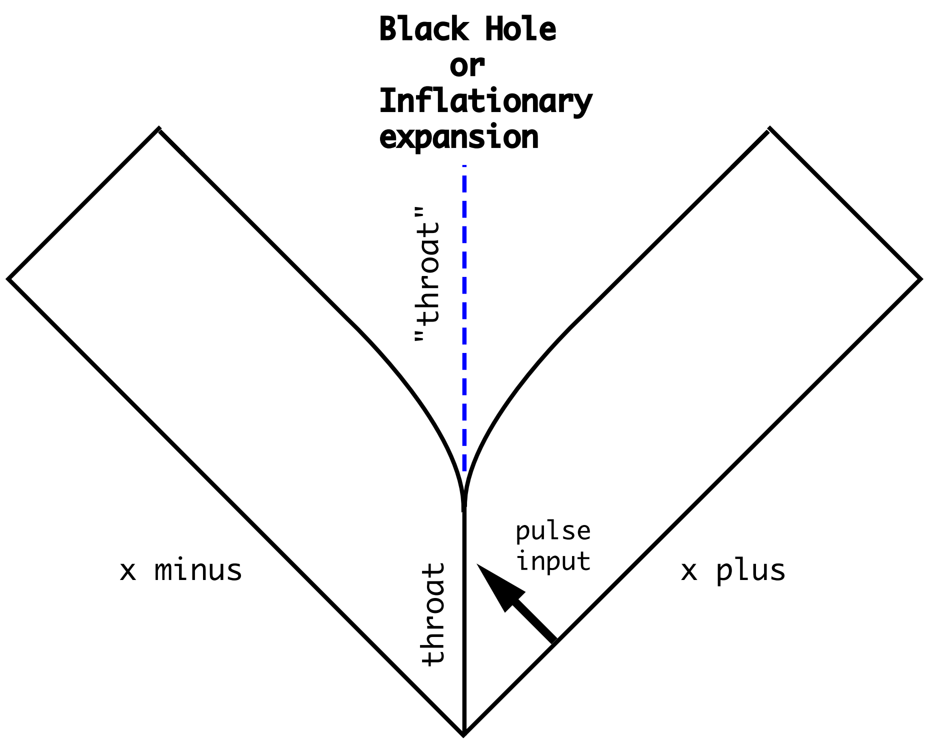


Figure 3: Horizon locations, $\vartheta_{\pm} = 0$, for perturbed wormhole. Fig.(a) is the case we supplement the ghost field, $c_a = 0.1$, and (b1) and (b2) are where we reduce the field, $c_a = -0.1$ and -0.01. Dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively. In all cases, the pulse hits the wormhole throat at $(x^+, x^-) = (3, 3)$. A 45° counterclockwise rotation of the figure corresponds to a partial Penrose diagram.

Bifurcation of the horizons – go to a Black Hole or Inflationary expansion



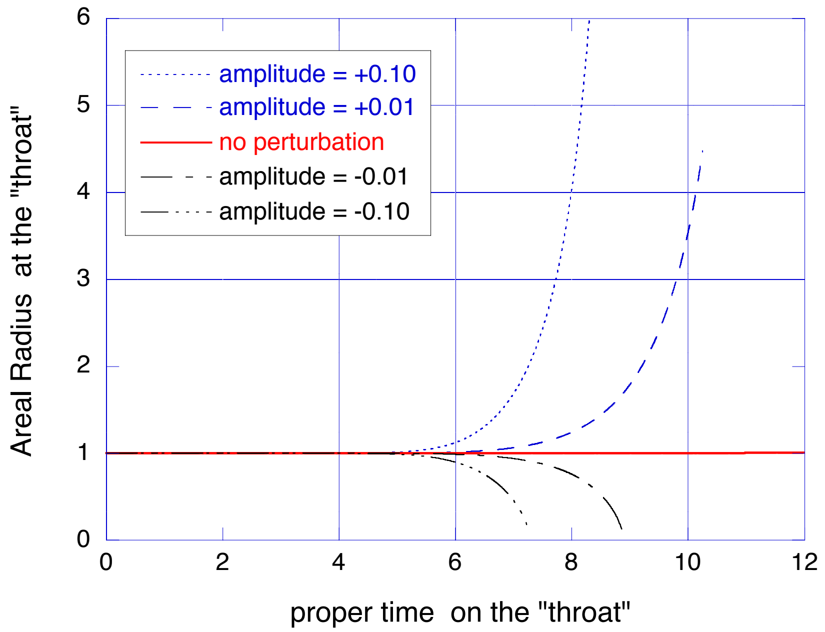


Figure 4: Partial Penrose diagram of the evolved space-time.

Figure 6: Areal radius r of the "throat" $x^+ = x^-$, plotted as a function of proper time. Additional negative energy causes inflationary expansion, while reduced negative energy causes collapse to a black hole and central singularity.

Local Energy Measure – Determination of the Black Hole Mass

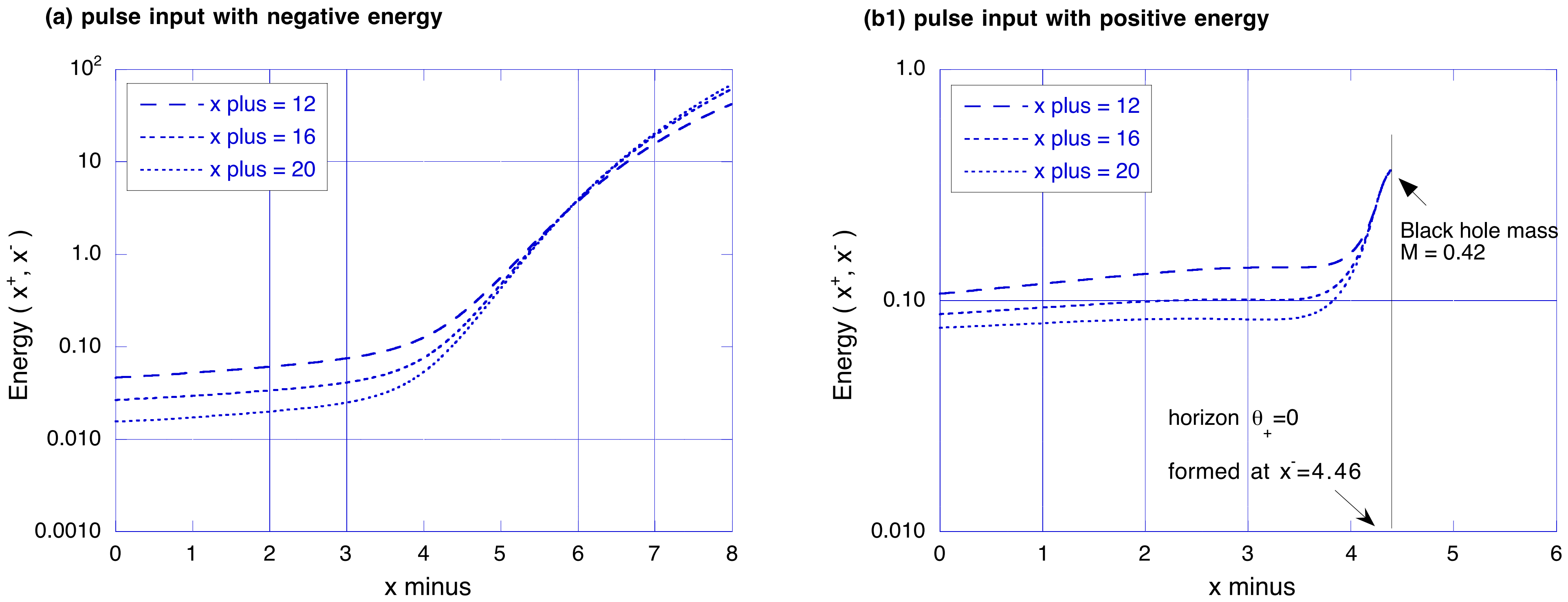


Figure 7: Energy $E(x^+, x^-)$ as a function of x^-, for $x^+ = 12, 16, 20$. Here c_a is (a) 0.05, (b1) -0.1 and (b2) -0.01. The energy for different x^+ coincides at the final horizon location x_H^-, indicating that the horizon quickly attains constant mass $M = E(\infty, x_H^-)$. This is the final mass of the black hole or cosmological horizon.

Is there a Minimum Black Hole Mass to be formed?

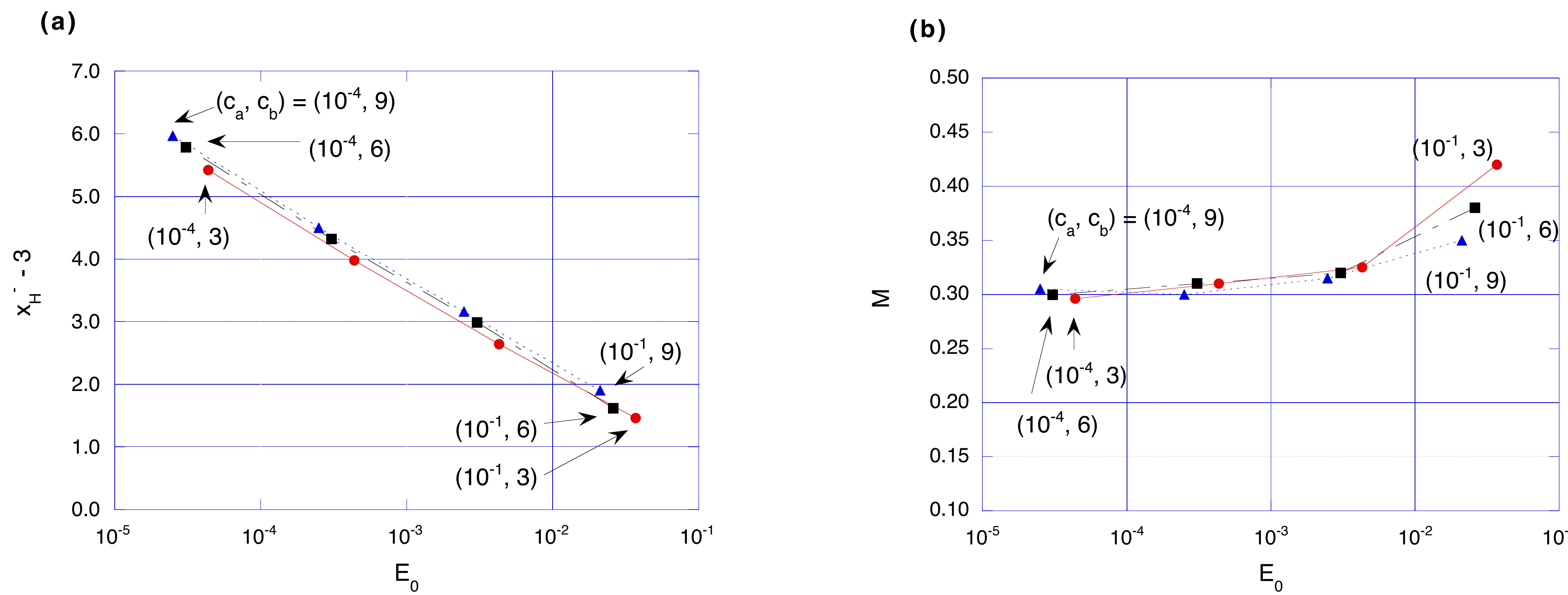


Figure 8: Relation between the initial perturbation and the final mass of the black hole. (a) The trapping horizon ($\vartheta_+ = 0$) coordinate, $x_H^- - 3$ (since we fixed $c_c = 3$), versus initial energy of the perturbation, E_0. We plotted the results of the runs of $c_a = 10^{-1}, \cdots, 10^{-4}$ with $c_b = 3, 6$, and 9. They lie close to one line. (b) The final black hole mass M for the same examples. We see that M appears to reach a non-zero minimum for small perturbations.

Normal Pulse (a traveller) Input – Forming a Black Hole

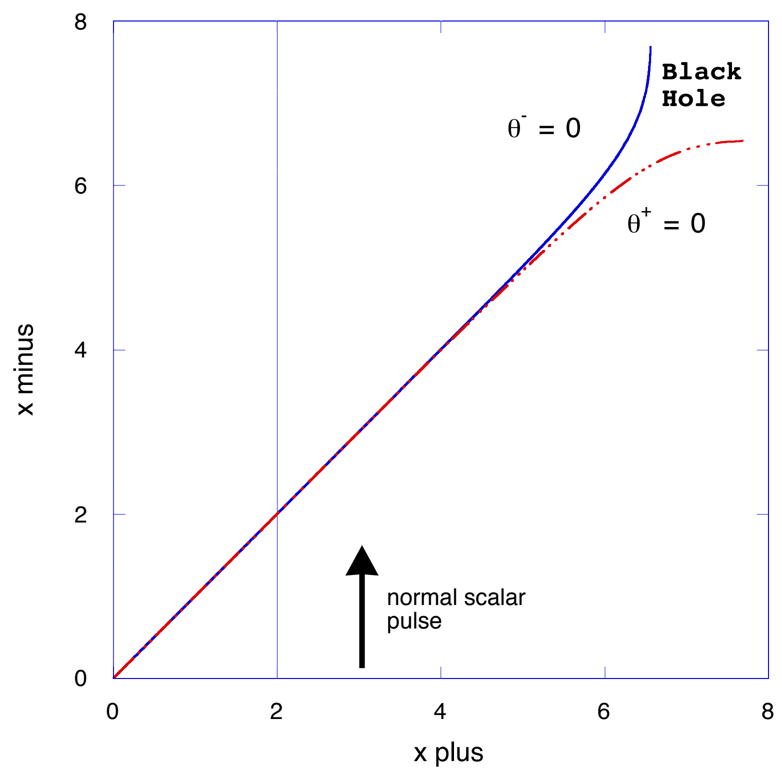


Figure 9: Evolution of a wormhole perturbed by a normal scalar field. Horizon locations: dashed lines and solid lines are $\vartheta_+ = 0$ and $\vartheta_- = 0$ respectively.

Critical Minimum Black Hole Mass again

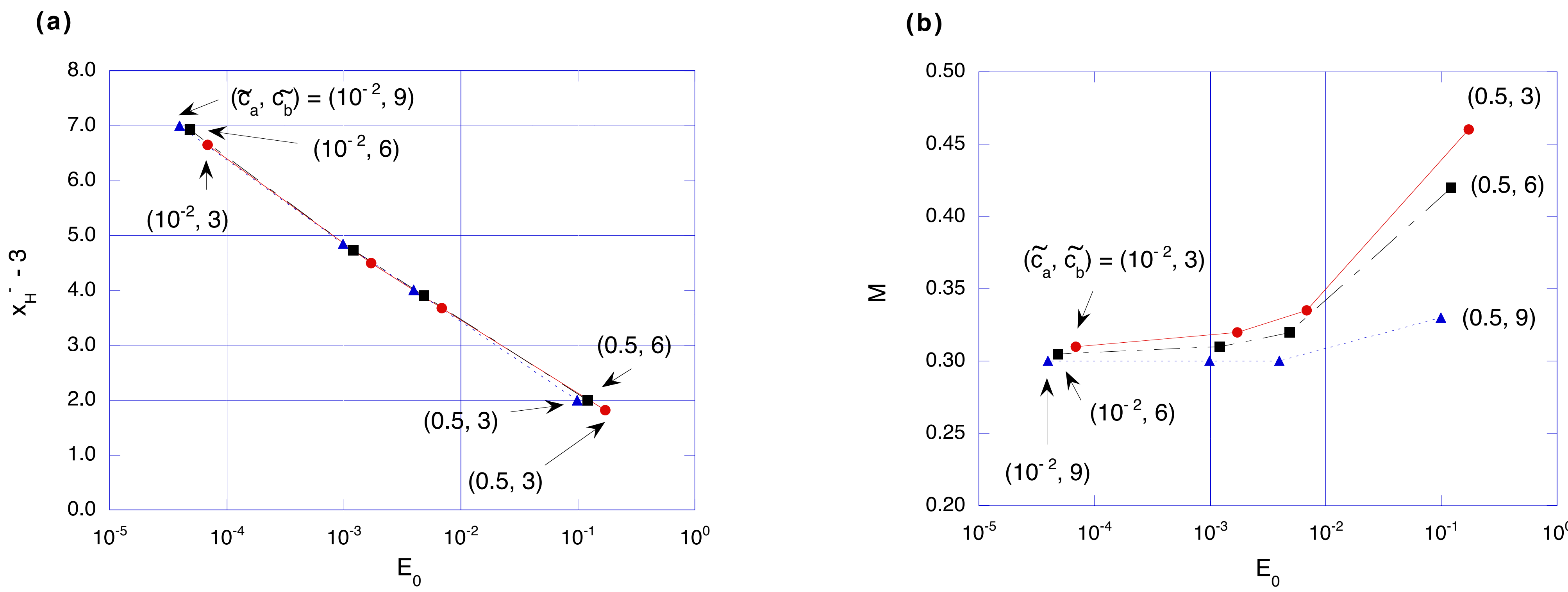


Figure 10: The same plots with Fig.**??** for the small conventional field pulses. (a) The trapping horizon ($\vartheta_+ = 0$) coordinate, $x_H^- - 3$ (since we fixed $\tilde{c}_c = 3$), versus initial energy of the perturbation, E_0. We plotted the results of the runs of $\tilde{c}_a = 0.5, \cdots, 10^{-2}$ with $\tilde{c}_b = 3, 6$, and 9. They lie close to one line. (b) The final black hole mass M for the same examples.

Travel through a Wormhole – with Maintenace Operations!

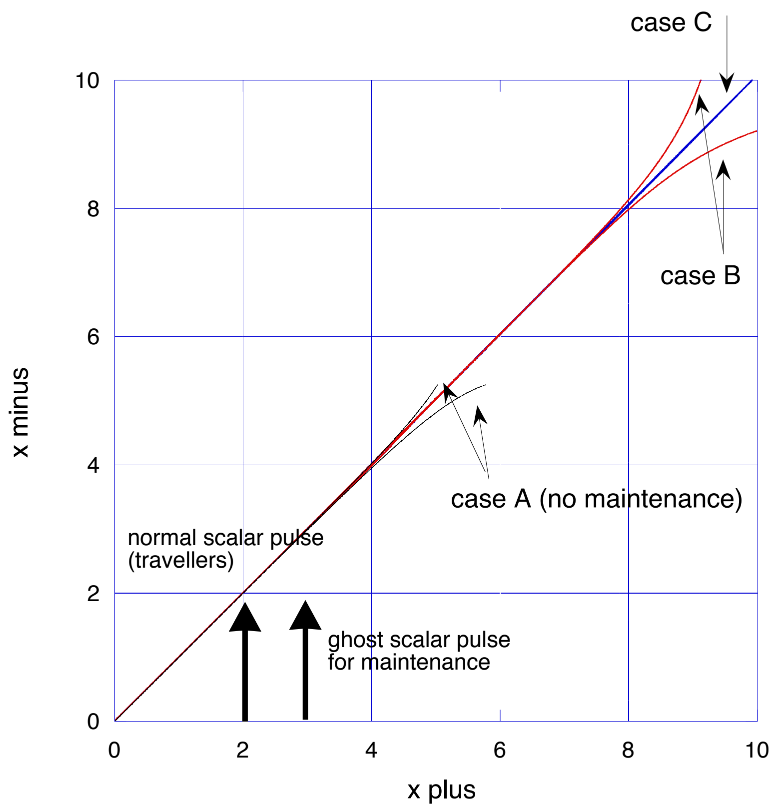


Figure 11: A trial of wormhole maintenance. After a normal scalar pulse, we signalled a ghost scalar pulse to extend the life of wormhole throat. The travellers pulse are commonly expressed with a normal scalar field pulse, $(\tilde{c}_a, \tilde{c}_b, \tilde{c}_c) = (+0.1, 6.0, 2.0)$. Horizon locations $\vartheta_+ = 0$ are plotted for three cases:
(A) no maintenance case (results in a black hole),
(B) with maintenance pulse of $(c_a, c_b, c_c) = (0.02390, 6.0, 3.0)$ (results in an inflationary expansion),
(C) with maintenance pulse of $(c_a, c_b, c_c) = (0.02385, 6.0, 3.0)$ (keep stationary structure upto the end of this range).

Discussion

Dynamics of the Ellis-Morris-Thorne traversible wormhole

$\Rightarrow$ WH is Unstable

(A) with positive energy pulse $\Rightarrow$ Black Hole

(B) with negative energy pulse $\Rightarrow$ Inflationary expansion

$\Rightarrow$ (A) confirms duality conjecture between BH and WH.

$\Rightarrow$ (B) provides a mechanism for enlarging a quantum wormhole to macroscopic size.

- **We answered to the question of : what happens if our hero (or heroine) attempts to traverse the wormhole.**
- **New discoveries of the critical behaviour.**

"Science can be stranger than science fiction."

NewScientist
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Quantum foot in the door

ALL around us are tiny doors that lead to the rest of the Universe. Predicted by Einstein's equations, these quantum wormholes offer a faster-than-light short cut to the rest of the cosmos—at least in principle. Now physicists believe they could open these doors wide enough to allow someone to travel through.

Quantum wormholes are thought to be much smaller than even protons and electrons, and until now no one has modelled what happens when something passes through one. So Sean Hayward at Ewha Womans University in Korea and Hisa-aki Shinkai at the Riken Institute of Physical and Chemical Research in Japan decided to do the sums.

They have found that any matter travelling through adds positive energy to the wormhole. That unexpectedly collapses it into a black hole, a supermassive region with a gravitational pull so strong not even light can escape.

But there's a way to stop any would-be traveller being crushed into oblivion. And it lies with a strange energy field nicknamed "ghost radiation". Predicted by quantum theory, ghost radiation is a negative energy field that dampens normal positive energy. Similar effects have been shown experimentally to exist.

Ghost radiation could therefore be used to offset the positive energy of the travelling matter, the researchers have found. Add just the right amount and it should be possible to prevent the wormhole collapsing—a lot more and the wormhole could be widened just enough for someone to pass through.

It would be a delicate operation, however. Add too much negative energy, the scientists discovered, and the wormhole will briefly explode into a new universe that expands at the speed of light, much as astrophysicists say ours did immediately after the big bang.

For now, such space travel remains in the realm of thought experiments. The CERN Large Hadron Collider in Switzerland is expected to generate one mini-black hole per second, a potential source of wormholes through which physicists could try to send quantum-sized particles. But sending a person would be another thing. To keep the wormhole open wide enough would take a negative field equivalent to the energy that would be liberated by converting the mass of Jupiter.

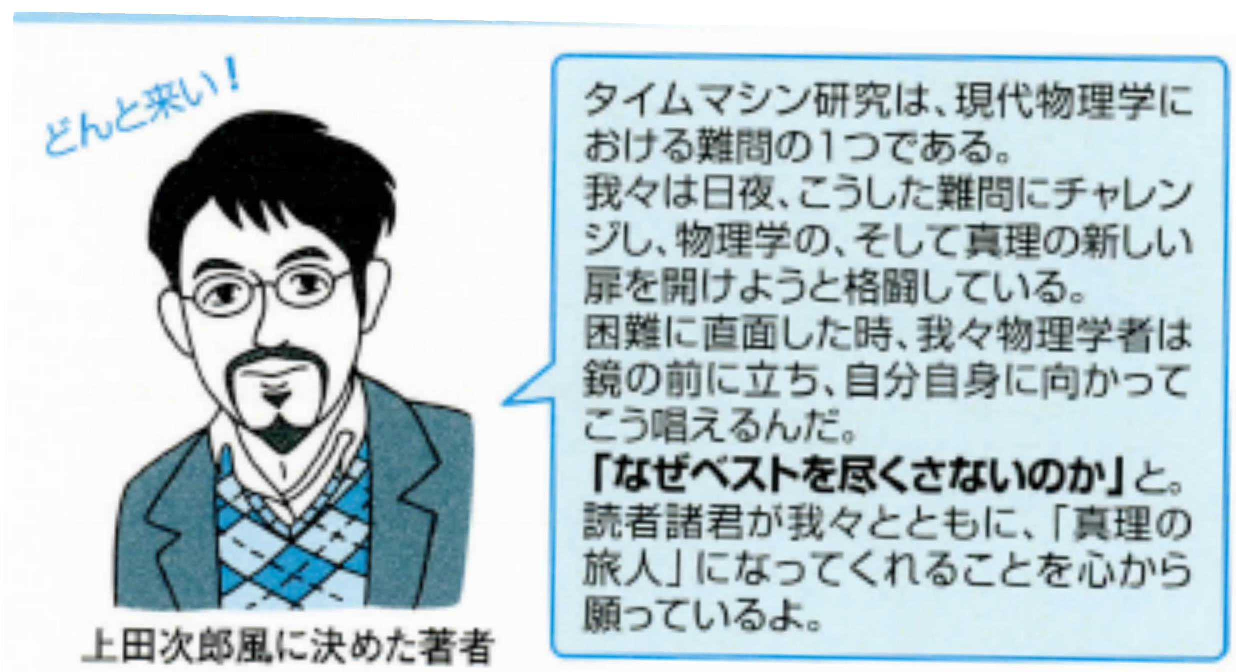
Charles Choi

More at: www.arxiv.org/abs/gr-qc/0205041

タイムマシンと時空の科学 (図解雑学) [単行本]

真貝 寿明 (著)

(1 カスタマーレビュー) いいね (4)



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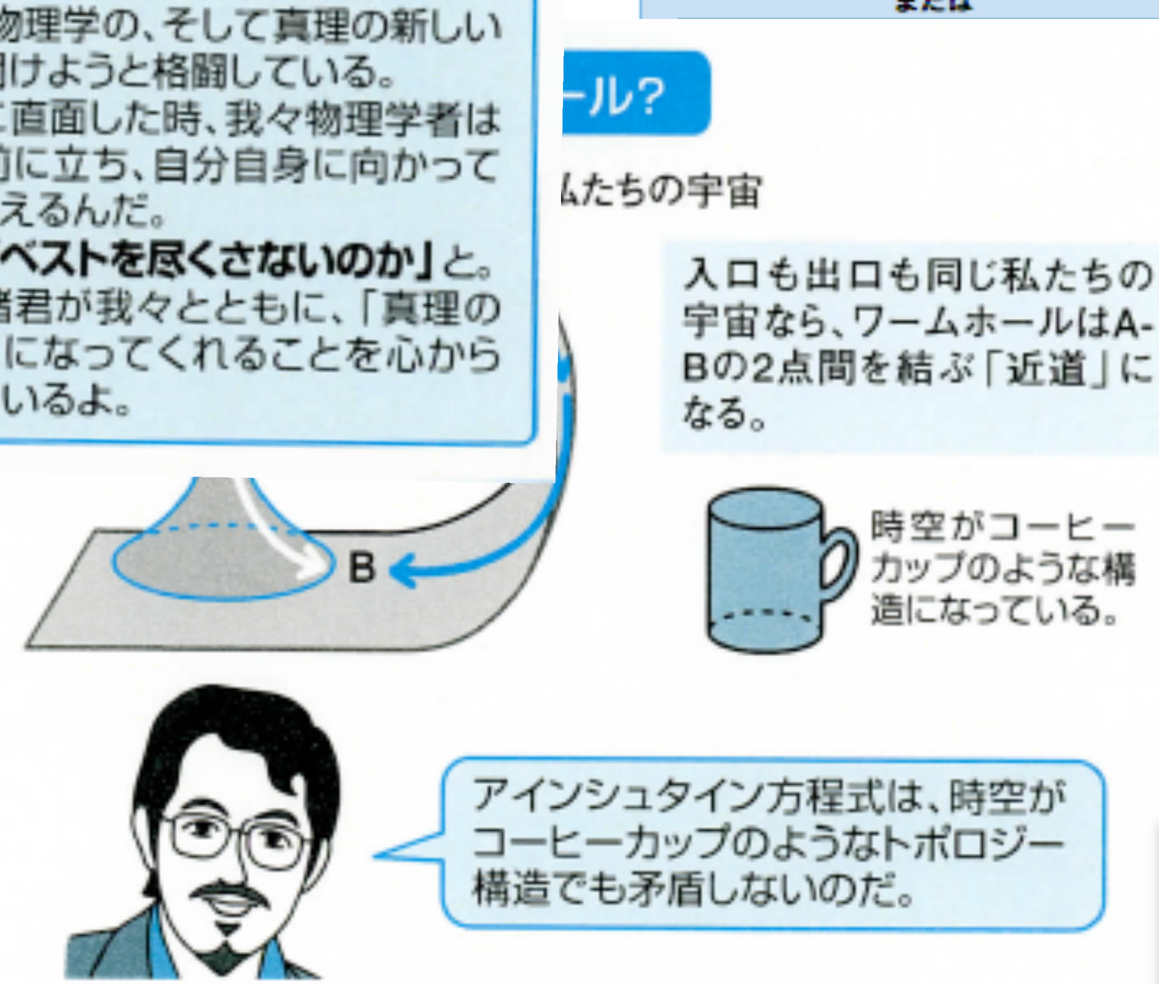
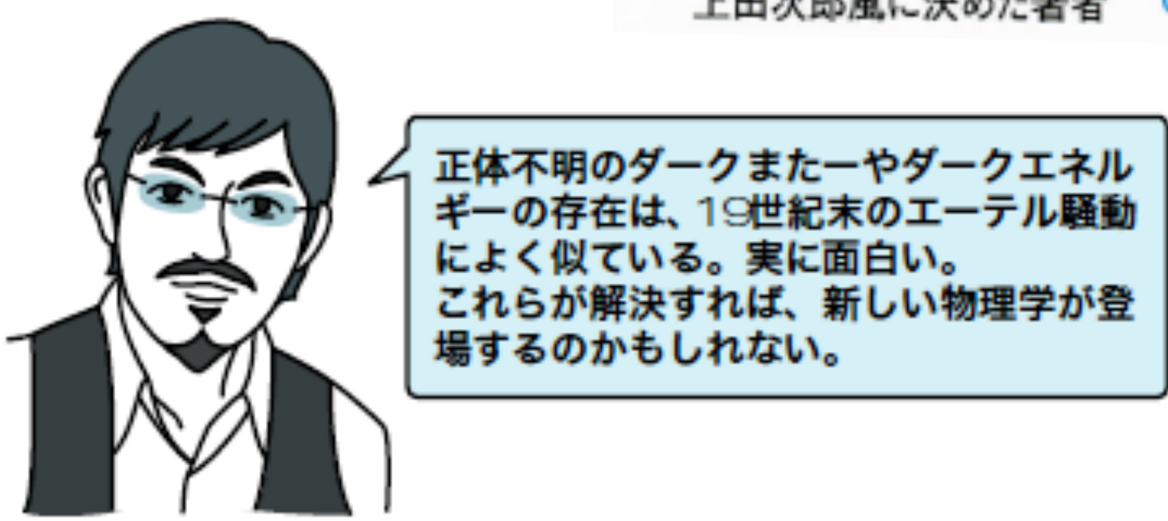
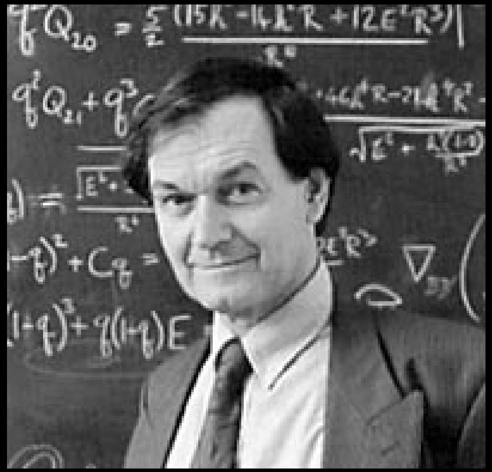




Some Unsolved Problem in Classical GR

R. Penrose,
in “Seminar on Differential Geometry”
(Princeton U Press, 1982)



1. Find a suitable quasi-local definition of energy-momentum in GR.
2. Find a suitable quasi-local definition of angular-momentum in GR.
3. Find an asymptotically simple Ricci-flat space-time which is not flat -- or at least prove that such space-times exist.
4. Are there restrictions on k for non-stationary k-asymptotically simple space-times, with non-zero mass, which are vacuum near $\mathcal{I}$ (Scri)?
5. Find conditions on the Ricci tensor R_{ab} throughout M which ensure that the generators of Scri are infinitely long.
6. Show that if a cut C of Scri-plus [or Scri-minus] can be spanned by a spacelike hypersurface along which an appropriate energy condition holds, then the Bondi-Sachs mass defined at C is non-negative.
7. Does the Bondi-Sachs mass defined on cuts of Scri-plus have a well-defined limit as the cuts recede into the past along Scri-plus, this limit agreeing with the mass defined at spacelike infinity?

R. Penrose,
in "Seminar on Differential Geometry"
(Princeton U Press, 1982)

8. Show that if the dominant energy condition holds, then the Bondi-Sachs energy-momentum, and also the energy-momentum defined at spacelike infinity, are future-timelike, the space-time being assumed not to be flat everywhere in the region of an appropriate spacelike hypersurface.
9. In an asymptotically simple space-time which is vacuum near $\mathcal{I}$ (Scri) and for which outgoing radiation is present and falls off suitably near i^0 and i^+, is it necessarily the case that i^0 and i^+ are non-trivially related? (At least, are there some examples i which i^0 and i^+ are non-trivially related?)
10. Find a good definition of angular momentum for asymptotically simple space-times.
11. If there is no incoming radiation and no outgoing radiation and the space-time M is vacuum near $\mathcal{I}$ and (in some suitable sense) near i^0, is M necessarily stationary near Scri?
12. Is Cosmic Censorship a valid principle in classical GR?
13. Let S be a spacelike hypersurface in M which is compact with boundary, the boundary consisting of a cut C of Scri-plus together with a trapped surface T. Let m be the Bondi-Sacks mass evaluated at C and let A be the area of T. Show that

$$A \leq 16\pi m^2$$

provided that the dominant energy condition holds throughout some neighbourhood of S.

14. Show that there is no vacuum equilibrium configuration involving more than one black hole.